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Anharmonic Bloch oscillations in a waveguide array

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Abstract. **Using the coupled-wave method, we consider anharmonic Bloch oscillations of light in an array of waveguides, taking into account the coupling between waveguides up to the third order. It is shown that the beam trajectory is periodic, with the trajectory oscillating within a single period.**

Keywords: *anharmonic Bloch oscillations, array of waveguides.*

1. Introduction

The great interest shown to date by researchers to arrays of waveguides is due to the fact that they allow the behaviour of signals propagating in them to be controlled and managed. Theoretical investigation of optical phenomena in the arrays of interacting waveguides is based on the use of the coupled-mode method. An important task is to study an array of waveguides whose optical parameters vary depending on the number and position of the waveguide in the array. Khadzhi et al. [1] studied the properties of the light propagation in planar semi-infinite waveguide arrays, whose propagation constants and coupling constants change according to the given laws, depending on the waveguide number. The possibility of creating Chebyshev arrays of the first and second kind, as well as Laguerre, Legendre, Hermitte, Jacobi and Gegenbauer arrays was also predicted. Purchel et al. [2] studied for the first time optical Bloch oscillations in an infinite array of waveguides, the propagation constant of which increases in proportion to the waveguide number. It was shown that the trajectory of the optical beam exciting the waveguide group from the end face periodically oscillates. In this case, each waveguide interacts only with the nearest neighbours.

At present, more complex optical structures such as zigzag waveguide arrays with the second-order coupling are of considerable interest. The authors of Refs [3–5] studied anharmonic Bloch oscillations in an array of waveguides, in which the propagation constant is proportional to the waveguide number in the array, taking into account the first- and second-order couplings. Gozman et al. [5] generalised the result obtained in [2] to the case of zigzag arrays. An analytical solution to the system of equations for the amplitudes of coupled modes was found and a formula for the trajectory of the optical beam was obtained.

In this paper we present an analytical solution of an infinite system of coupled-mode equations for an array in which the propagation constant contains a correction proportional to the waveguide number in the array, and in addition, the coupling of the first, second and third orders is taken into account. The results presented below are a generalisation of the results from Refs [2–5].

2. Statement of the problem. Basic Equations

The starting point of our study is the system of coupled-mode equations:

$$\left(\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}z}+\alpha j\right)a_j(z)+\gamma_1(a_{j-1}(z)+a_{j+1}(z))$$
$$+\gamma_2(a_{j-2}(z)+a_{j+2}(z))+\gamma_3(a_{j-3}(z)+a_{j+3}(z))=0, \quad (1)$$

where $a_j(z)$ is the modal amplitude of the jth waveguide as a function of the longitudinal coordinate z; α is the correction to the propagation constant; and γ_1, γ_2 and γ_3 are the first, second and third order coupling constants, respectively. Following [5], we move from the amplitudes $a_j(z)$ to their Fourier transforms and obtain the differential equation:

$$\left(\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}z}+\mathrm{i}\alpha\frac{\mathrm{d}}{\mathrm{d}k}\right)a(k,z)+2(\gamma_1\cos k+\gamma_2\cos 2k$$
$$+\gamma_3\cos 3k)a(k,z)=0. \quad (2)$$

Using the boundary condition $a(k,z)|_{z=0}=a^0(k)$, we find the general solution to this equation in the form

$$a(k,z)=a^0(k-\alpha z)\exp\Big\{2\mathrm{i}\frac{\gamma_1}{\alpha}[\sin k-\sin(k-\alpha z)]$$
$$+2\mathrm{i}\frac{\gamma_2}{\alpha}[\sin 2k-\sin 2(k-\alpha z)]+2\mathrm{i}\frac{\gamma_3}{\alpha}[\sin 3k-\sin 3(k-\alpha z)]\Big\}. \quad (3)$$

Assuming below that the function $a^0(k)$ has a sharp peak at $k=k_0$ and expanding expression (3) in a Taylor series in the vicinity $(k-\alpha z)-k_0$, we obtain

$$a(k,z)=a^0(k-\alpha z)\exp[\mathrm{i}\varphi(z)+\mathrm{i}(k-\alpha z)-k_0\psi(z)], \quad (4)$$

where

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$$\varphi(z) = 2\frac{\gamma_1}{\alpha}[\sin(k_0 + \alpha z) - \sin k_0] + 2\frac{\gamma_2}{\alpha} \times[\sin 2(k_0 + \alpha z) - \sin 2k_0] + 2\frac{\gamma_3}{\alpha}[\sin 3(k_0 + \alpha z) - \sin 3k_0]; \quad (5)$$

and

$$\psi(z) = 2\frac{\gamma_1}{\alpha}[\cos(k_0 + \alpha z) - \cos k_0] + 2\frac{\gamma_2}{\alpha} \times[\cos 2(k_0 + \alpha z) - \cos 2k_0] + 2\frac{\gamma_3}{\alpha}[\cos 3(k_0 + \alpha z) - \cos 3k_0]. \quad (6)$$

Substituting (4) into (2), we find solutions for the field amplitude $a_j(z)$ and the intensity $|a_j(z)|^2$, similar to solutions (20) and (21) from [5]. Then for the trajectory of the optical beam in the (j, z) plane for $j_0 = 0$ and $k_0 = 0$, we obtain

$$j(z) = 2\frac{\gamma_1}{\alpha}[1 - \cos\alpha z] + 2\frac{\gamma_2}{\alpha}[1 - \cos 2\alpha z] + 2\frac{\gamma_3}{\alpha}[1 - \cos 3\alpha z]. \quad (7)$$

It is seen from (7) that the function $j(z)$ is periodic and depends on the z coordinate along the waveguide with a period $z = 2\pi/\alpha$. Moreover, $j(z = 0) = j(z = 2\pi/\alpha) = 0$. The period decreases monotonically with increasing parameter α. We find the extrema of the function $j(z)$ within the same period $0 \leqslant z \leqslant 2\pi/\alpha$ by equating the derivative $\mathrm{d}j/\mathrm{d}z$ to zero. The position of the extrema is determined by the equations

$$\sin\alpha z = 0, \quad (8)$$

$$\gamma_1 - 3\gamma_3 + 4\gamma_2\cos\alpha z + 12\gamma_3(\cos\alpha z)^2 = 0. \quad (9)$$

It follows from (8) that, regardless of the values of the parameters γ_1, γ_2 and γ_3, one of the extrema of the trajectory $j(z)$ is found at $z = \pi/\alpha$, where $j = 4(\gamma_1 + \gamma_3)/\alpha$. Thus, beginning with the waveguide with $j = 0$, the beam first moves from waveguide to waveguide perpendicular to the waveguide axes, reaches the waveguide with the number $j = 4(\gamma_1 + \gamma_3)/\alpha$, then returns to the initial waveguide with $j = 0$, passing along its axis the distance $z = 2\pi/\alpha$. Figure 1 shows the trajectory of the beam with one maximum and one minimum within one period. Comparing this result with the analogous result from [5], we can conclude that when the third-order coupling is taken into account, the moving beam is additionally shifted to the right by the number of the waveguides, $\Delta j = 4\gamma_3/\alpha$.

From the equation (9) we obtain two solutions for the positions of the beam extrema:

$$\cos\alpha z = \frac{1}{6\gamma_3}\left(-\gamma_2 + \sqrt{\gamma_2^2 + 9\gamma_3^2 - 3\gamma_1\gamma_3}\right), \quad (10)$$

$$\cos\alpha z = -\frac{1}{6\gamma_3}\left(\gamma_2 + \sqrt{\gamma_2^2 + 9\gamma_3^2 - 3\gamma_1\gamma_3}\right). \quad (11)$$

The extremum positions essentially depend on the values of the parameters γ_1, γ_2, γ_3 and α. In the limit $\gamma_3 \to 0$, from (10) we arrive at the solution obtained in [5]: $\cos\alpha z = -\gamma_1(4\gamma_2)^{-1}$. Hence, it can be seen that additional solutions for extrema exist only for $\gamma_1/(4\gamma_2) < 1$, which, as shown in [5], are defined by formulas

$$z_{1,2} = \frac{1}{2}\left(\pi \pm \arccos\frac{\gamma_1}{4\gamma_2}\right).$$

In the general case, when $\gamma_3 \neq 0$ and the condition $\gamma_1 < \min[4\gamma_2 - 9\gamma_3^2;\ 3\gamma_3 + \gamma_2^2/(3\gamma_3)]$ is fulfilled, the solutions for the extremums are as follows:

$$z_{1,2} = \frac{1}{\alpha}\left\{\pi \pm \arccos\left[\frac{1}{6\gamma_3}\left(-\gamma_2 + \sqrt{\gamma_2^2 + 9\gamma_3^2 - 3\gamma_1\gamma_3}\right)\right]\right\}. \quad (12)$$

If $4\gamma_2 - 9\gamma_3^2 < \gamma_1 < 3\gamma_3 + \gamma_2^2/(3\gamma_3)$, then there are two additional extrema at

$$z_{3,4} = \frac{1}{\alpha}\left\{\pi \pm \arccos\left[\frac{1}{6\gamma_3}\left(\gamma_2 + \sqrt{\gamma_2^2 + 9\gamma_3^2 - 3\gamma_1\gamma_3}\right)\right]\right\}. \quad (13)$$

3. Discussion of results

Figure 1 shows the dependences $j(z)$ for different values of the parameters. It can be seen that within one period there can be either two, or four, or finally six extrema. Each extremum is found at certain values of the variable z and the waveguide number j, i.e., as the beam of light propagates, diffusion takes place in a direction perpendicular to the propagation direc-

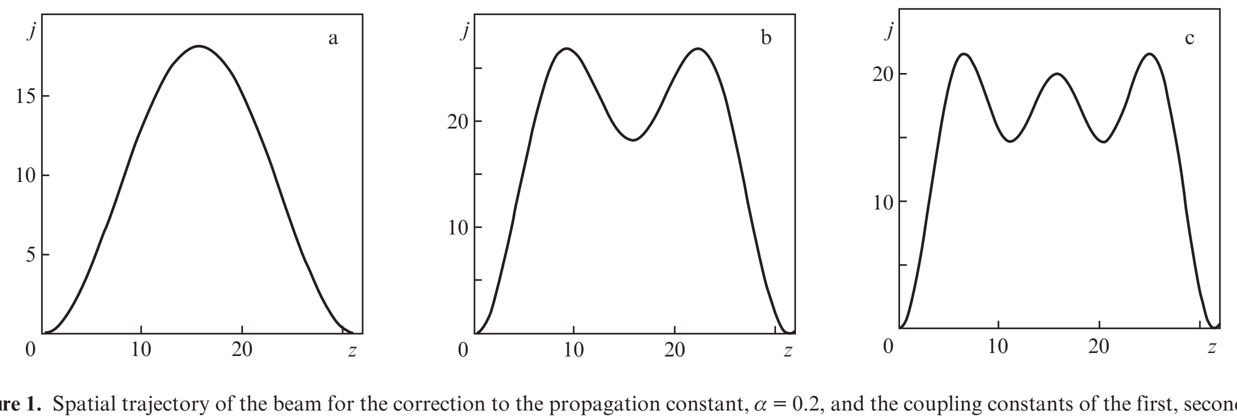

Figure 1. Spatial trajectory of the beam for the correction to the propagation constant, $\alpha = 0.2$, and the coupling constants of the first, second and third orders: (a) $\gamma_1 = 0.9$, $\gamma_2 = 0.012$, $\gamma_3 = 0.001$; (b) $\gamma_1 = 0.9$, $\gamma_2 = 0.83$, $\gamma_3 = 0.015$; and (c) $\gamma_1 = 0.5$, $\gamma_2 = 0.499$, $\gamma_3 = 0.05$.

tion. This result indicates that with allowance for the additional coupling between the waveguides (in this case, third-order coupling), the spatial structure of the beam trajectory is enriched by a pair of additional extrema. The result obtained allows us to state that when all couplings in the array are taken into account, up to the Nth order, the centre of the propagating beam will move in the space of variables (j, z) along the curve

$$j(z) = \frac{2}{\alpha}\sum_{s=1}^{N}\gamma_s(1 - \cos s\alpha z),$$

which is a natural generalisation of solution (7) and solution (24) from [5]. In this case, the curve (j, z) can have $2s$ extrema within the same period.

Thus, we have considered anharmonic Bloch oscillations in an array of optical fibers, taking into account the coupling between waveguides up to the third order. It is shown that the beam trajectory is a periodic function, and there exist oscillations of trajectories with six extrema within the same period.

References

1. Khadzhi P.I., Lyakhomskaya K.D., Orlov O.K. *Quantum Electron.*, **36**, 971 (2006) [*Kvantovaya Elektron.*, **36**, 971 (2006)].
2. Purchel U., Pertsch T., Lederer F. *Opt. Lett.*, **23**, 1701 (1998).
3. Wang G., Huang J.P., Yu K.W. *Opt. Lett.*, **35**, 1908 (2010).
4. Dreisow F., Wang G., Heinrich M., Keil R., Tünnermann A., Nolte S., Szameit A. *Opt. Lett.*, **36**, 3963 (2011).
5. Gozman M.I., Guseynow A.I., Kagan Yu.M., Pavlov A.I., Polishchuk I.Ya. arXiv:1501.06492 (26 Jan. 2015).