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Estimation of the signal gain in a linear-cavity ytterbium laser

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Abstract. **Based on the exponential representation of signal amplification along an active fibre, we construct an analytical approximation of the solution to a system of balance equations describing the dynamics of an average signal power and pump inside a linear cavity. The output power of the signal at the ends of the linear cavity is estimated. The output power is optimised for a linear-cavity ytterbium fibre laser.**

Keywords: *fibre laser, signal amplification, linear cavity.*

1. Introduction

Currently, there exist many types of fibre lasers, and their further development increasingly requires the use of numerical methods aimed at multiparametric optimisation of characteristics [1–3]. This optimisation requires very expensive computations, which, in spite of the active development of numerical modelling, from a practical point of view cannot be performed for a wide range of parameters even with the use of distributed computer systems [3].

An alternative to numerical modelling of laser systems is an analytical description of the evolution of the light in laser cavities of various types [4, 5]. In this paper, an analytical approximation is constructed for the output power of an optical signal in order to find the optimal regimes for the generation of a linear-cavity laser.

In this paper, we consider a continuous wave (cw) laser with a Fabry–Perot cavity [6]. Usually, in solving the problem of finding stable generation regimes in a fibre laser, various methods for solving the nonlinear Schrödinger equation are used in conjunction with a two-level model of signal amplification inside the cavity. The latter is a boundary value problem consisting of four first-order nonlinear differential equations,

$$\frac{\mathrm{d}S^{\pm}(z)}{\mathrm{d}z} = \pm\left[\alpha_{\mathrm{s}}\left(\frac{\mu P(z) + S(z)}{1 + P(z) + S(z)} - 1\right) - q_{\mathrm{s}}\right]S^{\pm}(z), \quad (1)$$

$$\frac{\mathrm{d}P^{\pm}(z)}{\mathrm{d}z} = \pm\left[\frac{\alpha_{\mathrm{p}}}{\mu}\left(\frac{\mu P(z) + S(z)}{1 + P(z) + S(z)} - \mu\right) - q_{\mathrm{p}}\right]P^{\pm}(z), \quad (2)$$

with the boundary conditions at the edges of the computational domain

$$P_0^+ = P^+(0), \quad P_L^- = P^-(L),$$
$$S_0^+ = S^+(0), \quad S_L^- = S^-(L), \quad (3)$$

where $S = S^+ + S^-$; $P = P^+ + P^-$; $P = P_{\mathrm{p}}/P_{\mathrm{p}}^{\mathrm{sat}}$; and $S = P_{\mathrm{s}}/P_{\mathrm{s}}^{\mathrm{sat}}$ are the pump and signal powers, respectively; the signs '+' and '–' indicate the light propagation direction; $P_{\mathrm{s}}^{\mathrm{sat}}$ and $P_{\mathrm{p}}^{\mathrm{sat}}$ are the signal and pump saturation powers; and L is the length of the active fibre.

The system of equations (1) and (2) is an alternative writing for the widely used equations given in [2]. Formulas (1) and (2) take into account in explicit form the dependences of the populations of the energy levels on the pump and signal powers to illustrate the development of saturation processes of signal amplification and pump depletion. Despite the fact that this system does not explicitly contain a number of parameters observed in experiments, it is more convenient for developing theoretical approaches. A detailed description of the relationship between theoretical coefficients and coefficients obtained in experimental studies is given in [6].

The signal wavelength at the output is $\lambda_{\mathrm{s}} = 1083$ nm, the pump wavelength is $\lambda_{\mathrm{p}} = 910$ nm, and the coefficients of the signal (α_{s}) and pump (α_{p}) absorption by the fibre cross section are chosen to be 0.062 and 0.115 m^{-1}, respectively [6]. The signal and pump saturation powers $P_{\mathrm{s}}^{\mathrm{sat}}$ and $P_{\mathrm{p}}^{\mathrm{sat}}$ are 0.055 and 4.9 W, respectively, and therefore, the dimensionless parameter is

$$\mu = \frac{\lambda_{\mathrm{s}}}{\lambda_{\mathrm{p}}}\frac{\alpha_{\mathrm{s}}}{\alpha_{\mathrm{p}}}\frac{P_{\mathrm{s}}^{\mathrm{sat}}}{p_{\mathrm{p}}^{\mathrm{sat}}} = 7\times10^{-3}.$$

Despite the fact that the presence of unsaturated losses q_{s} and q_{p} considerably complicates the system of equations in question, they significantly affect the result and, therefore, it is impossible to neglect these quantities. The values of unsaturated losses are chosen as follows: $q_{\mathrm{p}} = 0.8$ dB m^{-1} [7] and $q_{\mathrm{s}} = 0.25$ dB m^{-1} [8, 9].

For the presented boundary value problem, it is expedient to apply a theoretical analysis of the dynamics of the laser light and energy balance in an active medium with the aim of developing approximate analytical solutions that allow one to abandon laborious calculations.

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2. Formulation of the boundary value problem for a linear configuration

The system of equations (1) and (2) makes it possible to obtain the following integrable relations:

$$\frac{\mathrm{d}S^+(z)}{S^+(z)} = -\frac{\mathrm{d}S^-(z)}{S^-(z)}, \quad \frac{\mathrm{d}P^+(z)}{P^+(z)} = -\frac{\mathrm{d}P^-(z)}{P^-(z)}, \tag{4}$$

$$\frac{\mathrm{d}S^+}{S^+} - \zeta\frac{\mathrm{d}P^+}{P^+} = \phi\mathrm{d}z, \tag{5}$$

where

$$\phi = \alpha_\mathrm{s}(\mu - 1) + \zeta q_\mathrm{p} - q_\mathrm{s}; \ \zeta = \mu\frac{\alpha_\mathrm{s}}{\alpha_\mathrm{p}}; \text{ and } \mu = \frac{\lambda_\mathrm{s}}{\lambda_\mathrm{p}}\frac{\alpha_\mathrm{s}}{\alpha_\mathrm{p}}\frac{P_\mathrm{p}^\mathrm{sat}}{p_\mathrm{c}^\mathrm{sat}}.$$

We denote the distribution of the amplification of the signal power along the fibre through the function

$$G(z) = \frac{S^+(z)}{S^+(0)} = \frac{S^-(L)}{S^-(z)}.$$

Then the total gain of the signal per round trip along the active fibre will be determined by the expression $G_\mathrm{s} = G(L)$. We define the general form of the solution:

$$S^+(z) = S_0^+G(z), \ S^-(z) = S_L^-\frac{G_\mathrm{s}}{G(z)}, \tag{6}$$

$$P^+(z) = P_0^+G^{\zeta^{-1}}(z)\exp\left(-\frac{\phi}{\zeta}z\right),$$
$$P^-(z) = P_L^-\frac{G_s^{\zeta^{-1}}\exp\left(-\frac{\phi}{\zeta}L\right)}{G^{\zeta^{-1}}(z)\exp\left(-\frac{\phi}{\zeta}z\right)}. \tag{7}$$

From all that has been said above, the boundary conditions follow

$$G(0) = 1, \ G(L) = G_\mathrm{s}.$$

With allowance for all the notations introduced and the integral relations obtained, it is possible to reduce problem (1), (2), consisting of four nonlinear differential equations, to a boundary value problem of the form

$$\frac{\mathrm{d}G(z)}{\mathrm{d}z} = \left[\alpha_\mathrm{s}\left(\frac{\mu P(z) + S(z)}{1 + P(z) + S(z)} - 1\right) - q_\mathrm{s}\right]G(z), \tag{8}$$

$$G(0) = 1, \ G(L) = G_\mathrm{s},$$

where

$$S(z) = S_0^+G(z) + S_L^-\frac{G_\mathrm{s}}{G(z)};$$

$$P(z) = P_0^+G^{\zeta^{-1}}(z)\exp\left(-\frac{\phi}{\zeta}z\right) + P_L^-\frac{G_\mathrm{s}^{\zeta^{-1}}\exp\left(-\frac{\phi}{\zeta}L\right)}{G^{\zeta^{-1}}(z)\exp\left(-\frac{\phi}{\zeta}z\right)}.$$

Above we investigated the boundary-value problem (1), (2) with the boundary conditions (3). However, in practice, the laser output power is unknown. In other words, the values of $S^+(0)$ and $S^-(L)$ are unknown. On the other hand, it is possible to initially determine the signal gain G_s per round trip along the active fibre, which, generally speaking, is different for different configurations of the cavities.

We denote the total intracavity losses by Σ; this value describes the total power losses of the signal in the device: thermal losses on various structural elements, optical losses inside the fibre, losses on the output devices (WDM couplers, lenses and optical gratings), etc. Note that a necessary condition for a stable generation regime in an optical cavity, regardless of its type, is compensation for the loss by amplification.

In the active medium of a linear cavity, the signal propagates in both directions; therefore, for a configuration of this type, according to Fig. 1, the following relations hold: $S^+(0) = S^+(0)G_\mathrm{s}^2\Sigma = S^+(0)G_\mathrm{s}^2R^+R^-$ and $S^-(L) = S^-(L)G_\mathrm{s}^2\Sigma = S^-(L) \times G_\mathrm{s}^2R^-R^+$. Consequently,

$$G_\mathrm{s} = \frac{1}{\sqrt{\Sigma}} = \frac{1}{\sqrt{R^-R^+}}. \tag{9}$$

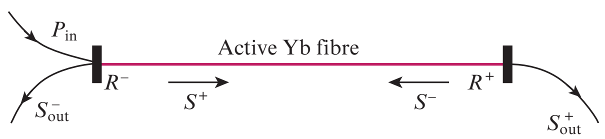

Figure 1. Scheme of a linear optical cavity.

In what follows we need the write (8) in the integral form:

$$\ln(G(z)) + (\alpha_\mathrm{s} + q_\mathrm{s})z + \zeta\Big[P_0^+\Big(G_\mathrm{p}(z) - 1 + q_p\int_0^z G_\mathrm{p}(z)\mathrm{d}z\Big)$$
$$+ P_L^-G_\mathrm{p}(L)\Big(1 - \frac{1}{G_\mathrm{p}(z)} + q_\mathrm{p}\int_0^z\frac{1}{G_\mathrm{p}(z)}\mathrm{d}z\Big)\Big]$$
$$+ S^+(0)\Big(G(z) - 1 + q_\mathrm{s}\int_0^z G(z)\mathrm{d}z\Big)$$
$$+ S^-(L)G_\mathrm{s}\Big(1 - \frac{1}{G(z)} + q_\mathrm{s}\int_0^z\frac{1}{G(z)}\mathrm{d}z\Big) = 0, \tag{10}$$

where $G_\mathrm{p}(z) = G^{\zeta^-}(z)\exp(-\phi z/\zeta)$.

Next, we define the boundary conditions for the linear configuration in Fig. 1. Using (9), the boundary conditions for the linear cavity are written in the form:

$$P_0^+ = P^+(0), \quad P_L^- = P^-(L),$$
$$S^+(0)G_\mathrm{s}R^+ = S^-(L), \quad S^-(L)G_\mathrm{s}R^- = S^+(0). \tag{11}$$

From equation (10) and boundary conditions (11) we obtain the expression

$$S^+(0) = -\Big\{(1 + G_\mathrm{s}R^+)\Big[G_\mathrm{s} - 1) + q_\mathrm{s}\int_0^L G(z)\mathrm{d}z\Big]\Big\}^{-1}$$
$$\times\Big\{\ln G_\mathrm{s} + (\alpha_\mathrm{s} + q_\mathrm{s})L + \zeta(P_0^+ + P_L^-)$$
$$\times\Big[G_\mathrm{s}^{\zeta^{-1}}\exp\Big(-\frac{\phi}{\zeta}L\Big) - 1 + q_\mathrm{p}\int_0^L G^{\zeta^{-1}}(z)\exp\Big(-\frac{\phi}{\zeta}z\Big)\mathrm{d}z\Big]\Big\}. \tag{12}$$

3. Construction of the analytic approximation

In Section 2, we describe two statements of the boundary value problem for equation (8); on their basis we construct numerical methods that make it possible to find solutions to these problems in an acceptable time. However, it is difficult to perform optimisation over many parameters using iterative methods with a large number of degrees of freedom that this model has. A good solution to this problem is the use of any suitable analytic approximation of the unknown function $G(z)$ described by Eqn (8). In this paper, an exponential approximation is chosen, and this choice is explained by an attempt to develop an analytical method that allows one to optimise the parameters of the model, significantly reducing the computation time, while maintaining acceptable accuracy.

Assume that the function $G(z)$ is exponential. Taking into account the conditions from equation (8), we obtain $G(z) = \exp(z \ln G_s/L)$. Such a choice of the functional dependence makes it possible to obtain analytically simpler expressions for the integrals in Eqn (12), eliminating the need for numerical integration. Substituting this approximation into (12), we obtain an expression for the signal power in the cavity:

$$S^+(0) = -\left[\left(1 + \frac{q_s L}{\ln G_s}\right)(G_s - 1)(1 + G_s R^+)\right]^{-1}$$

$$\times\left\{\ln G_s + \zeta(P_0^+ + P_L^-)\left[G_s^{\zeta-1}\exp\left(-\frac{\phi}{\zeta}L\right) - 1\right]\right.$$

$$\left.\times\left(1 + \frac{\zeta q_p L}{\ln G_s - \phi L}\right) + (\alpha_s + q_s)L\right\}. \quad (13)$$

Finally, this analytic approximation allows us to quickly approximate the solution to problem (8) for the conditions corresponding to different parameters of laser cavities.

To answer the question of the accuracy of the constructed analytic signal power approximation, we consider (12) in the absence of unsaturated losses, i.e., for $q_s = q_p = 0$. In this case, system (8) has an analytic solution [10, 11]:

$$S^+(0) =$$

$$-\frac{\ln G_s + \zeta(P_0^+ + P_L^-)\left[G_s^{\zeta-1}\exp\left(-\frac{\phi}{\zeta}L\right) - 1\right] + (\alpha_s + q_s)L}{(1 + G_s R^+)(G_s - 1)}.$$

This solution coincides exactly with the constructed approximation (13) for zero linear losses ($q_s = q_p = 0$).

Below we compare the analytical expression obtained with the results of direct numerical simulation of the initial system. We define the relative error of the signal power by the expression

$$\varepsilon_s = \frac{|S_{\rm num} - S_{\rm anal}|}{S_{\rm num}},$$

where $S_{\rm num} = S^+(0) + S^-(L)$ and $S_{\rm anal} = S^+(0) + S^-(L)$ are the sums of the signals at the boundaries, calculated using a numerical algorithm and an analytical approximation, respectively. Figure 2 shows the dependences characterising the accuracy of the approximation obtained using the example of a linear cavity.

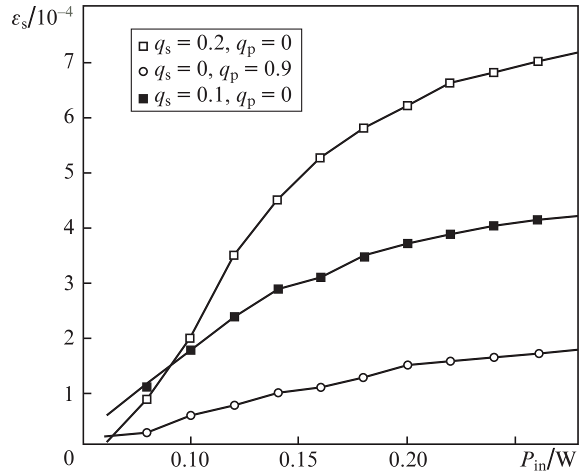

Figure 2. Uncertainties of the analytical approximation for a linear cavity as functions of the input pump power for various values of unsaturated losses and $L = 2.5$ m.

One can see that the magnitude of the uncertainty depends on the unsaturated loss, with q_s more significantly affecting the uncertainty than q_p. This fact is explained by the difference in the values of the analytical expressions for the integrals in front of q_s and q_p, which introduce th uncertainty. Comparison of the obtained analytical expression with direct numerical simulation of the initial system showed the correspondence of the results to an accuracy of 0.1%, which indicates the possibility of effective application of analytical results in solving optimisation problems.

4. Optimisation of a cw laser with a Fabry–Perot cavity

The goal of optimisation is to maximise the output power of the signal. Therefore, we solve the boundary value problem (8) with boundary conditions of form (11) corresponding to the linear configuration. A pump device with a power of up to 150 mW is used in the simulation [10]. The output power of the light for the resonator schematically shown in Fig. 1 is determined by expressions

$$S_{\rm out}^- = P_s^{\rm sat} S^+(0) G_s^2 R^+(1 - R^-),$$

$$S_{\rm out}^+ = P_s^{\rm sat} S^+(0) G_s(1 - R^+).$$

One can see from Fig. 3a that at a low pump power, stable lasing is possible only at small losses. The maximum output power obtained is also small and in this case is 6.02 mW at the right end of the cavity. For the simulated pump device, the position of the optimum point is also quite obvious: it corresponds to the minimum total losses and the maximum pump power in the cavity and is in the upper right corner of the lasing region in Fig. 3a.

During the second stage of optimisation, use is made of the pump device from [6], with a maximum power being equal to 23 W. One can see from Fig. 3b that an increase in the pump power allows not only an increase in the output power of the signal, but also a significantly expansion of the generation area, which makes it possible to achieve stable lasing at

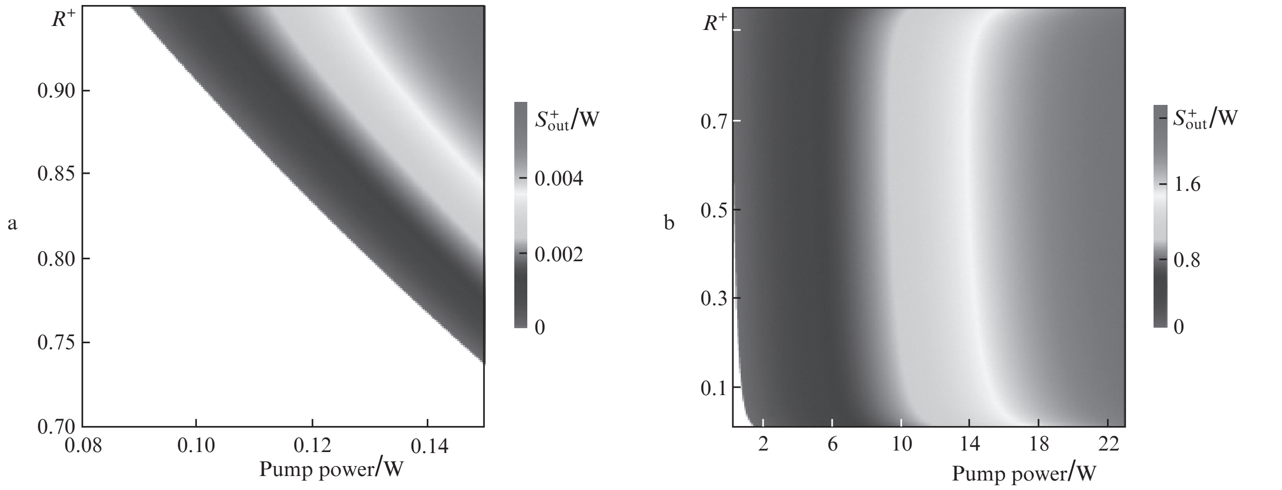

Figure 3. Dependences of the output power S^+_{out} of the linear cavity at its right end on the input pump power $P_{in} = P_p^{sat} P_0^+$ and the reflection coefficient R^+ of the light inside the cavity at the right end for $R^- = 0.99$ and $L = 2.5$ m.

larger losses than previously. In addition, the maximum output power of the signal increases to 2.55 W. The optimum output power is achieved at a maximum pump power and the reflection coefficient $R^+ = 0.7$ at the right end.

Figure 4 shows the dependence of the output power of the signal as a function of the length of the active ytterbium fibre segment in a linear cavity. With an optimum reflection coefficient $R^+ = 0.7$ and a pump power of 23 W, the maximum output power is reached, which is twice as large as the analogous experimental value from [6].

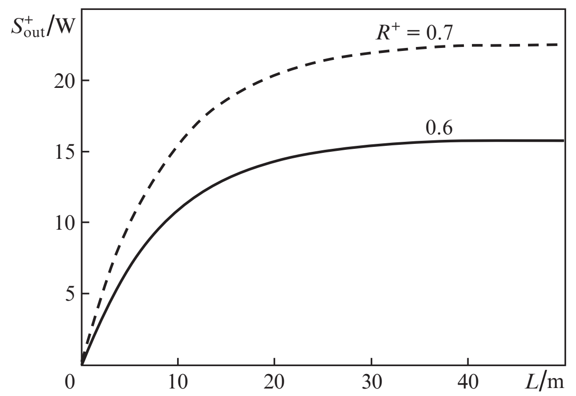

Figure 4. Dependences of the output power S^+_{out} of the linear cavity at its right end on the length of the ytterbium active fibre L for two values of R^+ at $R^- = 0.99$ and $P_{in} = 23$ W.

5. Conclusions

We have constructed an analytical approximation of the output signal power. The accuracy of this approximation is estimated and its applicability in determining the lasing region and optimal laser light parameters in cavities of different configurations is shown. On its basis, a region of stable lasing for a linear laser configuration is determined and the losses at the cavity coupler, pump radiation power and active fibre length are optimised in order to achieve the maximum output signal power.

The obtained analytical results can be successfully used to optimise laser systems and also construct various numerical algorithms for solving complex problems on finding the dynamics of the average power of the pump and the signal in the active medium inside the laser cavity.

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