

https://doi.org/10.1070/QEL16551

Theorem of comparative sensitivity of fibre sensors

M.I. Belovolov, V.M. Paramonov, M.M. Belovolov

Abstract. **We report an analysis of sensitivity of fibre sensors of physical quantities based on different types of interferometers. We formulate and prove the following theorem: under the time-dependent external physical perturbations at nonzero frequencies (i.e., except the static and low-frequency ones) on the sensitive arms of an interferometer in the form of multiturn elements (coils), there exist such lengths *L* of the measuring arms of the fibre interferometers at which the sensitivity of sensors based on the Sagnac fibre interferometers can be comparable with the sensitivity of sensors based on Michelson, Mach–Zehnder, or Fabry–Perot fibre interferometers, as well as exceed it under similar other conditions (similar-type perturbations, similar arm lengths and single-mode fibre types). The consequences that follow from the theorem, important for practical implementation of arrays of fibre sensors for measurement purposes and the devices with stable metrological properties, are discussed.**

Keywords: *fibre-optic sensor, fibre interferometer, Sagnac interferometer, Michelson interferometer, sensitivity, transform function.*

1. Introduction

The present paper is devoted to the issue of comparative sensitivity of fibre-optic sensors of different physical quantities, in which the sensitive elements are multiturn elements or coils of single-mode fibre, incorporated in the measuring arms of fibre interferometers. The most frequently used interferometers are the Fabry–Perot (Fig. 1a), Michelson (Fig. 1b) and Mach–Zehnder (Fig. 1c) interferometers [1–5]. During the last 30 years a large number of papers have appeared devoted to the distributed and 'point' fibre-optic sensors, based on the schemes of Sagnac fibre interferometers with multiturn sensitive elements or coils (see papers [6–14] and references therein). However, in spite of the development of fundamentals of fibre-optic sensor operation, it is still unclear which of the schemes can provide high sensitivity of the sensor to the required physical perturbation under the condition of acceptable metrological characteristics of the sensor or measuring device.

This general question can be hardly answered. It is common to think that it is necessary to develop high-sensitivity sensors using one of the fibre interferometer schemes (Fabry–Perot, Michelson, or Mach–Zehnder, Fig. 1) since this choice assures high sensitivity, while the Sagnac interferometer should serve mainly in gyroscopes. In a more particular form, the question of choosing the optical scheme for developing a fibre sensor sounds as follows: what length should the single-mode optical fibre have and what will the expected zero point stability, transform function linearity and possible dynamic range of the detected physical perturbations be? The theorem about the comparative sensitivity of fibre-optic interferometric sensors formulated and proved in the present paper should answer these questions with a certain degree of generality and

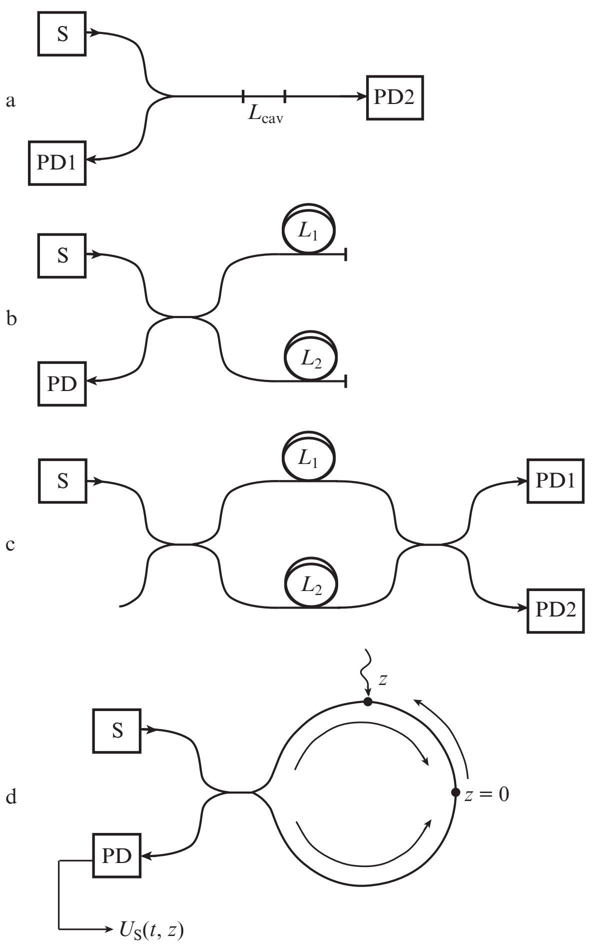

Figure 1. Most widespread schemes of fibre-optic sensors based on (a) Fabry–Perot, (b) Michelson, (c) Mach–Zehnder and (d) Sagnac interferometers; S is the radiation source, PD is the photodetector, L_{cav} is the length of the Fabry–Perot cavity.

M.I. Belovolov, V.M. Paramonov, M.M. Belovolov Fiber Optics Research Center, Russian Academy of Sciences, ul. Vavilova 38, 119333 Moscow, Russia; e-mail: bmi@fo.gpi.ru

Received 24 October 2017
Kvantovaya Elektronika **47** (12) 1128–1134 (2017)
Translated by V.L. Derbov

facilitate the construction for a fibre-optic sensor or a distributed measuring system based on interferometric principles. The illustrative materials and the numerical values of the parameters of the sensors, in particular, the fibre acoustic sensors (hydrophones) are presented here for better understanding of comparative sensitivity, estimation of its values and explanation of the theorem.

The comparative sensitivity theorem in the form of a definite statement of general character is formulated and proved with mathematical rigor. It is physically substantiated based on the known properties of the dependence of fibre interferometric sensor sensitivity on the fibre length L in the sensitive elements, as well as on the present experience of design and testing of operating samples of fibre sensors and measuring systems on their base.

2. Basic schemes of fibre-optic interferometric sensors and initial conditions

After the appearance of single-mode optical fibres, it was found that they can preserve coherence properties of transmitted radiation. This important property of single-mode fibres based on fused silica (SiO_2) stimulated extensive studies and design of the sensors of various physical quantities based on high-sensitivity fibre-optic interferometers. As early as in 1983 the review papers appeared summarising the experience of research and development of fibre-optic sound detectors and hydrophones [1, 2]. To relate numerically the sensitivity of different sensor schemes, we use the data on the sensitivity of fibre-optic sound detectors. Similar comparative sensitivity values can be used for other types of external physical perturbations. The analogy does not affect the generality of consideration and proof of the theorem, which is mainly related to the dependence of the output signal amplitude or the steepness of the transform function (Fig. 2) on the length L of the single-mode fibre in the sensitive multiturn element.

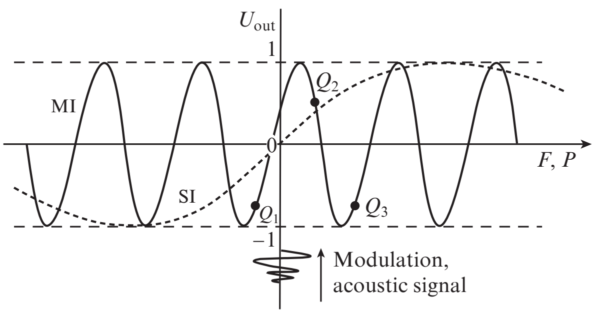

Figure 2. Qualitative form of the normalised transform functions U_{out} for the fibre-optic sensors based on the Michelson interferometer (MI) and Sagnac interferometer (SI) as a function of the external perturbation F or pressure P; Q_i is the position of the working point (to the substantiation of the choice of Sagnac fibre-optic interferometer).

It is known that in the most sensitive sensors of various physical quantities, use is made of single-mode optical fibres and interferometers based on them (Fabry–Perot, Michelson and Mach–Zehnder interferometers). These schemes are equivalent in physical principles of functioning and possess a sinusoidal function transforming the external perturbation into the electric output signal (Fig. 2) and the sensitivity of similar order of magnitude. The modulation of the interferometer phase $\Delta\varphi$ is converted into the output electric signal U_M (mV) by the detector photodiode in correspondence with the sinusoidal transform function, shown in Fig. 2 in the normalised form. It is also known that the phase ($\Delta\varphi$) and the output voltage (U_M) sensitivity of the sensors based on Michelson interferometers are proportional to the fibre length L in the measuring arm of the interferometer

$$U_M = aL, \tag{1}$$

where a is the proportionality factor that depends on the type of physical perturbation, outer coatings and geometry of fibre winding on the spool (the numerical values of a can be found in Table 2 from Ref. [3]). In the case of recording acoustic signals (sound pressure), the parameter $a \approx 2$ mV m^{-1}, and the noise level U_n of the recording instrumentation commonly amounts to 0.1–1 mV. For the amplitude of the output acoustic signal $U_M \approx 100$ mV and digital processing, one can detect signals with a signal-to-noise ratio no smaller than 1000 (60 dB).

Figure 3 shows the sensitivity dependences adopted from Refs [1, 2] for fibre-optic sensors of sound in water (hydrophones) on the sound frequency for different lengths L of the single-mode optical fibres without outer coatings in the measuring arm of the Mach–Zehnder interferometer. It is seen that already for the length L = 1 m of single-mode optical fibre in the measuring arm of the interferometer sensor, its sensitivity appears to be at the level of the best piezoelectric hydrophones and nearly corresponds to the audibility threshold of human ear ($\sim 2 \times 10^{-5}$ Pa). For the single-mode fibre lengths 1, 100 and 100 m, the sensitivity can be increased by one, two and three orders of magnitude, respectively. This

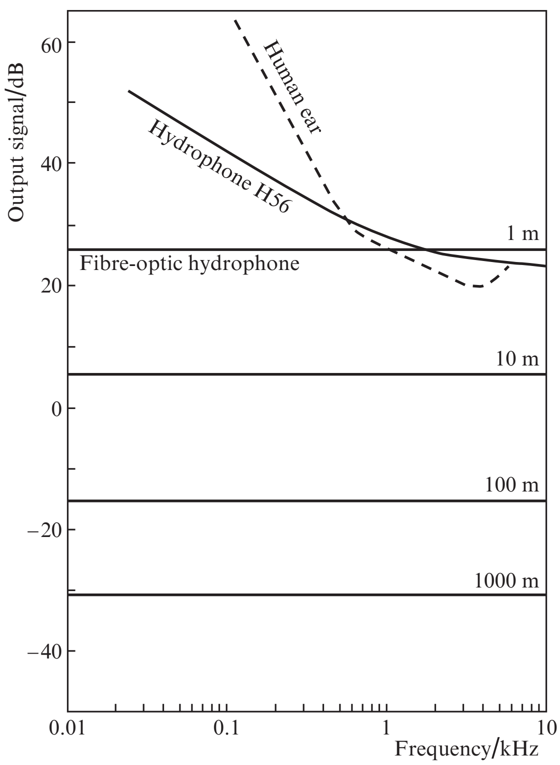

Figure 3. Sensitivity of fibre detectors of sound (hydrophones and piezoelectric hydrophones) compared to the sensitivity of human ear (adopted from [1, 2]) normalised to 1 μPa.

increase in the sensitivity and reduction of the minimal threshold of acoustic signal detection cannot be implemented practically because the level of technical noises or the sea noise is usually higher than the minimal detection threshold.

Using long optical fibres in sensitive elements leads, in particular, to an increase in noises due to the temperature fluctuations, vibrations and various instabilities that directly affect the position of the working point of an interferometric sensor, which becomes floating. Active stabilisation of the working point of the interferometric sensor is possible, but difficult in the case of many sensors in the measuring systems. Thus, the sensitivity of the sensors linearly increases with increasing length L of single-mode fibre in the measuring arm of the interferometer sensor in correspondence with formula (1). This fact determines the need to use multiturn elements made of single-mode fibres wound on spools with a diameter d. The diameter of the acoustical coil antenna limits the upper frequency f_u of the range of the detectable acoustic frequencies, which is determined by the 'in-phase condition': the antenna size d should not exceed half the wavelength of the detected acoustic waves λ_s:

$$d < \lambda_s/2. \quad (2)$$

Ideally, the condition should be $d \ll \lambda_s/2$ to provide more precise recording of the acoustic wave phases and to have a possibility to design phased acoustic antenna arrays for direction finding and determination of coordinates of the sound sources in space.

The coatings of the optical fibres and the environment surrounding the multiturn sensitive elements of the sensor can essentially affect both the sensitivity and the amplitude–frequency characteristic (AFC), as shown in Ref. [5] for the acoustic sound detectors in the air environment. In Fig. 3 the dependence of sensitivity on the sound frequency does not manifest itself among the dependences obtained for single-mode uncoated fused silica fibres and in this case serves to evaluate it by the order of magnitude.

The in-phase condition in form (2) for the acoustic receiving antenna with the diameter d will evidently cause the fall of sensitivity at high frequencies, starting from the frequency $f_u = V_s/(2d)$, when the antenna size becomes smaller than a half-period of the acoustic wave and the antiphase subtraction of the effects occurs under the simultaneous action of half-waves of hydrostatic compression and rarefaction. The estimates for the aqueous medium with $V_s = 1500$ m s^{-1} and $d = 8$ cm yields $f_u \approx 9.4$ kHz, and in the first-order approximation this estimate is independent of the fibre length L of the sensor sensitive element. The increase in the length of the optical fibre in the coil will lead to greater uncertainty in the position of the working point Q_i (see Fig. 2) when using optical schemes of Fabry–Perot, Michelson, or Mach–Zehnder interferometers. The arm lengths of the fibre Michelson and Mach–Zehnder interferometers can be equalised to the difference of lengths $\Delta L \leqslant 1$ mm. This will decrease the level of noise related to the fluctuations of the carrier frequency of the laser source radiation and draw the working point Q_i of the interferometric sensor closer to the zero position within one–two periods of the response amplitude variation. However, without special measures, there will be no automatic stabilisation of the working point at the zero position and it will remain unstable.

The analysis of response specific features in fibre sensors with multiturn elements and Michelson, Mach–Zehnder and Fabry–Perot interferometer schemes has shown that they are close to each other in functional principles and possess practically similar transform functions in the sinusoidal form (see Fig. 2) with the transform steepness similarly depending on L according to the linear law (1). The linear dependence of the sensitivity of the acoustic response amplitude of fibre hydrophones U_M on the fibre length L in the sensitive multiturn element (coil) is presented in Fig. 4. Let us use this dependence of U_M for mathematical and graphical proof of the comparative sensitivity theorem, which we formulate as the statement on the relationship of sensitivities of the fibre sensor using a Sagnac interferometer and sensors using Fabry–Perot, Michelson and Mach–Zehnder interferometers as functions of length L. Since in practice the fibre length L in the sensors can amount to 1–1000 m, they have the form of multiturn elements or small-size coils with the external diameter 30–100 mm.

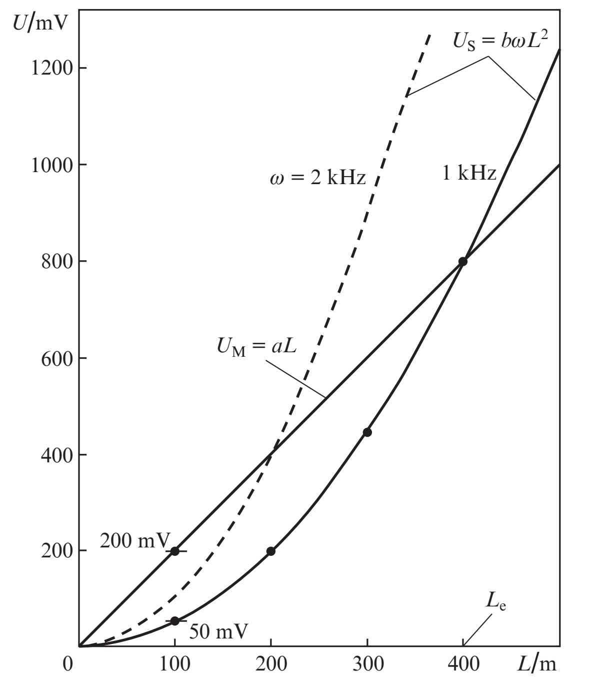

Figure 4. Dependences of the voltage sensitivity U of fibre acoustic sensors (hydrophones) based on Michelson interferometers (U_M) and Sagnac interferometers (U_S) on the length L of single-mode fibre in the sensitive arm (coil); L_e is the fibre length corresponding to equal sensitivities of the sensors (to the sensitivity theorem for fibre sensors).

3. Sensitivity and AFC of fibre sensors based on Sagnac interferometers

The sensors based on fibre-optic Sagnac interferometers are distinguished among all fibre interferometric sensors by the unique functional properties, related to the stable zero working point, the linearity of the function that transforms the external modulating perturbation into the output electric signal, and the wide dynamic range of admissible external berturbations (see Fig. 2). Multiple design and research works devoted to fibre sensors based on Sagnac interferometers started from Ref. [6], where the dependence of the response sensitivity $U_S(t, z)$ on the coordinate z of the external local physical perturbation was determined. The latter should be time-dependent (neither static, nor low frequency) and possess an additional property of quasi-stationarity: the time of variation of the external perturbation at the upper boundary of the registered acoustic frequency band $\tau = 1/f_u$ must be much greater than the time of the radiation propagation along the Sagnac interferometer contour with the length $2L$:

$$f_u \ll c/(2\pi nL), \tag{3}$$

where c is the speed of light in vacuum; n is the refractive index of the single-mode fibre core material; and L is the half-length of the fibre contour of the Sagnac interferometer.

Following Ref. [6], we present the derivations necessary to understand the physics of the Sagnac fibre interferometer operation as a distributed or local sensor of physical perturbations. If z is the coordinate of the acoustic impact on one of the interferometer arms, measured from the middle of the contour with the zero coordinate ($z = 0$, see Fig. 1d), then the phase difference $\Delta\varphi(t)$ of interfering signals at the output of the Sagnac interferometer will be proportionally transformed by the interferometer photodiode into the output electric signal

$$\Delta U_S(t,z) = \Delta\varphi(t) = \phi\left(t - \frac{L-z}{V_g}\right) - \phi\left(t - \frac{L+z}{V_g}\right), \tag{4}$$

where $V_g = c/n \approx 2 \times 10^8$ m s^{-1} is the the group velocity of the light wave propagation through the core of the single-mode fibre.

If the signals are observed and registered by both arms in the quasi-stationary regime (3) with the frequencies $f \leqslant f_u$, then the additions to the time argument t in formula (4) are small, i.e., $2z/V_g$ is small [7]. Expanding expression (4) into the Taylor series and neglecting the terms of higher orders of smallness, we arrive at the expression for the signal $U_S(t,z)$, proportional to the phase change $\Delta\varphi(t)$ in the Sagnac interferometer

$$\Delta U_S(t,z) = \left[\phi(t) - \frac{L-z}{V_g}\frac{d\phi}{dt}\right] - \left[\phi(t) - \frac{L+z}{V_g}\frac{d\phi}{dt}\right]. \tag{5}$$

This formula in the form

$$U_S(t,z) \approx \Delta\phi_S(t,z) = \frac{2z}{V_g}\frac{d\phi}{dt} \tag{6}$$

is the key one when the fibre interferometer is used as a distributed fibre-optic sensor of physical quantities.

It is seen that the signal of the Sagnac interferometer contains the coordinate z. To determine it one can use, e.g., the signal $U_M(t) \sim \varphi(t)$ of the Michelson interferometer, also included in this contour (see [6, 8]). The method of simultaneous implementation of two fibre interferometers in one closed contour for the determination of coordinates of physical perturbations is referred to as the method of superposed interferometers [5]. Other methods and algorithms for determination of the external perturbation coordinates using a combination of Sagnac and Michelson interferometers in a system of perimeter protection or defence against unapproved perturbations can be found in Refs [6, 7, 11–13].

In the present case, we are interested in the regime of registration when the external perturbation affects the entire Sagnac contour in the form of a multiturn element or coil. To understand the physics of detecting such perturbations, including the detection of acoustic waves by means of fibre hydrophones as quasilocal sensors, it is reasonable to present the Sagnac contour as two halves divided by the central point with the coordinate $z = 0$ (Fig. 5). The fibre Sagnac interferometers in the form of a compact coil with a directed X-coupler are often used as separate sensors of acoustic signals (microphones or hydrophones). A half of the closed contour should be isolated from the sound or made essentially asymmetric in sensitivity [5]. The question about the amplitude–frequency characteristic of such sensor naturally arises. To determine the AFC of a half-contour of the Sagnac fibre interferometer, let us assume that it is affected by the acoustic signal $A_{ac} = A_0\sin(\omega t + \varphi)$ with the acoustic wavelength $\lambda_{ac} \gg 2d$, exceeding the diameter d of the spool carrying the fibre. Integrating the response over the half of the Sagnac interferometer contour from 0 to L, we obtain the general form of the fibre sensor response

$$U_S(t) \propto (L^2\omega/V_g)A_0\cos(\omega t + \varphi). \tag{7}$$

It is seen that the response amplitude is proportional to the square of the fibre length L^2 in the contour and linearly grows with increasing sound frequency ω. Therefore, high acoustic frequencies are better detected by the fibre Sagnac interferometer as a sensor, while low frequencies, low-frequency vibrations and drifts are not detected.

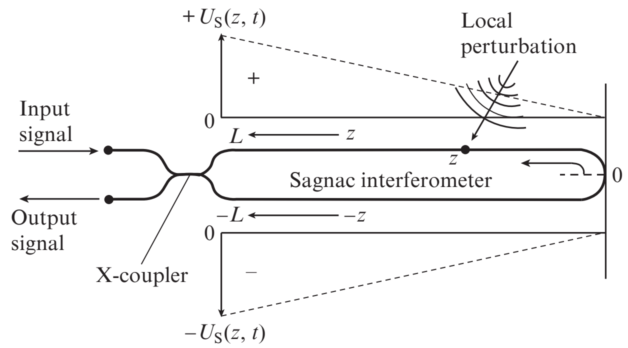

Figure 5. Fibre Sagnac interferometer as a distributed coordinate-sensitive sensor of physical quantities with a linear dependence of the response amplitude $U_S(z,t)$ on the coordinate z [to the derivation of Eqn (8) for the dependence of the integral response $U_S(\omega, L)$].

The insensitivity to low-frequency signals and drifts is known to be favourable for the operation of fibre-optic gyroscopes. From the schematic diagram in Fig. 5 many technical aspects are clear concerning the operation of both fibre gyroscopes and distributed and local fibre sensors. In gyroscopes under the external low-frequency impact on both halves of the fibre contour it is important to choose the point with $z = 0$, fold two fibres of the contour together and wind them parallel to each other with maximal accuracy. Under the external perturbation, any asymmetry of winding will produce uncompensated difference signal of interference and noise. The Sagnac contour becomes a sensor with maximal sensitivity, if a half of it, e.g., the one with conditionally negative response amplitudes (Fig. 5), is physically isolated from external perturbations. The maximal sensitivity will be inherent in the segments of the fibre contour, located closer to the X-coupler of the fibre circuit. Near the middle of the contour with $z = 0$, the most insensitive and 'deaf' segments of the fibre line are located. Since these parts of the fibre line or the sensor coil weakly contribute to the integral sensitivity of the fibre sensor, they should be better used as lead-in fibre channels, not caring much about the exact determination of the point $z = 0$. In the process of the coil winding, these layers of the fibre should be better located at the hydrophone spool from below.

To confirm formula (7), the amplitude part of which represents the AFC of the fibre hydrophone with a Sagnac interferometer, we manufactured and studied a mock-up, for which the schematic and the general appearance are shown in Fig. 6. The hydrophone based on a Sagnac interferometer with a sensitivity-asymmetric closed contour was fabricated using a single-mode optical fibre with the length $2L = 100$ m. A half of the fibre was wound on a separate spool and acoustically isolated from the external environment inside the sensor case. The second half is a sensitive part of the hydrophone. The 50/50 directional X-coupler, according to the scheme in Fig. 6a, is placed in the isolated container box.

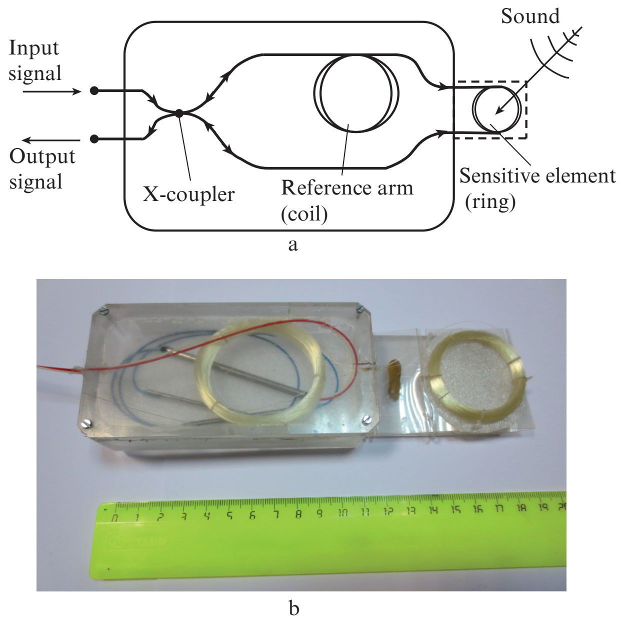

Figure 6. (a) Optical scheme and (b) general appearance of the fibre hydrophone sensor based on the Sagnac interferometer with a sensitivity-asymmetric closed contour having the length $2L$=100 m.

Figure 7 presents the shape of the AFC of the fibre hydrophone based on the Sagnac interferometer obtained point by point in the range of the acoustic frequencies from 10 Hz to 15 kHz. Confirming formula (7), the AFC begins from the zero frequencies and linearly grows with ω up to the values determined by the spool diameter $d = 8$ cm in correspondence with in-phase condition (2). A considerable decrease in the acoustic response amplitude begins from the frequency $f_u \approx$ 9.4 kHz. This experiment confirms the linear dependence of the Sagnac interferometer hydrophone response on the sound frequency ω given by Eqn (7).

According to Eqn (7), we obtain the following expression for the amplitude of the maximal integral response U_S from the entire multiturn sensor element based on the Sagnac interferometer:

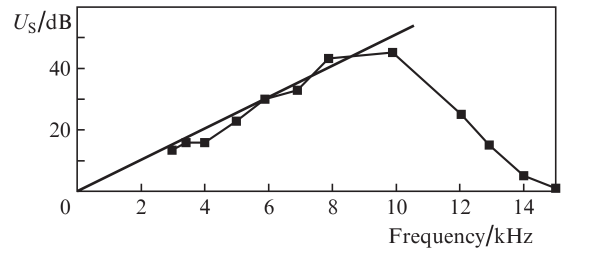

Figure 7. Example of the AFC plotted point by point for the fibre Sagnac interferometer hydrophone with the coil of single-mode CS980 fibre 50 m long having a spool diameter $d = 8$ cm as a sensitive element. The upper boundary of the frequency band of sensitivity is $f_u = V_s/(2d) \approx 9.4$ kHz.

$$U_S = b\omega L^2, \qquad (8)$$

where the parameter b is approximately equal to 5×10^{-6} mV Hz^{-1} m^{-2} for fibre hydrophones made of single-mode fibre. The quadratic dependence of the voltage sensitivity U_S of the Sagnac fibre interferometer hydrophone on L is shown in Fig. 4 for two values of the sound frequency ω (1 and 2 kHz).

4. Formulation and proof of the comparative sensitivity theorem for fibre interferometric sensors

The above analysis of specific features and operation principles of fibre interferometric sensors, in which multiturn elements and coils of single-mode optical fibre play the role of sensitive elements, allow the formulation of the comparative sensitivity theorem for the sensors based on Sagnac interferometers and classical Michelson interferometers, as well as on Mach–Zehnder and Fabry–Perot interferometers.

4.1. Theorem

Under time-dependent external physical perturbations with nonzero frequencies ω (except the static and low-frequency ones) on the sensitive arms of the interferometers in the form of multiturn coil elements, the length of the fibre interferometer measuring arms L exists, for which the sensitivity of the sensors based on the fibre Sagnac interferometers can be comparable with that of the sensors based on the Michelson, Mach–Zehnder and Fabry–Perot interferometers or exceed it, provided that the rest conditions are similar (single-type perturbations, similar arm lengths and same single-mode fibre type). The mathematical content of the comparative sensitivity theorem is the existence of such a length of the optical fibre L, at which the validity of the inequality $U_S \geqslant U_M$ is achieved at the positive parameters a, b, and ω in formulas (1) and (8).

4.2. Proof of the theorem

The proof of the sensitivity theorem for fibre sensors is the following. According to the established dependences of the sensitivity U of fibre-optic sensors with Michelson and Sagnac interferometers presented by Eqns (1) and (8), independent of the numerical values of the parameters a and b ($a > 0$, $b > 0$)

for the nonzero frequencies ω of the external physical perturbations, there exists a length L_e of the fibre forming the sensitive multiturn coil element, for which the sensitivity of the sensors based on a Sagnac interferometer is comparable with or equal to the sensitivity of the sensor based on Michelson (Mach–Zehnder, Fabry–Perot) interferometers $L_e = a/(b\omega)$. For $L > L_e$ the sensitivity of a fibre-optic sensor with a Sagnac interferometer can exceed that of the sensor based on a Michelson interferometer. The theorem is proved with mathematical rigor and the generality of assumptions, determined by the form of formulas (1) and (8) that express the dependence of the sensitivity on the length of single-mode fibre in the coil sensors. The physical conditions for the existence of the above dependences in form (1) and (8) are the sufficiently large wavelengths of the physical impacts compared to the size of the coil antennas (the in-phase condition) and the quasi-stationary condition (the expansion in the small phase modulation parameter).

For $L > L_e \approx 400$ m the sensitivity of the Sagnac interferometer sensors exceeds that of the sensors with Michelson (Mach–Zehnder) interferometers for the sound frequency $\omega = 1$kHz. For the tone signals with $\omega = 2$ kHz the amplitude of the response signal of the sensor with the Sagnac interferometer will exceed the response amplitude of the sensor with the Michelson interferometer for the length 200 m of fibre in the sensitive coil element. The practical sense of the proved sensitivity theorem consists in the fact that in fibre hydrophones the length L_e of single-mode fibre, required for the coil winding, is not enormously large (a few hundred meters). When the optical fibre and the cables based on them are used in distributed ground-based systems of security and monitoring, the fibre contour lengths of Sagnac interferometers can be as large as 100–200 km. The time of the optical signal propagation along the contour amounts to ~1 ms. This fact means that the frequency spectrum of the external physical perturbations or sound is limited by the upper frequencies ~1 kHz. Such interferometers are suitable for the systems protecting against the unapproved access or for monitoring the vibration of machines and mechanisms in natural conditions.

A drawback of fibre sensors with Sagnac interferometers is the large length of single-mode fibre, required to achieve higher sensitivity of hydrophones as compared to the hydrophones with Michelson and other interferometers. This is actually seen from the performed analysis and the comparison of the sensitivities of the Sagnac interferometer-based sensor and other types of sensors. For the length $L \approx 500$ m of single-mode fibre in the sensor coil, one can expect an increase in sensitivity of hydrophones by more than two orders of magnitude as compared to the sensitivity of human ear or the best piezoelectric hydrophone (see Fig. 2). This sensitivity may appear too high, and the use of such sensors will be difficult, since the technical noises may exceed the sensitivity thresholds. For practical use, it appears more important to implement the stability of the working point of the sensor at the transformation zero, as well as the linearity of the transform function and wider dynamic range of admissible external perturbations. Therefore, it is not reasonable to wind a very long fibre (20–50 m) onto the spool of the hydrophone sensor. The sensor should be a part of the closed contour of the Sagnac interferometer with the enlarged length, e.g., 500 m or 1 km. According to the scheme in Fig. 5, the sensitive element of the hydrophone must be placed closer to the directional X-coupler, where the response amplitude is maximal. A less sensitive part of the closed contour of the Sagnac interferometer can be placed in an isolating container to avoid the registration of noise signals. As a result, the sensitivity of the hydrophone becomes somewhat smaller, but it can approach the sensitivity of a piezoelectric hydrophone in the absolute value, and this will be enough for simultaneous conservation of the working point stability at zero, the linearity of the transform function and the increased dynamic range of the recorded external perturbations.

Figure 8 presents the results of experiments on the registration of acoustic signals by means of fibre-optic hydrophones with a 20-m-long single-mode fibre in the sensor coil and a 1-km-long Sagnac interferometer contour. The experimentally found threshold levels of detecting the acoustic signals at the frequency $\omega = 8$ kHz amounted to $\sim 10^{-5}$ Pa. For the Sagnac interferometer contour 40 m in length and the sensitive multiturn element of the hydrophone 20 m in length, the threshold values of the sound registration amounted to nearly 10^{-2} Pa, which is suitable for some technical applications of hydrophones, but not for detecting the sound sources at large distances. The obtained data on the high sensitivity of hydrophones with small lengths of single-mode sensitive fibre ($L \approx 20$ m) in the schemes with Sagnac interferometers indirectly confirm the sharper L^2 dependence (8) of the sensitivity on the fibre length, as compared to the linear growth of sensitivity (1) in the case of Michelson interferometers and similar ones.

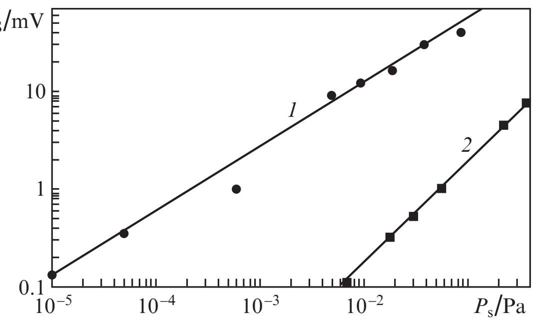

Figure 8. Experimental dependences of the sensitivity U_S of the fibre hydrophone using (*1*) 20 and (*2*) 40 m of fibre in the coil at the end of the Sagnac interferometer contour 1 km long on the sound intensity P_s at the frequency 8 kHz. The noise level in the system is 0.01–0.1 mV, the sensitivity threshold is $\sim 10^{-5}$ Pa. The sensitivity threshold of the hydrophone based on the Sagnac interferometer with the full contour length 40 m and the length of the fibre in the sensitive element 20 m amounts to $\sim 10^{-2}$ Pa.

5. Consequiences from the theorem and discussion of results

The mathematical rigor of the theorem proof follows from the functional dependence of the signal amplitudes in form (1) and (8) for any positive values of the parameters a, b and ω.

One can see from Fig. 4 that for small fibre lengths of the sensor coils ($L = 10$–100 m) due to the quadratic dependence of the response amplitude on the fibre length in the Sagnac interferometers the response sensitivity is a quantity of the second order of smallness, as compared to the sensitivity of the sensors based on Michelson interferometers. However, with the growth of the fibre length the response amplitudes in the hydrophones based on Sagnac interferometers can quickly become equal in absolute value to the response amplitudes of the sensors based on Michelson interferometers, or even exceed them. In the sensor based on a Sagnac interferometer, the working point position at the zero of the transform function (see Fig. 2) is conserved, as well as the linearity and the wide dynamic range of admissible external perturbations. We demonstrate the possibility of designing measuring instruments for external perturbations with metrologically calibrated scale and stable position of zero. As to detecting such sign-changing physical effects as sound, one can choose the similar orientation of the coil loops with respect to the hydrophone case and match the polarisations to implement inphase detection of the acoustic signals using two and more practically similar hydrophone sensors, which will operate with the responses synchronously, like gyroscopes of similar construction.

The main physical consequences of the theorem are:

– The high absolute sensitivities of fibre sensors due to the large lengths of the single-mode fibres in the Sagnac interferometer contours $L \sim 100-1000$ m.

– The stable position of the working point at zero of the transform function for the sensors based on Sagnac interferometers.

– The linearity of the function transforming the modulating perturbation into the output signal for the sensor based on the Sagnac interferometer.

– The large range of admissible external physical perturbations on the fibre Sagnac sensor.

– The possibility to arrange pair and arrays of similar fibre-optic sensors with synchronous (phased) response. In the case of detecting tone and harmonic signals, it is possible to determine the direction towards the source of sound or other physical perturbation from the external space.

– The possibility to make measuring instruments with calibration with respect to the types of physical perturbations (if the problem of matching the polarisation of the signals in the fibre circuit of the sensor is solved and the effect of polarisation fading is eliminated) due to the stability of the zero working point and the linear output characteristic.

Note that for small fibre length L the fibre sensor based on the Sagnac interferometer will yield to the sensors based on Michelson and other types of interferometers. The sensitivity advantage of the sensors based on Sagnac interferometers appears for the fibre length $L \geqslant 400$ m (as shown in Fig. 4 for fibre hydrophones). High threshold sensitivities ($\sim 10^{-5}$ Pa for the sound) can be achieved using sensitive elements with small ($L \sim 20-100$ m) fibre length, if they are included into the Sagnac contours with the total length 500 m – 1 km, and the connection to the optical circuit is near the X-coupler (see the schematic in Fig 5 and the example in Fig. 8).

The possibilities of increasing the sensitivity of the fibre sensors based on Sagnac interferometers with the conservation of the working point zero stability and the response linearity, revealed and confirmed in the present paper, are consequences of the proved theorem of comparative sensitivity of fibre interferometric sensors. They offer new possibilities in the design and construction of detecting sensors and measuring instruments for different physical quantities with definite metrological characteristics, which will be calibrated.

References

1. Lyamshev L.M., Smirnov Yu.Yu. *Sov. Phys. Acoust.*, **29**, 169 (1983) [*Akust. Zh.*, **29** (3), 289 (1983)].
2. Butusov M.M., Tarasyuk Yu.F., Urvantsev N.L. *Zarubezhn. Elektron.*, (5), 38 (1983).
3. Kul'chin Yu.N. *Raspredelennye volokonno-opticheskiye datchiki i izmeritel'nye seti* (Distributed Fibre-Optic Sensors and Measuring Networks) (Vladivostok: Dal'nauka, 1999)].
4. Kirkendall C.K., Dandridge A. *J. Phys. D: Appl. Phys.*, **37**, R197 (2004).
5. Belovolov M.I., Paramonov V.M., Turtaev S.N. *Foton-Ekspress*, (6), 27 (2014).
6. Udd E. *Proc. SPIE*, **425**, 90 (1983).
7. Dakin J.P., Pearce D.A.J., Strong A.S., Wade C.A. *Proc. SPIE*, **838**, 325 (1987).
8. Chtcherbakov A.A., Swart P.L. *Proc. SPIE*, **2838**, 301 (1996).
9. Vakoc B.J., Digonnet M.J.F., Kino G.S. *Proc. SPIE*, **3860**, 276 (1999).
10. Fang X. *J. Lightwave Technol.*, **14** (10), 2250 (1996).
11. Russell S.J., Brady K.R.C., Dakin J.P. *Proc. 14-th Intern. Conf. Optical Fiber Sensors* (Vinice, Italy, 2000) paper FR 2-3.
12. Kondrat M., Szustakowski M., Palka N. *Proc. 18-th Intern. Conf. Optical Fiber Sensors* (Cuncun, Mexico, 2006) ThE5.
13. Kondrat M., Szustakowski M., Palka N., Ciurapinski W., Zyczkowski M. *Opto-Electron. Rev.*, **15** (3), 127 (2007).
14. Belovolov M.I., Belov V.S., Zykov-Myzin K.A., Orlov A.P., Gladyshev A.V., Gorskii M.A., Dianov E.M. *Proc. SPIE*, **5855**, 948 (2005).