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Self-modulation oscillations in a solid-state coupled-cavity ring laser

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Abstract. **We propose a theoretical model describing the radiation dynamics in a solid-state coupled-cavity ring laser. Based on the numerical simulation conducted within the framework of this model, we investigate antiphase harmonic oscillations of counterpropagating wave intensities. It is shown that the amplitudes and frequencies of self-modulation oscillations in cases of in-phase and antiphase cavity couplings differ significantly. In the case of antiphase coupling, a new possibility of increasing the scale factor with increasing perimeter of the additional cavity is found. The dependence of the frequency of self-modulation oscillations on the optical nonreciprocity produced in the main cavity by a constant magnetic field is studied experimentally. Comparison of experimental and theoretical results shows good agreement between theory and experiment.**

Keywords: *solid-state ring laser, coupled cavities, self-modulation oscillations, optical nonreciprocity, scale factor, laser gyroscopy.*

1. Introduction

The sensitivity of a ring laser to rotation and the possibility of increasing it are important for problems related to the applications of laser gyroscopes (LGs). One way to increase the scale factor and the sensitivity of an LG is based on the use of a strong anomalous dispersion medium placed inside a laser cavity (see [1 – 3] and references therein). Eliseev [1] theoretically investigated the possibility of increasing the scale factor in a semiconductor laser whose resonator is filled with a dispersive medium. Shahriar et al. [2] predicted theoretically the possibility of increasing the scale factor by a factor of 10^6 with the use of intracavity anomalous dispersion media. A detailed analysis of the possibilities of using gas anomalous dispersion media to increase the scale factor in an LG based on helium – neon lasers was carried out by Salit at al. [3], who showed that the use of such linear gas media for this purpose is unpromising.

Another approach to increasing the scale factor of an LG is based on the use of coupled ring lasers (without the use of any intracavity highly dispersive medium). Laser gyroscopes include systems that employ optical angular rotation velocity sensors of two types: 1) ring lasers that generate counterpropagating waves with different frequencies inside the laser cavity, and 2) sensors in which the external laser radiation is transmitted in counterpropagating directions through a Sagnac interferometer (or a ring resonator). The LGs with a sensor of the first type are called active, and the LGs with a sensor of the second type are called passive. For passive LGs, the possibility of increasing the scale factor by means of coupled ring resonators was shown theoretically in [4 – 6]; This conclusion was confirmed in [4] by experimental results.

In the case of active LGs, in accordance with the theoretical studies carried out in Refs [7 – 9], one can expect that the use of coupled cavities will allow a substantial increase in the scale factor *K*. In these works it was shown that coupled ring resonators make it possible to control intracavity dispersion and realise conditions, which arise under anomalous dispersion and lead to an increase in *K*. However, as far as we know, these results have not been verified experimentally to date.

Previous studies have shown that harmonic antiphase self-modulation oscillations of the intensities of counterpropagating waves with a frequency that depends on the angular rotation velocity are excited in solid-state ring lasers (SRLs), in particular in miniature ring chip lasers, due to the competition of counterpropagating waves. This generation regime was called the self-modulation regime of the first kind (SMR1). The first experimental and theoretical studies of this regime in diode-pumped annular chip lasers were performed in Refs [10 – 13].

Using SRLs operating in the self-modulation regime, it is possible in principle to create one of the versions of an active LG, which differs from the conventional method of measuring the rotational velocity. In a conventional LG, a beat signal, which arises during interference of counterpropagating waves as a result of their mixing outside the cavity, is processed, and in the variant using SMR1 it is necessary to measure the frequency of intensity self-modulation of one of the counterpropagating waves emerging from the laser cavity. The advantage of this type of sensors is the absence of a lock-in zone (self-modulation oscillations are retained even at zero rotation velocity). SMR1 is the main regime of operation for miniature monolithic ring-shaped chip lasers, but the possibilities of using such sensors for navigation applications are limited by the small value of the scale factor *K*, due to the small size of the chip laser. The situation could change with increasing *K* by means of coupled ring resonators.

The purpose of this paper is a theoretical and experimental study of self-modulation oscillations of radiation in a coupled-cavity SRL.

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2. Theory

2.1. System of equations for an external optical feedback SRL

Figure 1 illustrates a schematic of a coupled-cavity ring laser. Inside the main ring resonator containing an active element (AE), two counterpropagating waves with complex amplitudes $E_{1,2}$ propagate (in Fig. 1 only the wave is shown). The radiation emitted from the main resonator through the partially transmitting coupling mirror (M) excites optical fields $E_{c1,c2}$ in the external ring resonator and returns again to the main resonator through the same mirror (Fig. 1 shows only the wave E_{c1}).

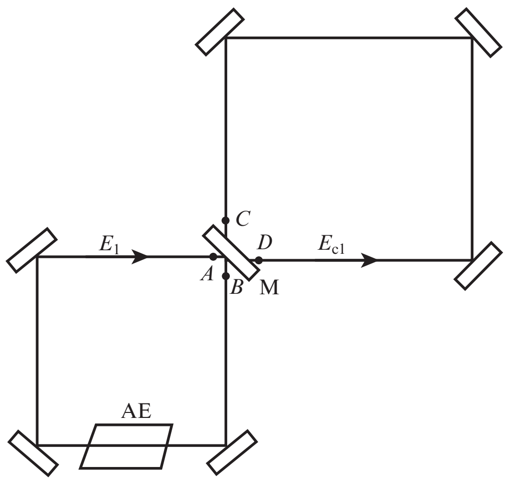

Figure 1. Scheme of a coupled-cavity ring laser.

We consider a one-dimensional model of a laser in which the optical electric fields of the counterpropagating waves $E_{1,2}^{\rm opt}$ in the main resonator are represented in complex form as functions that depend on time and coordinate z directed along the cavity axis:

$$E_{1,2}^{\rm opt} = E_{1,2}(z,t)\exp[{\rm i}\omega(t \mp z/c)], \tag{1}$$

where ω is the frequency of optical vibrations; and c is the speed of light. The complex amplitudes $E_{1,2}(z,t)$ are slow functions that vary little over the period of optical oscillations. These functions describe the counterpropagating travelling waves, and outside the active medium we represent them in the form

$$E_{1,2}(z,t) = E_{1,2}(t \mp z/c). \tag{2}$$

The cyclicity conditions (uniqueness of the field) in the ring cavity have the form

$$E_{1,2}^{\rm opt}(z,t) = E_{1,2}^{\rm opt}(t,z+L), \tag{3}$$

where L is the optical length of the resonator.

Using (1)–(3), we can obtain the following difference equations describing the variation of the complex amplitudes $E_{1,2}(z,t)$ during the round-trip time $T = L/c$ of the wave inside the ring cavity:

$$E_{1,2}(t) = k_{1,2}r_{\rm tot}\exp(-{\rm i}\omega T)E_{1,2}(t-T), \tag{4}$$

where $k_{1,2}$ are the coefficients of wave amplification during passage through the active medium; and $r_{\rm tot}$ is the product of the amplitude reflection coefficients of all the mirrors of the main resonator.

Equations (4) refer to the case when there is only one main resonator in the ring laser. Below, we obtain a system of difference equations, which is valid for a laser with two coupled resonators. To this end, we consider the transformation of the electric field $E_1(t)$ on the coupling mirror M (Fig. 1). We denote by $E_A(t)$ and $E_B(t)$ the value of the field $E_1(t)$ at points A and B (before and after reflection from the coupling mirror) inside the main resonator. Passing through the coupling mirror, the wave $E_1(t)$ enters the additional resonator and causes a change in the amplitude of the wave field $E_{c1}(t)$ propagating in this cavity. We denote by $E_C(t)$ and $E_D(t)$ the values of the field $E_{c1}(t)$ at points C and D (before and after reflection from the coupling mirror) inside the additional resonator. The field conversions on the coupling mirror have the form

$$E_B(t) = rE_A(t) + t_{\rm r}E_C\exp({\rm i}\varphi), \tag{5}$$

$$E_D(t) = rE_C(t) + t_{\rm r}E_A\exp({\rm i}\varphi). \tag{6}$$

Here r and $t_{\rm r} = \sqrt{1-r^2}$ are the amplitude coefficients of reflection and transmission for the coupling mirror; and the factor $\exp({\rm i}\varphi)$ takes into account the phase shift φ of the transmitted wave with respect to the reflected wave.

Taking into account the propagation of the fields inside the main and additional cavities, as well as the amplification of light with the coefficient k_1 as it passes through the active medium, we obtain the expressions:

$$E_A(t) = k_1 r_{\rm e} E_B(t-T), \tag{7}$$

$$E_C(t) = r_{\rm c} E_D(t-T_{\rm c}). \tag{8}$$

Here $r_{\rm e}$ is the effective coefficient, equal to the product of the reflection coefficients of all the mirrors of the main resonator, with the exception of the coupling mirror, and the coefficient that takes into account the field attenuation, which arises from all other losses in the main resonator; $r_{\rm c}$ is an analogous value for an additional resonator; and $T_{\rm c}$ is the round-trip time of light inside the additional resonator.

For a coupled-cavity laser, taking into account transformations (5)–(8), the system of difference equations (4) takes the form:

$$E_{1,2}(t) = k_{1,2}r_{\rm e}\exp(-{\rm i}\omega T) \times[rE_{1,2}(t) + t_{\rm r}\exp({\rm i}\varphi)E_{c1,c2}(t-T)], \tag{9}$$

$$E_{c1,c2}(t) = r_{\rm c}\exp(-{\rm i}\omega T_{\rm c}) \times[rE_{c1,c2}(t) + t_{\rm r}\exp({\rm i}\varphi)E_{1,2}(t-T_{\rm c})]. \tag{10}$$

Here equations (9) determine the generation of counterpropagating waves $E_{1,2}$ inside the main resonator, taking into account the effect of the fields $E_{c1,c2}$, and equations (10) – the excitation of counterpropagating waves in the external resonator by waves $E_{1,2}$. This system of equations must be supplemented by equations for the inverse population in the active medium. Using them, we can calculate the coefficients $k_{1,2}$.

Equations (9), (10) are applicable for the analysis of both single-mode and multimode generation with a large number of axial modes. We consider below single-mode generation, and the losses in the main resonator per single round trip are assumed to be small. In this case, the gains are close to unity and can be written in the form $k_{1,2} = 1 + \sigma Nl/2$, where σ is the stimulated emission cross section at the laser transition; N is the population inversion density; and l is the length of the active element. After the difference equations (9) are replaced by differential equations, the system of equations for a coupled-cavity SRL, generalising a system of analogous equations for a single-cavity SRL [14, 15] takes the form:

$$\dot{E}_{1,2} = -\frac{\omega}{2Q}E_{1,2} \pm \mathrm{i}\frac{\Omega}{2}E_{1,2} + \frac{\mathrm{i}}{2}\tilde{m}_{1,2}E_{2,1}$$

$$+\frac{\sigma l}{2T}(N_0E_{1,2} + N_{\pm}E_{2,1}) + \frac{t_{\mathrm{r}}\exp(\mathrm{i}\varphi)}{T}E_{\mathrm{c1,c2}},$$

$$T_1\dot{N}_0 = N_{\mathrm{th}}(1+\eta) - N_0 - N_0a(|E_1|^2+|E_2|^2) \qquad (11)$$

$$-N_+aE_1E_2^* - N_-aE_1^*E_2,$$

$$T_1\dot{N}_+ = -N_+ - N_+a(|E_1|^2+|E_2|^2) - N_0aE_1^*E_2,$$

$$E_{\mathrm{c1,c2}}(t) = r_{\mathrm{c}}\exp[\mathrm{i}(\Phi \mp \Omega_{\mathrm{c}}T_{\mathrm{c}}/2)]$$

$$\times[rE_{\mathrm{c1,c2}}(t - T_{\mathrm{c}}) + t_{\mathrm{r}}\exp(\mathrm{i}\varphi)E_{1,2}(t - T_{\mathrm{c}})]. \qquad (12)$$

The following notations are used in equations (11) and (12): ω/Q is the bandwidth of the main resonator (the losses inside the cavity are assumed equal for counterpropagating waves); Q is its Q-factor; T_1 is the longitudinal relaxation time; $a = T_1c\sigma\times(8\hbar\omega\pi)^{-1}$ is the saturation parameter; Ω and Ω_{c} are the frequency nonreciprocities of the main and additional resonators, arising from the Sagnac effect during rotation; $\Phi = \omega_nT_{\mathrm{c}}$; and ω_n is the eigenfrequency of the main resonator for counterpropagating waves in the absence of rotation. The pump rate is given in the form $N_{\mathrm{th}}(1 + \eta)/T_1$, where N_{th} is the threshold density of the inverse population; and η is the excess of the pump power above the threshold. The linear coupling of counterpropagating waves is determined by phenomenologically introduced complex coupling coefficients

$$\tilde{m}_1 = m_1\exp(\mathrm{i}\vartheta_1), \quad \tilde{m}_2 = m_2\exp(-\mathrm{i}\vartheta_2), \qquad (13)$$

where $m_{1,2}$ are the moduli of coupling coefficients; and $\vartheta_{1,2}$ are their phases.

The inverse population density is expanded in a series of spatial harmonics

$$N(z,t) = N_0(t) + N_+(t)\exp(\mathrm{i}2kz)$$

$$+ N_-(t)\exp(-\mathrm{i}2kz), \;\; N_+ = N_-^* \qquad (14)$$

taking into account the zero harmonic N_0 and the second harmonics $N_{\pm}$. Because of the interference of counterpropagating waves, the radiation intensity inside the resonator changes periodically in space (along the z axis of the resonator), and as a result of the population inversion saturation, lattices are formed in the active medium, the amplitudes of which are determined by the harmonics $N_{\pm}$.

Note that equations (11), (12) are written for the case of generation at the frequency of the gain line centre. In addition, in these equations the optical frequency ω is set equal to ω_n ($\omega = \omega_n$).

2.2. Results of numerical simulation

In the absence of external optical coupling, as was established above, the SMR1 arises in a wide range of laser parameters in a SRL. As shown in this paper, this regime can also be observed in a coupled-cavity SRL. The characteristics of self-modulation oscillations were found by solving numerically equations (11) and (12).

The SRL parameters were set as follows. It was assumed that the main resonator is a monolithic ring resonator cut from a YAG:Nd crystal (a resonator of a ring chip laser [13, 16]). The perimeter of the main resonator was $L = 5$ cm, and the reflection coefficients were $r = 0.97$ and $r_{\mathrm{e}} = 0.93$, which corresponds to the main resonator bandwidth ω/Q equal to 4.5×10^8 s^{-1}. The coupling coefficients were assumed equal: $m_1 = m_2 = m = 1.3 \times 10^6$ s^{-1}. In this case, the frequency of self-modulation oscillations in the absence of optical nonreciprocity ($\Omega = 0$) in a chip laser without an additional resonator is 206 kHz. The excess of the pump above the threshold was $\eta = 0.09$. These parameters refer to an Nd:YAG ring chip laser, which was used in the present work in experimental studies. The perimeter of the additional ring cavity L_{c} in these experiments was 86.5 cm. The coefficient r_{c}, which determines the losses in the additional cavity, was not experimentally measured, and its value (0.35) was chosen so that the parameters of the self-modulation oscillations (amplitude and frequency) were consistent with those measured in the experiment. The system of equations (11), (12) also includes two parameters characterising the phases of the optical coupling of the main and additional resonators: $\Phi = \omega_nT_{\mathrm{c}}$ and the phase shift φ between the reflected and transmitted waves on the coupling mirror. The results of numerical simulation presented below were obtained for $\varphi = 0$.

Numerical simulation showed that at $\Phi = \omega_nT_{\mathrm{c}} = 2\pi p$, where p is an integer (when the lengths of the main and additional cavities differ by an integer number of times), an external optical coupling leads to a decrease in losses in the main resonator and to an increase in the amplitude of self-modulation oscillations. This coupling of the resonators will be called in-phase coupling. At $\Phi = 2\pi p \pm \pi$, the external optical coupling increases losses in the main resonator and reduces the amplitude of self-modulation oscillations. This optical coupling of resonators will be called antiphase coupling.

Figure 2 shows the time dependences of the counterpropagating wave intensities $I_{1,2} = a|E_{1,2}|^2$ in the case of in-phase coupling, calculated in the absence of frequency nonreciprocity ($\Omega = \Omega_{\mathrm{c}} = 0$) and for the values of other laser parameters indicated above. As can be seen from the figure, there is an antiphase sinusoidal modulation of the counterpropagating wave intensities, characteristic of the SMR1.

Numerical simulation has shown that the frequencies of self-modulation oscillations, $f_{\mathrm{m}} = \omega_{\mathrm{m}}/2\pi$, in the case of in-phase and antiphase couplings differ. In the absence of frequency nonreciprocity, the frequency f_{m} for the in-phase coupling and the above parameters is the smallest: $f_{\mathrm{m}}^{\mathrm{s}} = 183$ kHz. In the case of antiphase coupling, f_{m}, on the contrary, assumes the largest value: $f_{\mathrm{m}}^{\mathrm{a}} = 217$ kHz.

In numerical simulations, we calculated the dependence of the frequency of self-modulation oscillations on the optical

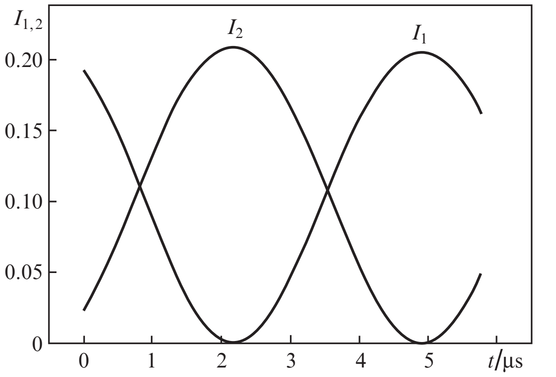

Figure 2. Time dependences of the dimensionless intensities of counter-propagating waves, calculated for a SRL with in-phase coupling of the cavities at $\eta = 0.09$ (the values of other parameters are given in the text of the paper).

nonreciprocity Ω of the main cavity. Figure 3 shows the dependence of f_m on Ω in cases of in-phase and antiphase couplings of the cavities. These dependences are measured at the above parameters, when the main resonator is a monolithic ring cavity based on a YAG : Nd crystal with a perimeter L = 5 cm and a perimeter L_c = 86.5 cm of an additional ring cavity. The dependence for the case of an antiphase coupling of the cavities at L_c = 7 m is also presented.

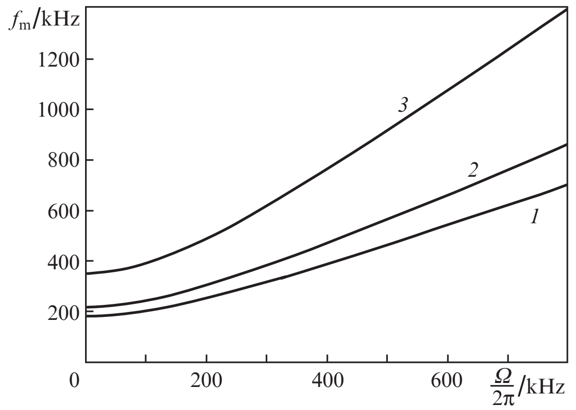

Figure 3. Dependences of the frequency of self-modulation oscillations on the optical nonreciprocity of the main cavity for (*1*) in-phase and (*2, 3*) antiphase resonator couplings. The lengths of the additional resonator are (*1, 2*) 86.5 cm and (*3*) 7 m (the values of other parameters are given in the text of the paper).

Based on the calculations performed, we can determine the change in the scale factor $K = \mathrm{d}f_m/\mathrm{d}\Omega$ when use is made of an additional resonator. In the case of an antiphase coupling, the value of K normalised to the scale factor of the chip laser without an additional cavity, with a perimeter L_{c1} = 86.5 cm of the additional cavity, virtually does not change (K_1 = 1.05) due to the optical coupling. With the increase in the perimeter of the additional resonator to L_{c2} = 7 m, the scale factor increases to K_2 = 1.7.

3. Results of experimental studies

In the present work, we studied experimentally a coupled-cavity SRL. The main cavity was a monolithic ring resonator made of a high-quality YAG : Nd single crystal in the form of a complex polyhedral prism. Such a monolithic resonator is usually used in ring-shaped chip lasers [13, 16]. The laser radiation emitted from the monolithic resonator through the dichroic mirror coating applied to the face (M) was returned to the monolithic ring cavity with the help of mirrors M1, M2 and M3, which are part of the additional ring cavity (Fig. 4). These mirrors had reflection coefficients close to unity. Mirrors M1 and M2 are plane mirrors, mirror M3 is spherical with a radius R = 50 cm. The perimeter L_c of the additional ring cavity was 86.5 cm. The ring laser was excited by a semiconductor laser diode whose radiation was focused and directed to the monolithic cavity through the dichroic mirror M on the face.

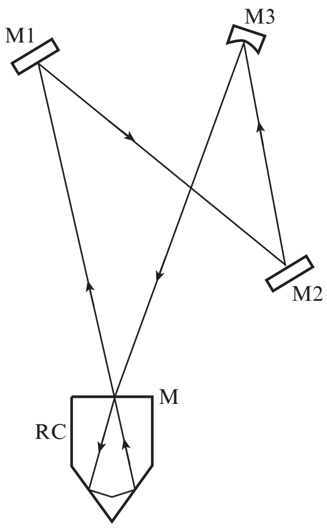

Figure 4. Scheme of a coupled-cavity SRL: (RC) monolithic ring cavity; (M1, M2) plane mirrors of the external cavity; (M3) spherical mirror.

The time and spectral characteristics of the radiation of the coupled-cavity SRL were studied experimentally. In the absence of an additional cavity, the ring-shaped chip laser operated in the SMR1. When external optical coupling was introduced with the help of an additional cavity, this regime was maintained. By tuning the perimeter of the additional cavity by a value of the order of the laser radiation wavelength, it is possible to change the amplitude of the self-modulation oscillations. To conduct experimental studies, the perimeter L_c was tuned so that the amplitude of the self-modulation oscillations became maximal (at a fixed pump power). We assume that this tuning of the additional cavity corresponds to the in-phase coupling of the two resonators.

During the experiments we found that the amplitude of self-modulation oscillations is unstable: for times on the order of several milliseconds, there are irregular changes in the amplitude from the initial (maximum) value that is approximately three times greater than in the chip laser without external optical coupling, which is about 1/3 less than the maximum. In a coupled-cavity SRL, high-frequency antiphase sinusoidal modulation of self-modulation oscillations has an irregular low-frequency envelope, which is an alternating sequence of intervals with a maximum amplitude of self-modulation (in-phase intervals) and intervals with a minimum amplitude (antiphase intervals). The appearance of such alternating intervals is apparently due to the instability of the

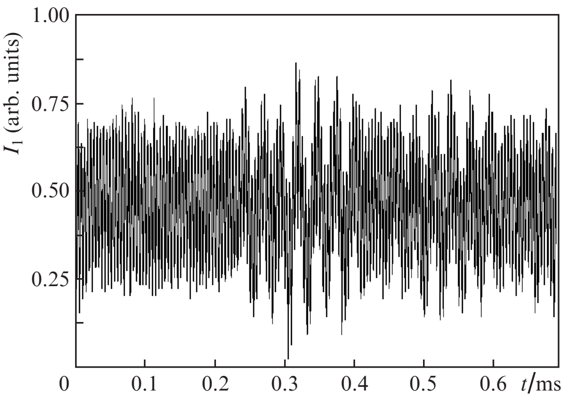

Figure 5. Time dependence of the radiation intensity of the wave E_1.

phase parameters $\Phi = \omega_n T_c$ and φ in the SRL in question. Figure 5 illustrates an oscillogram of the radiation intensity of one of the waves, $I_1(t) = |E_1(t)|^2$, which shows a transition from the antiphase interval to the in-phase one.

As was already noted in the discussion of the results of numerical simulation, the frequencies f_m of self-modulation oscillations depend on their amplitude. In the case of in-phase coupling of the cavities, the self-modulation amplitude is maximal, and the frequency f_m is the smallest. At a minimum self-modulation amplitude, the frequency f_m, on the contrary, assumes the greatest value.

The experimental measurements of f_m were carried out as follows. At a time interval of ~1 ms (measurement interval), the spectrum of the laser radiation intensity was measured. Each measurement of f_m consisted of a series including 20 recorded spectra. In most of the spectra included in the series, the self-modulation frequency corresponded to the in-phase coupling, and in the other spectra of this series (no more than 20%) the coupling was antiphase.

During the experiment we measured the dependence of the frequency f_m of self-modulation oscillations on the optical nonreciprocity $\Omega/2\pi$, created in the main cavity by a constant magnetic field superimposed on a monolithic ring resonator. The magnetic field H was produced using a solenoid and was proportional to the current I in the coil of the solenoid ($H = kI$). In a ring-shaped chip laser without an additional cavity, the frequency of self-modulation oscillations is determined by the approximate formula:

$$f_m = [f_{m0}^2 + (\Omega/2\pi)^2]^{1/2} = [f_{m0}^2 + (k_1 I)^2]^{1/2}, \quad (15)$$

where f_{m0} is the self-modulation frequency for $\Omega = 0$; and k_1 is the proportionality coefficient. Having chosen, as in [17], the value of the coefficient k_1, we can calculate the frequency nonreciprocity Ω.

Figure 6 shows the experimentally measured dependences of $f_m(\Omega)$ on in-phase (Fig. 6a) and antiphase (Fig. 6b) intervals. Empty circles indicate the results related to a ring-shaped chip laser without external optical coupling, and the filled circles to a coupled-cavity SRL. The solid curves correspond to a ring chip laser without an additional cavity [they were calculated from Eqn (15)], and the dashed curves were obtained in numerical simulation in cases of in-phase and antiphase couplings.

As shown in Fig. 6a, the frequencies of self-modulation oscillations measured at in-phase and antiphase intervals are in good agreement with the calculated values obtained by

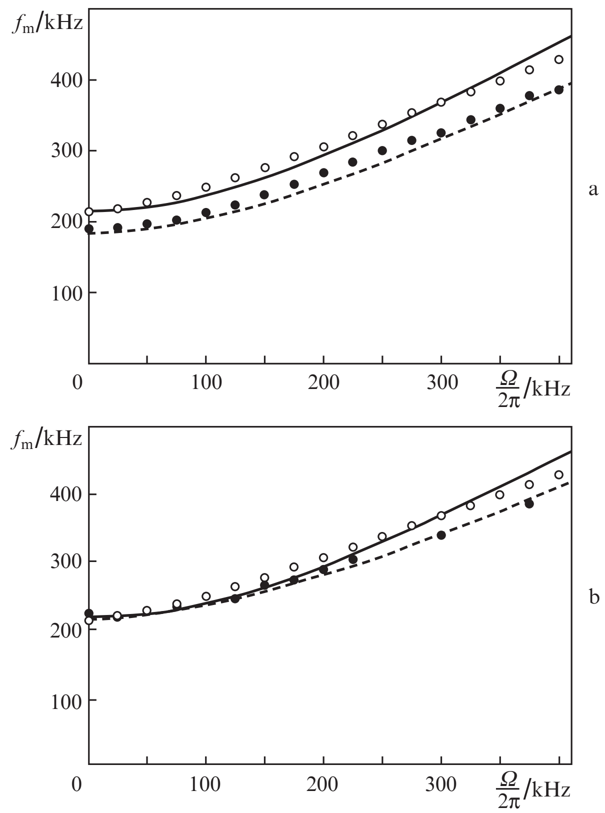

Figure 6. Dependences of the frequency of self-modulation oscillations on optical nonreciprocity at (a) in-phase and (b) antiphase intervals (see the text).

numerical simulation. As was already noted, it was assumed in numerical simulation that the phase shift φ between the reflected and transmitted waves on the coupling mirror is zero. The agreement between the results of the numerical simulation and the experiment does not give grounds for asserting that the condition $\varphi = 0$ was realised in the experiment. It can be shown that for $\varphi \neq 0$ the condition of the in-phase coupling of the cavities is $\Phi = \omega_n T_c - 2\varphi = 2\pi p$, and the antiphase coupling condition is $\Phi = \omega_n T_c - 2\varphi = 2\pi p + \pi$. It follows that when the perimeter of the additional resonator changes by a value on the order of the wavelength, it is possible to implement in-phase and antiphase couplings even at $\varphi \neq 0$.

4. Discussion of results

A theoretical model describing the radiation dynamics in a coupled-cavity SRL is proposed. Based on the numerical simulation carried out within the framework of this model and experimental studies performed with a coupled-cavity SRL, we can conclude that the proposed theoretical model gives a correct qualitative description of self-modulation oscillations in a coupled-cavity SRL. One of the conclusions obtained using the proposed model is that the amplitudes and frequencies of self-modulation oscillations in a coupled-cavity SRL with in-phase and antiphase coupling of the cavities differ substantially.

To improve the quantitative agreement of theory with experiment, it is required to refine the parameters of the coupled-cavity SRL: the values of the losses in the additional cavity and the phase shift between the reflected and transmitted waves on the coupling mirror. In the experiments carried out, despite the large reflection coefficients of the mirrors of the additional cavity, the losses were large. At $r_c = 0.35$, the losses per round trip of the light inside the resonator are $1 - r_c^2 = 88\%$. The transverse dimensions of the fields (modes) of the main and additional cavities differ significantly, and this leads to large diffraction losses on the coupling mirror. To reduce losses, it is necessary to match the transverse modes of the main and additional resonators.

The use of coupled cavities, as shown in Refs [7–9], makes it possible to control the intracavity dispersion and realise an effective dispersion (close to anomalous) in the main resonator. In the present paper, because of the large losses in the additional cavity, the control capabilities of the intracavity dispersion turned out to be very limited and it was not possible to approach the case of anomalous dispersion. Nevertheless, even with large losses in the additional cavity, the change in the dispersion properties of the main resonator can contribute to an increase in the scale factor. In the case of antiphase coupling of the cavities, a new possibility of increasing the scale factor with increasing perimeter L_c of the additional resonator is found. It can be significantly increased by using an optical delay line in the additional cavity (for example, an optical fibre).

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