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Laser autodyne registration of nanodisplacements under laser wavelength modulation

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Abstract. **The formation of an autodyne interference signal under the current modulation of the laser radiation wavelength is studied. For the first time we propose a method for determining the stationary phase of the autodyne signal based on the analysis of the low-frequency spectrum of the laser autodyne system. The results of measuring nanodisplacements of the electromagnetic translator by the ratio of the autodyne signal spectral components under harmonic current modulation of the laser autodyne wavelength are presented.**

Keywords: *laser interferometry, autodyne, semiconductor laser, laser radiation modulation, distance measurement, microvibration, spectral analysis of signals.*

1. Introduction

Modern systems for ranging and displacement measurements are based on recording the phase of reflected laser radiation [1–4], on applying the methods of laser interferometry with variation of the laser diode radiation wavelength [5–7], and on using the gauge dependences of the controlled physical quantity on the distance to the probed surface [8–11]. However, these systems suffer from a number of drawbacks, caused by the necessity to know the gauge dependence of the measured quantity, by the insufficient resolution of laser rangers, and by the bulkiness of measuring systems.

In this connection, it is interesting to measure distances using semiconductor laser autodynes, which are compact and possess low weight [12, 13]. By the term ‘autodynes’, we understand open oscillatory systems capable of responding to external impact [14]. In Russian and foreign literature they also use the terms “delayed feedback laser” [15], “self-mixing laser” [16], etc.

In order to measure nanodisplacements with such autodyne interference laser systems, use is made of the alternating component of the recorded signal due to the excitation of reflector vibrations with the amplitude, exceeding a half of the laser radiation wavelength [17]. Instead, we propose to use the harmonic current modulation of the laser radiation wavelength for obtaining the alternating recorded signal [18, 19]. As an example, we consider the possible application of such an autodyne meter for controlling the displacements of the probe of the scanning near-field microwave microscope aimed to determine its measurement accuracy.

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2. Autodyne detection in semiconductor lasers

In the USSR, I.L. Bershtein, a distinguished researcher in the field of self-excited oscillator theory [20], developed the autodyne theory. For the first time he formulated and solved in a rather correct way the problem how the eigensignal reflected from a moving target affects a self-excited oscillatory system [21]. The papers by N.G. Basov et al. [22], R.F. Kazarinov, R.A. Suris, A.A. Tager [23, 24], and E.M. Gershenzon et al. [25, 26] significantly contributed to the theory of autodyne detection in semiconductor lasers.

In Ref. [23] the function, describing the external perturbation by the electromagnetic wave coupled to the cavity of a semiconductor laser, is introduced into the system of equations for the electromagnetic field amplitude and the balance of injected electrons in the active layer of a semiconductor laser. In Ref. [22] this system of equations together with the equation for the susceptibility was used to calculate the amplitude and frequency of the generated oscillations under the action of the reflected electromagnetic wave and to describe the effect of the phase incursion of the wave returned into the cavity on the shape of the autodyne signal. The modification of the rate equations proposed in Refs [27, 28], in which the contribution of spontaneous emission into the generated modes is taken into account, appeared applicable to the description of processes near the oscillation threshold, which is important for autodyne systems, since near the oscillation threshold the autodyne gain has a maximal value [12, 28].

Because the system of rate equations is not suitable for the description of coherent autodyne reception that requires the analysis of phase relations between the direct wave and the reflected one, R. Lang and K. Kobayashi [29] proposed a system of differential equations for the amplitude and the phase of the electromagnetic field, as well as the concentration of charge carriers. The equation for the field amplitude comprises the term describing the external optical feedback. Using the Lang–Kobayashi equations, the authors of Refs [15, 30, 31] performed further studies of the oscillation regimes in a diode laser with an external mirror. Since the analysis of the laser radiation dynamics was carried out simultaneously with the analysis of operation of microwave autodynes, the term “laser with external feedback” has come into use alongside with the term “autodyne” [32–34].

In the interference measurements, the level of external feedback is an important parameter of an autodyne system. To exclude the mode hopping and to reduce the laser diode frequency shift due to the change in the charge carrier concentration in the active region, it was proposed to reduce the level of feedback [34–36]. Obviously, this level will also affect the shape of the autodyne signal formed in the process of modulating the laser radiation wavelength. However, the influence of the external optical feedback on the shape and the spectrum of the autodyne signal can be neglected, if the feedback level $C < 0.1$, since in this case the autodyne signal $P(t)$ will coincide in shape with the interference signal in the system decoupled from the radiation source [37, 38].

3. Formation of an autodyne signal under laser radiation wavelength modulation

In the case of modulating the laser radiation wavelength in an autodyne system, the expression for the interference signal will differ from the analogous expression in systems, decoupled from the radiation source. In conventional Michelson and Mach–Zehnder interferometers, it is possible to equalise the intensities in two arms. For an external-feedback semiconductor laser the equation for the laser autodyne radiation power follows from the Lang–Kobayashi composite cavity model [29] and can be presented in the form

$$P(j(t)) = P_1(j(t)) + P_2\cos[\omega(j(t))\tau_0(t)], \qquad (1)$$

where $P_1(j(t))$ is the power component independent of the distance to the external reflector; P_2 is the amplitude of the power component depending on the phase incursion of the wave $\omega(j(t))\tau_0(t)$ in the system with external reflector; τ_0 is the time of laser radiation propagation through the distance L to the external reflector; and $\omega(j(t))$ is the semiconductor laser radiation frequency depending on the pump current density $j(t)$ and the feedback level.

When the radiation wavelength of the semiconductor laser is modulated, its frequency and power amplitude are determined by the relations

$$\omega(j(t)) = \omega_0 + \omega_A\sin(2\pi\nu t), \quad P_1(j(t)) = I_1\sin(2\pi\nu t),$$

where ω_0 is the eigenfrequency of the semiconductor laser radiation; ω_A is the frequency deviation of the semiconductor laser diode; ν is the modulation frequency of the pump current of the laser diode; and I_1 is the current modulation amplitude of the component $P_1(j(t))$.

Thus, the expression for the radiation power of the frequency-modulated semiconductor laser (1) will have the form

$$P(j(t)) = I_1\sin(2\pi\nu t) + P_2\cos[\omega_0\tau_0 + \omega_A\tau_0\sin(2\pi\nu t)]. \qquad (2)$$

To describe the low-frequency spectrum of the autodyne signal under harmonic modulation of the wavelength of the laser diode radiation, the power of this signal can be presented according to [19] in the form of a series expansion in Bessel functions of the first kind J_n:

$$P(t) = I_1\sin(\Omega t) + P_2(\cos\theta)J_0(\sigma)$$
$$+ 2P_2(\cos\theta)\sum_{n=1}^{\infty}J_{2n}(\sigma)\cos(2n\Omega t)$$
$$- 2P_2(\sin\theta)\sum_{n=1}^{\infty}J_{2n-1}(\sigma)\sin[(2n-1)\Omega t], \qquad (3)$$

where $\theta = \omega_0\tau_0$ is the stationary phase of the autodyne signal; $\sigma = \omega_A\tau_0$; and $\Omega = 2\pi\nu$ is the circular frequency of the pump current modulation of the laser diode.

The expressions for the amplitudes S_n of the spectral components of the series expansion in Bessel functions have the form

$$S_1 = I_1 - (\sin\theta)P_2J_1(\sigma), \qquad (4)$$

$$S_{2n} = 2(\cos\theta)P_2J_{2n}(\sigma), \qquad (5)$$

$$S_{2n+1} = 2(\sin\theta)P_2J_{2n+1}(\sigma), \qquad (6)$$

$$S_{2n+2} = 2(\cos\theta)P_2J_{2n+2}(\sigma), \qquad (7)$$

$$S_{2n+3} = 2(\sin\theta)P_2J_{2n+3}(\sigma). \qquad (8)$$

Equations (4)–(8) characterise the relation between the spectral components of the frequency-modulated autodyne signal and the Bessel functions of the first kind.

To determine the nanodisplacements of the reflector, let us use the following relations:

$$S_{2n}/S_{2n+2} = J_{2n}(\sigma)/J_{2n+2}(\sigma), \qquad (9)$$

$$S_{2n+1}/S_{2n+3} = J_{2n+1}(\sigma)/J_{2n+3}(\sigma). \qquad (10)$$

The solution to Eqns (9) and (10) with respect to the unknown parameter σ allows writing the expression for the stationary phase of the autodyne signal in the form

$$\theta = \arctan\left[\frac{S_{2n+1}J_{2n}(\sigma)}{S_{2n}J_{2n+1}(\sigma)}\right]. \qquad (11)$$

Keeping in mind that $\theta = \omega_0\tau$, and the time of passing the length of the reflector displacement by the radiation is $\tau = \Delta L/c$, we arrive at the expression for the reflector nanodisplacement ΔL

$$\Delta L = \theta\frac{c}{\omega_0}. \qquad (12)$$

Thus, in order to determine the reflector nanodisplacements under current modulation of the laser radiation wavelength from the amplitudes of the Fourier transform of the autodyne signal spectrum S_{2n}, S_{2n+2}, S_{2n+1}, and S_{2n+3}, it is necessary to calculate the value of the parameter σ using Eqns (9), (10) and to find the stationary phase of the autodyne signal θ using Eqn (11). Then from Eqn (12), with the periodicity of the function $\arctan(x)$ taken into account, we determine the value of the reflector displacement.

4. Modelling the autodyne signal under reflector nanodisplacements

The computer simulation of the autodyne signal under nanodisplacements of the reflector was performed with the following parameters: the laser radiation wavelength $\lambda = 650$ nm, the separation between the laser and the reflecting surface $L =$ 10 cm, the wavelength modulation amplitude $\Delta\lambda = 0.002$ nm, the modulation frequency of the laser diode pump current $\nu = 100$ Hz, the step of reflector displacement $\delta L = 20$ nm.

Figures 1 and 2 present the results of computer modelling of the shape and spectrum of autodyne signals under current

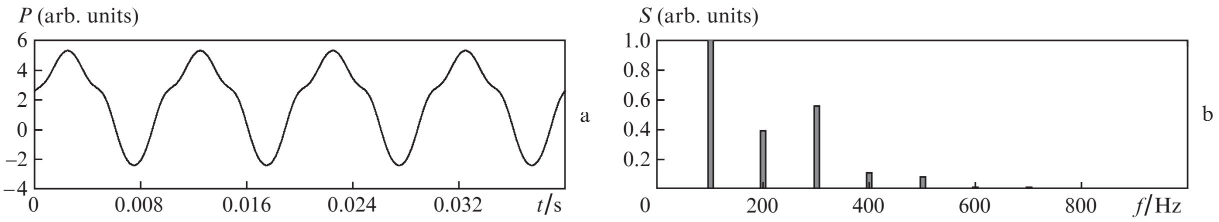

Figure 1. (a) Autodyne signal and (b) its spectrum for the initial position of the reflector ($\Delta L = 0$).

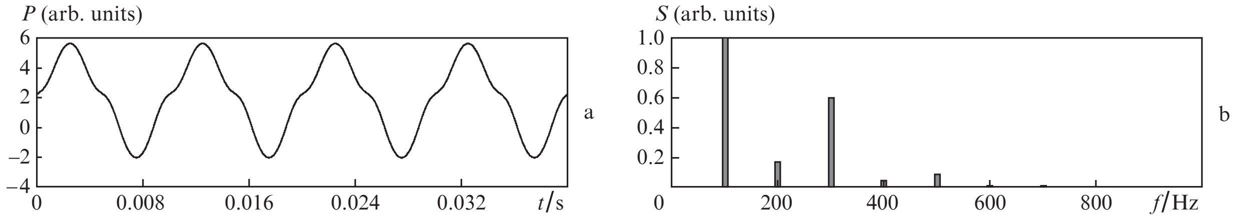

Figure 2. (a) Autodyne signal and (b) its spectrum for the displacement of the reflector by $\Delta L = 20$ nm from its initial position.

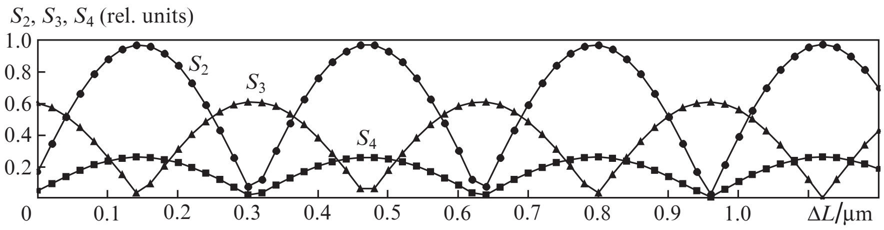

Figure 3. Variation in the amplitude of the second (S_2), third (S_3), and fourth (S_4) spectral harmonics of the autodyne signal due to the displacement of the reflector over the distance 1200 nm with the step $\delta L = 20$ nm.

modulation of the laser radiation wavelength and the reflector positions, corresponding to $\Delta L = 0$ (Fig. 1) and $\Delta L = 20$ nm (Fig. 2). One can see the dependence of the signal shape and spectrum on the reflector position. The change in the autodyne signal spectrum is expressed more clearly as compared to the change of its shape.

For the solution of the inverse problem using Eqn (11) for the autodyne signal stationary phase, the computer modelling of spectral components (9) and (10) was performed for the reflector displacement by 1200 nm. The dependences of even and odd amplitudes of the autodyne signal Fourier spectrum on the reflector displacement (with the step $\delta L = 20$ nm over the distance 1200 nm) are presented in Fig. 3 by the example of the second, third and fourth amplitudes. In Fig. 3, one can observe a periodic variation in the amplitudes of the autodyne signal spectral components, caused by the properties of the standing wave in the external cavity of the autodyne system. Within $\Delta L = 0-160$ nm the autodyne signal amplitude is a monotonic function of the reflector displacement, which corresponds to the shift of the laser radiation wavelength by $\lambda/4$.

5. Experimentally observed autodyne responses of the semiconductor laser system to the reflector motion

Figure 4 presents a block diagram of the experimental setup. The radiation from a frequency-modulated semiconductor laser autodyne (*1*) [laser diode RLD-650(5) based on quantum-dimensional structures with diffraction-limited single spatial mode] having the wavelength 650 nm, fixed at a holder (*3*) of a near-field microwave microscope, was incident on the surface of a reflector (*4*), driven by a translator (*6*). Via an amplifier (*7*), the photodetector signal arrived at an ADC (*8*). The digital signal from the ADC was saved in the memory of a PC (*9*) for further processing. The laser diode was pumped using a stabilised current source (*2*).

The radiation wavelength was modulated at the frequency $\nu = 100$ Hz by varying the laser operation current. The radiation of the laser diode was focused with a lens having a numerical aperture NA = 0.25. The reflector was placed in the focusing plane of the laser beam.

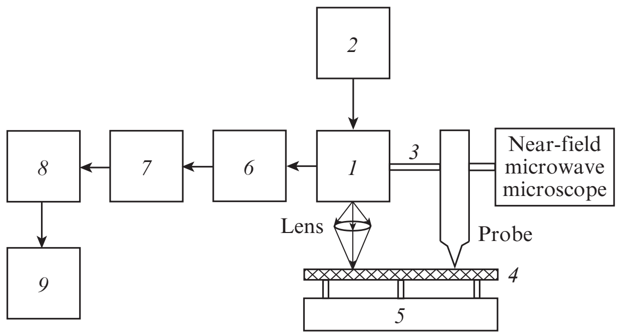

Figure 4. Block diagram of the experimental setup (see the text).

As known, the maximal coefficient of the autodyne gain corresponds to the threshold values of the semiconductor laser operation current. In this connection, the operation current of the laser autodyne was set to be ~1.2 of the threshold current value (I_{th} = 25 mA). As a result, the power of the laser diode radiation decreased to 2 mW from the nominal power of 5 mW.

To reduce the feedback level we used controlled laser beam defocusing, described in Ref. [34]. Using the sets of different spectral components of the autodyne signal the feedback level C was determined, which did not exceed 0.15 during the measurements. The effect of the beam width that approached ~0.3 mm was estimated with the insignificant defocusing of the beam taken into account. This led to minor transverse phase incursion. However, since only the alternating component of the autodyne signal was detected, this incursion did not introduce errors into the measured value of the reflector displacement. The measurements of nanodisplacements were carried out using a Standa electromagnetic translator (8MVT40-13 model) being a part of the operating model of a near-field microwave microscope [39, 40]. The main parameters of the translator are the following: the resolution 80 nm (the total step), the maximal translation distance 13 mm, the maximal translation velocity 0.416 mm s^{-1}.

To reduce the noise component of the laser autodyne signal we used filtering at the frequencies exceeding 10 kHz. Figure 5 presents the measured autodyne signal $P(t)$ after the filtering and its spectrum $S(f)$ under current modulation of the laser wavelength. The signal was obtained for the case of a reflecting surface placed at a distance of 10 cm from the radiator.

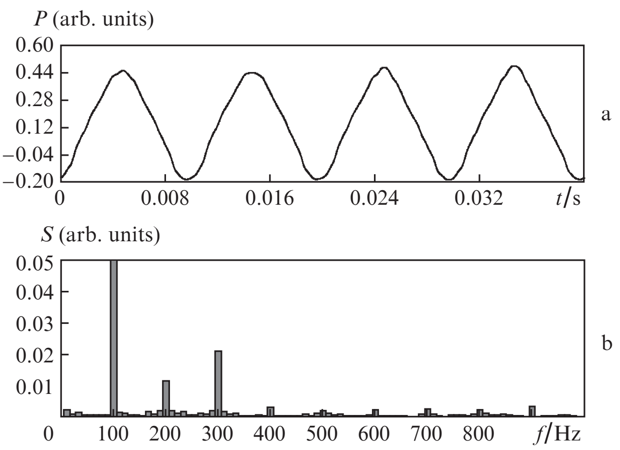

Figure 5. (a) Autodyne signal and (b) its spectrum.

The results of measuring the amplitude of the second, third and fourth spectral components of the autodyne signal of the semiconductor laser system, when the reflector moves with a step of 80 nm, are presented in Fig. 6. The periodicity of variation of the spectral component amplitudes due to the formation of a standing wave in the external cavity of the laser autodyne is clearly seen. The size of the region in which the amplitude of the autodyne signal monotonically depends on the reflector displacement amounts to ~160 nm for a radiation wavelength of 650 nm. Obviously, the use of laser diodes with the radiation wavelength 1550 nm would allow enlarging the region of monotonic variation by more than two times. However, in this case, the resolution of the system will be reduced and the problem of visualising the laser radiation focusing region will arise.

Figure 7 presents the reflector nanodisplacement ΔL measured using the above technique with the fixed translator step. The periodicity of variation of the autodyne signal stationary phase was taken into account.

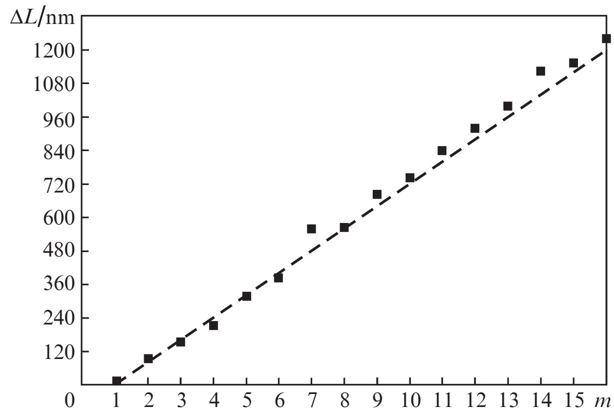

Figure 7. Measured reflector nanodisplacement vs. the number of steps m for the fixed step δL = 80 nm.

To process the information we used the linear regression method. In our case, the determination coefficient was R^2 = 0.995, or ~99.5%. The root-mean-square deviation amounted to 6.4%. This is an acceptable level for the measurement error of the method, which confirms the adequacy of the theoretical model.

6. Conclusions

We have demonstrated the possibility of using a semiconductor laser autodyne with current modulation of the radia-

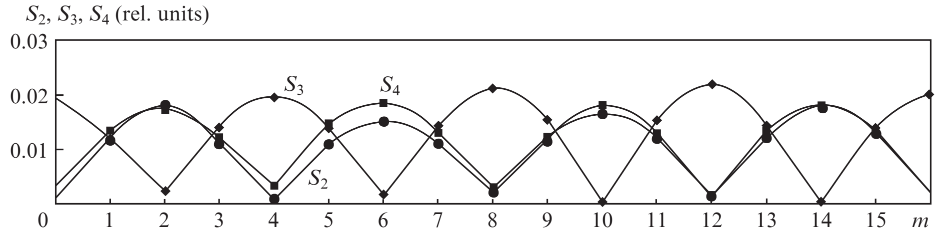

Figure 6. Amplitudes of the second (S_2), third (S_3), and fourth (S_4) spectral components of the autodyne signal of the semiconductor laser system varying due to the translator displacement with the step δL = 80 nm (m is the number of steps).

tion wavelength for monitoring nanodisplacements of the translator of a near-field microwave microscope. The analysis of the autodyne signal and its low-frequency spectrum formation under harmonic current modulation of the wavelength is carried out. It is found that in the autodyne signal spectrum the periodic variation in the amplitudes of its spectral components is observed, which is due to the formation of a standing wave in the external cavity of the autodyne system. The region of the monotonic dependence of the ratio of spectral component amplitudes on the reflector displacement under current modulation of the laser diode radiation wavelength is determined. The autodyne signal under current modulation of the radiation wavelength and the reflector translation with the step 80 nm is experimentally observed. The nanodisplacements of the translator of the near-field microwave microscope are measured.

The resolution of the method can be increased using lasers with a shorter wavelength. However, for increasing the measurement accuracy the main goal is to improve the stability of oscillation in laser autodynes, which can be achieved by using thermally stabilised laser diodes [5].

Thus, in the present work we report the first successful attempt to use the radiation wavelength modulation in a laser autodyne for measuring nanodisplacements instead of exciting additional mechanical vibrations of the object, as was earlier proposed in Ref. [17].

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