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Ultra-fast photon counting with a passive quenching silicon photomultiplier in the charge integration regime

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Abstract. **An ultra-fast photon counting method is proposed based on the charge integration of output electrical pulses of passive quenching silicon photomultipliers (SiPMs). The results of the numerical analysis with actual parameters of SiPMs show that the maximum photon counting rate of a state-of-art passive quenching SiPM can reach ~THz levels which is much larger than that of the existing photon counting devices. The experimental procedure is proposed based on this method. This photon counting regime of SiPMs is promising in many fields such as large dynamic light power detection.**

Keywords: *silicon photomultiplier, photon counting, charge integration, optical power meter.*

1. Introduction

In recent years, the demand for high photon counting capabilities has been increasing rapidly in many fields such as free space and fiber-based quantum cryptography [1 – 5], fast quantum random number generators [6], reflectometry [7, 8], light power detection [9] and astronomy [10]. It has been found that GHz-count rates can be obtained with superconducting single photon detectors [11]. However, superconducting photon-counting modules require complex refrigeration units, which are not as practical as avalanche photodiode (APD) based detectors. The maximum count rate of a single APD is about 100 MHz which is limited by the recovery time [12]. It has been proposed to reduce the recovery time by combining several individual APDs in parallel [13]; however, it is needed to use optical switches which makes it inefficient. Eraerds et al. [14] obtained a 430 MHz maximum counting rate with a silicon photomultiplier (a spatial multiplexing detector) by using a high passing amplifier to cut off the long recovery tails of avalanche pulses from the SiPM; unfortunately, if the pulse rate is higher, this method does not work anymore. Akiba et al. [15] obtained a 1 GHz photon counting rate by applying two types of the baseline correction algorithm to the signal from the multi-pixel photon counters (MPPCs). In this paper, we propose a photon counting method based on the charge integration of the output electrical pulse of a SiPM. The numerical analysis shows that the THz photon counting can be realised in principle.

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2. Preliminary remarks

On the one hand, since a SiPM is a spatial multiplexing photon counter, when one APD pixel is fired by a photon, the other pixels are still ready for detecting photons, merely partially superposed with the previous avalanche pulse (Fig. 1). One can see that there is a 'valley' between the two pulses. When the amplitude of the middle 'valley' is larger than the threshold of the discriminator, the two pulses are judged as one count; therefore, there will be a regular photon counting loss due to the threshold discriminating photon counting, which limits the maximum photon counting rate. The total output charge of these two avalanches is exactly twice of eG (e is the elementary charge, and G is the gain of one pixel of the MPPC). Thus, if we record the total charge Q of the output pulse of the SiPM, the detected photon number can be obtained by dividing Q by eG.

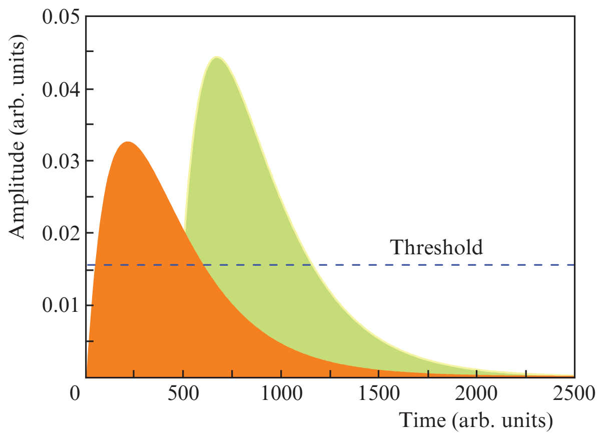

Figure 1. Waveform of two partially superposed avalanche pulses of the SiPM. The pulses from two individual pixels of the SiPM have the same area (or charge) and are different from after-pulse.

On the other hand, although a single passive quenching Geiger mode APD pixel has a relatively long recovery time to detect another photon, but it does not mean that it has no response to photons at all. One should not confuse the dead time and the recovery time of APD based photon counters [16, 17]. Actually, during the recovery time of an APD, the amplitude of the output pulse is smaller [17], which depends on the time interval between the previous pulse and the current pulse. If we can correct this effect, the dynamic range for photon counting will be extended.

3. Theoretical model

The model is based on the probability theory and recursion method with the following preconditions: (1) the incident light intensity is homogeneous at the SiPM surface which is easy to satisfy; (2) during the recovery time of an APD pixel, the APD can respond to the incoming photons, but with a smaller amplitude of the output pulse, which has been proved in many works [13–17]; and (3) the after-pulse effect is negligible, which is true for the state-of-the-art SiPMs operating at a normal bias voltage [15, 16, 18]. We firstly discuss the short pulsed light situation and then discuss the continuous light case.

3.1. Short pulsed light situation

Hereinafter, the short pulsed light means that the duration of the light pulse is smaller than the pixel recovery time constant τ_{rec} of the SiPM, i.e. the time needed for the voltage across the pixel junction to recover to the bias voltage of the SiPM [18]. Let us introduce the following notations: T is the period between two adjacent light pulses (the repetition frequency $F = 1/T$); M is the total number of pixels of a SiPM; η is the photon detection efficiency (PDE) of the SiPM, which is the same as the PDE of one APD pixel; $f(n) = Q(n)/(eG)$ is the number of fired pixels when n photons have already come; and p_{ct} is the optical crosstalk probability of the SiPM [17], i.e. the probability that the emitted near-infrared photons from the avalanche region in one APD pixel would fire another neighbouring pixel to the avalanche.

If $T \geqslant 5\tau_{rec}$, which guarantees that the fired pixel is completely recovered, then when the n + 1th photon comes, the function $f(n + 1)$ can be expressed as

$$f(n+1) = f(n) + \eta(1 + p_{ct})\left[1 - \frac{f(n)}{M}\right], \tag{1}$$

where $1 - f(n)/M$ is the probability that one APD pixel has not been fired (triggered) yet; and $1 + p_{ct}$ is the average fired pixels after a preliminary one-pixel-triggering event. Then $f(n)$ can be obtained by recursion:

$$f(n) = M\left\{1 - \left[\frac{M - \eta(1 + p_{ct})}{M}\right]^n\right\}. \tag{2}$$

As for the steady short pulsed light, the photon numbers per pulse can be well approximately described by the Poisson distribution in most practical cases [19, 20]:

$$p(n) = \frac{\exp(-\mu)\mu^n}{n!}, \tag{3}$$

where μ is the average input photon number per pulse. Thus the mathematical expectation of Eqn (2) is

$$E(f(n)) = f(\mu) = \sum_{n=1}^{\infty} p(n) f(n) \sum_{n=1}^{\infty} p(n)\, M\left[1 - \left(1 - \frac{\varepsilon}{M}\right)^{n+1}\right]$$
$$= M\left\{1 - \exp\left[-\frac{\mu\eta(1 + p_{ct})}{M}\right]\right\}. \tag{4}$$

Equation (4) is similar to the well known dynamic range formula for short pulsed light [21]. From this equation one can see that the maximum number of fired pixels is equal to M, which limits the dynamic range of the pulsed light photon counting regime. Then the total detected photon number per second is

$$N(\mu, F) = f(\mu)F = M\left\{1 - \exp\left[-\frac{\mu\eta(1 + p_{ct})}{M}\right]\right\}F. \tag{5}$$

From Eqn (5) we can see that by measuring N, the μ can be reconstructed if F is given.

If $\tau_{dis} < T < 5\tau_{rec}$, where τ_{dis} is the Geiger discharge time of a fired pixel, i.e. the duration of avalanching in the high electric field region [17, 22], then some of the APD pixels of the SiPM can be fired several times in one T. Let us suppose in a certain light pulse event $f(n)$ pixels have been fired, so the number of non-fired pixels is $M - f(n)$. Among these $M - f(n)$ pixels, some have not been fully recovered, and the average number of fired pixels per light-pulse event is

$$f(n) = M\left\{1 - \exp\left[-\frac{\mu\eta(1 + p_{ct})}{M}\right]\right\}$$

[according to Eqn (4)]. Then the probability that one pixel has been fired in the last pulse event is $f(\mu)/M$. And the probability that this pixel has not been fired in the first $k - 1$ pulse events is $[1 - f(\mu)/M]^{k-1}$. Thus, when the n + 1th photon comes, the number of fired pixels is

$$f(n+1) = f(n) + \eta\left[1 - \frac{f(n)}{M}\right]$$
$$\times \sum_{k=1}^{\infty} \frac{f(\mu)}{M}\left[1 - \frac{f(\mu)}{M}\right]^{k-1}\left[1 - \exp\left(-\frac{kT}{\tau_{rec}}\right)\right], \tag{6}$$

where the summation includes all the cases that a certain pixel has not fully recovered yet. The expression $1 - \exp(-kT/\tau_{rec})$ determines the recovery extent of a recovering pixel. Then $f(n)$ can also be obtained by recursion:

$$f(n) = M\left\{1 - \left\{1 - \frac{\exp\left(\frac{\eta\mu}{M}\right)\left[-1 + \exp\left(\frac{T}{\tau_{rec}}\right)\right]\eta}{\left[-1 + \exp\left(\frac{\eta\mu}{M} + \frac{T}{\tau_{rec}}\right)\right]M}\right\}^n\right\}. \tag{7}$$

The mathematical expectation of Eqn (7) is

$$E(f(n)) = f(\mu) = \sum_{n=0}^{\infty} f(n)\frac{\exp(-\mu)\mu^n}{n!}$$
$$= M\left\{1 - \exp\left\{\frac{\left[1 - \exp\left(\frac{T}{\tau_{rec}}\right)\right]\eta\mu(1 + p_{ct})}{\left[\exp\left(\frac{T}{\tau_{rec}}\right) - \exp\left(-\frac{\eta\mu}{M}\right)\right]M}\right\}\right\}. \tag{8}$$

Then the total detected photon number per second is

$$N(\mu, F) = M\left\{1 - \exp\left\{\frac{\left[1 - \exp\left(\frac{1}{F\tau_{rec}}\right)\right]\eta\mu(1 + p_{ct})}{\left[\exp\left(\frac{1}{F\tau_{rec}}\right) - \exp\left(-\frac{\eta\mu}{M}\right)\right]M}\right\}\right\}F. \tag{9}$$

From Eqn (9) we can see that if $F \ll 1/\tau_{rec}$ or $T \gg \tau_{rec}$, i.e. the repetition frequency of the light pulse is slow enough for the APD pixels to completely recover, the condition $\exp[1/(F\tau_{rec})] \gg 1 > \exp(-\eta\mu/M)$ is satisfied. In this case Eqn (9) can be simplified and reduced to Eqn (5).

If $T < \tau_{dis}$, the APD pixels have not finished discharging yet and thus have no response to photons.

3.2. Continuous light case

In the case of continuous light, the arriving time of a photon is random, whereas the photon number in a fixed time also follows the Poisson distribution. Hereinafter we set the average incident photon number per second equal to μ. Let us suppose that n photons arrived in τ_{rec}, and the number of fired pixels is $f(n)$, then when the $n+1$th photon comes, the function $f(n+1)$ can be expressed as

$$f(n+1) = \eta(1+p_{ct}) \times \left\{\left[1 - \frac{f(n)}{M}\right] + \frac{f(n)}{M}\left[1 - \exp\left(-\frac{1}{\mu\tau_{rec}}\right)\right]\right\}. \tag{10}$$

Here $1 - \exp[-1/(\mu\tau_{rec})]$ is the average recovery extent of a pixel that has been fired. After recursion, the function $f(n)$ can be obtained in the form:

$$f(n+1) = \exp\left(\frac{1}{\mu\tau_{rec}}\right) \times M\left\{1 - \left[\frac{M - \eta(1+p_{ct})\exp[-1/(\mu\tau_{rec})]}{M}\right]^n\right\}. \tag{11}$$

The mathematical expectation of Eqn (11) is

$$E(f(n)) = f(\mu) = \sum_{n=1}^{\infty} p(n) f(n) = \exp\left(\frac{1}{\mu\tau_{rec}}\right) \times M\left\{1 - \exp\left[\frac{-\mu\eta(1+p_{ct})\exp[-1/(\mu\tau_{rec})]}{M}\right]\right\}. \tag{12}$$

Equation (12) determines the expected detected photon number per second in a continuous light wave.

4. Numerical analysis with actual SiPM parameters

4.1. Selection of parameters

In order to numerically analyse the reasonability of the above model, parameters of an actual SiPM (Hamamatsu MPPC, S12571-010C) were selected [23]. The dark count rate (DCR) is $\nu_{DCR} \approx 100$ kHz mm^{-2} at 25 °C [or 100 kcps (kilo counts per second)], optical crosstalk probability is $p_{ct} \approx 6\%$ and the peak photon detection efficiency is $\eta \approx 10\%$ at a recommended operating voltage. The recovery time constant (τ_{rec}) of one pixel of the SiPM is about 1.5 ns [24] and the gain (G) of one pixel of this SiPM is about 1.35×10^5 (at a recommended operating voltage) [8]. The above mentioned parameters are substituted into Eqns (9) and (12) to obtain the numerical results. The numerical calculation is based on the Mathematica software.

4.2. Numerical analysis

Figure 2 shows the dependences of the detected photon number N on the repetition frequency F of short light pulses calculated by Eqn (9) at different average input photon numbers. Figure 2a shows that the detected photon number increases monotonically with increasing repetition frequency of short light pulses. The maximum detected photon number is influenced by the average incident photon number per pulse. One can also see that if μ and F are small, there is an approximately linear relationship between N and F. Thus, according to Fig. 2, the THz repetition frequency photon counting can be expected when μ is small. As μ becomes larger and larger, the N tends to saturate with increasing F. Whereas, this phenomenon does not mean that the MPPC cannot detect photons any more, on the contrary, one can reconstruct μ by Eqn (9) if F is given.

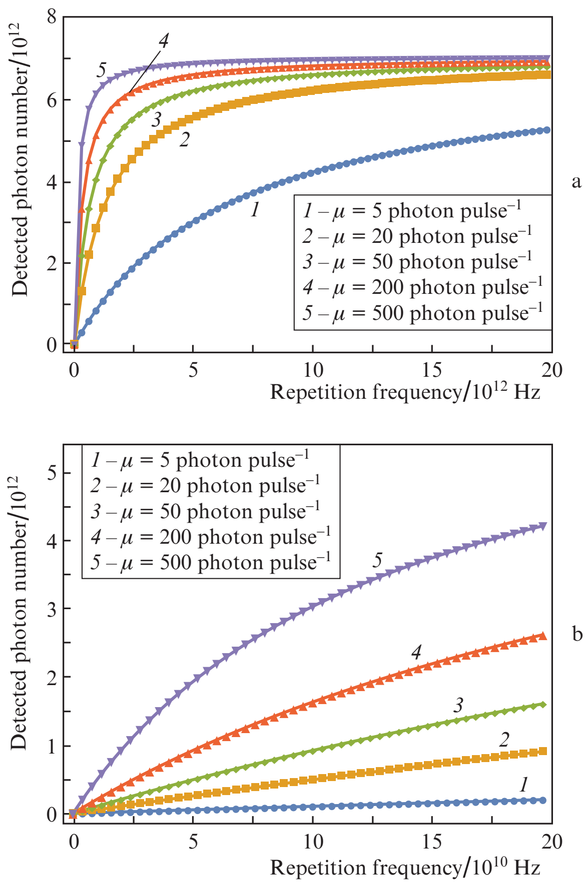

Figure 2. Dependences of the detected photon number on the repetition frequency of short light pulses. Figure 2b is the local zoom of the left side of Fig. 2a.

Figure 3 shows the dependence [calculated by Eqn (12)] of the detected photon number on the incident photon flux which irradiates the SiPM surface. One can see that the maximum detected photon number per second (N_{max}) happens when the incident photon flux is about 6×10^{13} photon s^{-1} and the N_{max} reaches up to about 2.4×10^{12}, which is much larger than the maximum photon counting rate (~1 GHz [25]) of the existing detectors. When the incident photon flux is larger than 5×10^{13} photon s^{-1}, most of the APD pixels can hardly recover anymore, and thus the effective detected photon number decreases. In this case, the detected photon number distorts, so that 2.4×10^{12} photon s^{-1} is the upper photon counting limit in this case. In practice, the photon counting rate deviates from linearity when it is of the same order of N_{max}; in this case, one should do the calibration by Eqn (12) to reconstruct μ.

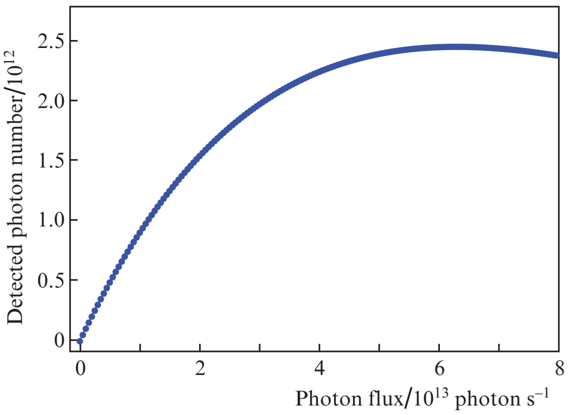

Figure 3. Dependence of the detected photon number on the incident photon flux.

5. Proposed experimental method

Figure 4 shows the experimental schematic for the above proposed photon counting method. The experiment procedure is as follow. Step 1: turn off the light source, power the SiPM at a normal operating voltage, accumulate data in a certain integration time (usually 1 s is enough) to obtain the dark count rates ν_{DCR} caused by the SiPM itself. Step 2: turn on the light source and do the same as step one at a different light intensity or a different light repetition frequency to obtain total output charges (Q_{total}). Step 3: divide Q_{total} by eG_{total} (the total gain including the gain of one APD pixel in the SiPM and the gain of the amplifier) to obtain the total pulse number (N_{total}). Step 4: subtract ν_{dcr} from N_{total} to obtain the detected photon number N, which corresponds to Eqn (9) and (12) after dividing by the integration time. After calibrating the incident light intensity, we can obtain the plot of N versus incident photon number like in Fig. 3 and the plot of N versus repetition frequency of pulsed light at different light intensities like in Fig. 2. Both formulas, (9) and (12), could be verified by comparing the calculation and experimental results which is our next task.

6. Discussion

Note that in this charge integration photon counting regime, one should use a dc-coupled amplifier with a smooth gain at a wide bandwidth, rather than an ac-coupled one, otherwise the dc current component will not be amplified, which will distort the total output integration charge. In addition, the SiPM should be connected to the voltage source directly without large series resistors to avoid current limiting. As for the minimum detectable photon flux in the continuous light case, it is limited by the dark count rate of the SiPM. Here the it is 100 kHz mm^{-2} at 25 °C, so that the dynamic range in the continuous wave case covers at least 7 orders of magnitude (i.e. from 10^5 to 10^{12} Hz), which greatly expands the upper limit of the photon counting rate. One limitation of this method is that the after-pulse effect has not been considered yet, although it has a small effect at a normal operating voltage of a SiPM, it would be more precise to take the after-pulse effect into account.

7. Conclusions

Charge integration photon counting scheme is proposed and the photon counting equation is found. The THz level photon counting rate can be obtained in principle by pulse charge integration of a passive quenching SiPM. The dynamic range of continuous wave photon counting covers at least 7 orders of magnitude, which greatly expands the upper limit of the photon counting rate.

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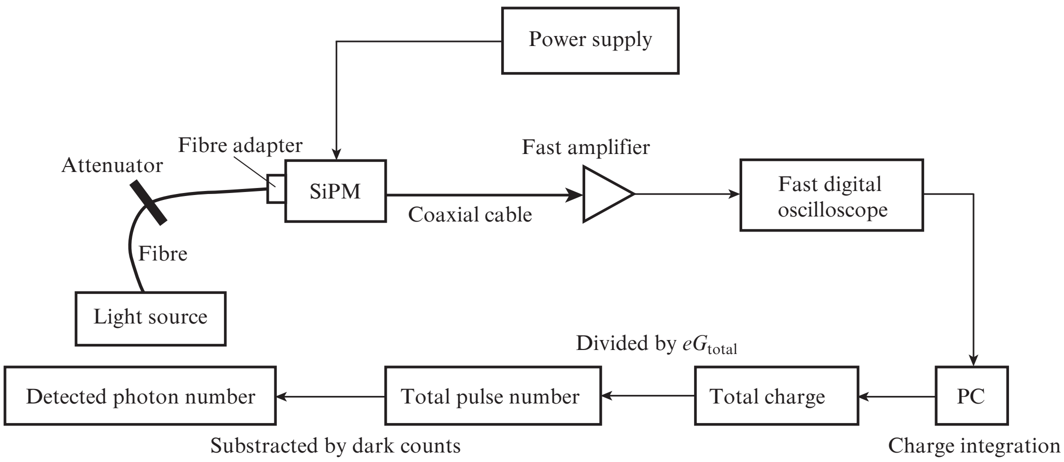

Figure 4. Experimental schematic for the proposed photon counting method.

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