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Amplification and generation of surface plasmon polaritons in a semiconductor film–dielectric structure

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Abstract. **The peculiarities of propagation and amplification of surface waves of plasmon polariton type in a planar semiconductor film–dielectric structure are considered for the THz frequency region, with allowance for dissipation in a semiconductor. Two spectral regions are found, where the group velocity of surface plasmon polaritons is negative. It is shown that in these regions the structure can be considered as an amplifying waveguide with distributed feedback and a high gain with respect to the reflected and transmitted signals. The possibility of generation of electromagnetic radiation in such structures is established.**

Keywords: *surface plasmon polariton, negative group velocity, current pumping, phase-matching condition, amplification and generation of THz radiation, distributed feedback laser.*

1. Introduction

Surface electromagnetic waves propagating along the interface between media (one of which in the considered spectral interval has a negative permittivity) have been called surface plasmon polaritons (SPPs) in the literature [1]. It is known that the SPP fields are strongly localised at the interlayer boundary and exponentially decay when they move away from it. In this case, the penetration depth of the field into both media is comparatively small and is on the order of the radiation wavelength [2–4]. The wave characteristics of SPPs are largely determined by type of dispersion of the material parameters of the adjacent media [5–7].

The behaviour of SPPs in metal–dielectric waveguide structures was investigated in sufficient detail [6–10]. Use of conducting media as a waveguide layer inevitably leads to ohmic losses and a significant decrease in the length of the wave propagation in a structure, due to which there are significant limitations on the application of SPPs in quantum optoelectronic devices. The compensation schemes proposed to date consist, in particular, in the creation of an inverted population in an active medium located near a metal surface [8]. Such methods are characterised by extremely low efficiency, require the use of an external laser and are suitable only for a pulsed regime, which does not make it possible to rely on their wide practical application. As an alternative to bulky optical pumping, Fedyanin and Arsenin [11] proposed to use electrical pumping of the active region by injecting charge carriers into a semiconductor.

We have studied the conditions of SPP amplification and generation by direct energy transfer of the drift current by a surface THz electromagnetic wave propagating along the interface between a dielectric and a semiconductor film. A similar amplification mechanism, an analogue of which is realised in travelling-wave tubes for amplifying microwave waves [12], does not require the presence of an active medium/mediator. To transfer directly the energy of the current wave to the electromagnetic wave, the phase matching conditions should be fulfilled, i.e. the drift velocity of charge carriers in a material medium should be comparable in order of magnitude with the phase velocity of a surface electromagnetic wave [6, 13]. In this paper, we determine the parameters of a planar semiconductor film–dielectric structure, which ensure a transition from the amplification regime to the generation of surface electromagnetic waves. The structure can be considered as a resonator due to the formation of an inverse positive coupling in the spectral region, where the group velocity of SPPs takes negative values. This structure is a model of an electrically pumped compact generator of surface electromagnetic waves of spaser type [5, 8], the physical principles of which are the same as that of the backward-wave tubes used in microwave electronics.

2. Material parameters and dispersion relation

Propagation and subsequent amplification of surface waves will be considered in a planar structure consisting of a thin semiconductor layer of thickness d, which is sandwiched between two nonmagnetic media with permittivities ε_1 and ε_3. A vacuum layer with $\varepsilon_1 = 1$ is chosen as a cover layer, and the substrate is an insulator with $\varepsilon_3 = 9$. The $NdGaO_3$ compound has, for example, a similar permittivity in the THz frequency range. As a waveguide layer of the structure, use is made of a doped p-type AlGaAs semiconductor, in which the permittivity in the framework of the Drude approximation is described by the expression [14]

$$\varepsilon_2(\omega) = \varepsilon_0\left[1 - \frac{\omega_p^2}{\omega(\omega + i\nu)}\right]. \qquad (1)$$

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Here ε_0 is the high-frequency value of the permittivity; $\omega_\mathrm{p} = [4\pi\times e^2n/(m^*\varepsilon_0)]^{1/2}$ is the plasma frequency; n and m^* are the concentration of impurity charge carriers (holes) and their effective mass; and ν is the relaxation parameter for charge carriers. The real part of the semiconductor permittivitty takes negative values in the region $\omega < \omega_\mathrm{p}$ (which corresponds to the THz range for $\omega_\mathrm{p} \approx 3.42\times 10^{13}$ s^{-1}).

In the structure in question, surface TM modes with wave field components $F_\alpha = E_x, H_y, E_z$ that propagate along the x axis directed along the media interface can be excited at the semiconductor–dielectric interface. The dependence of the indicated field components on time and coordinates has the form

$$F_\alpha(x,z,t) = F_\alpha(z)\exp[\mathrm{i}(\omega t - \beta x)], \qquad (2)$$

where $F_\alpha(z)$ are the profile functions of the corresponding components of the wave field; and $\beta = \beta' - \mathrm{i}\beta''$ is the SPP propagation constant. The relationship between these components of the field is determined from Maxwell's equations:

$$\frac{\partial^2 H_y}{\partial z^2} - q_j^2 H_y = 0,\quad E_x = \frac{\mathrm{i}}{k_0\varepsilon_j}\frac{\partial H_y}{\partial z},\quad E_z = -\frac{\beta}{k_0\varepsilon_j}H_y, \qquad (3)$$

where $q_j = q_j' - \mathrm{i}q_j'' = \sqrt{\beta^2 - k_0^2\varepsilon_j}$ are the transverse components of the SPP wave vector in each medium (j = 1, 2, 3); $q_j = q_j' - \mathrm{i}q_j'' = \sqrt{\beta^2 - k_0^2\varepsilon_j}$; and c is the speed of light in vacuum.

Taking into account the complexity of the SPP wave vector, the distribution of the magnetic field along the z coordinate in each of the three regions of the structure can be written in the form:

$$H_y(z) = \begin{cases} H_0\exp[-q_1'(z-d/2)]\exp[\mathrm{i}q_1''(z-d/2)], & z > d/2, \\ B[\cosh(q_2''z) + \mathrm{i}\sinh(q_2''z)]\cos(q_2'z) + C[\cosh(q_2''z) & \\ \quad - \mathrm{i}\sinh(q_2''z)]\sin(q_2'z), & |z| < d/2, \\ D\exp(q_3'z)\exp(-\mathrm{i}q_3''z), & z < -d/2, \end{cases} \qquad (4)$$

where H_0 is the magnetic field amplitude at the interface $z = d/2$. The distribution of the tangential component of the wave electric field E_x is found from Eqns (3) with allowance for (4). Using the continuity conditions for the tangential components of the wave field at the interfaces $z = \pm d/2$, we arrive at a system of two equations relating the parameters B, C and D with the amplitude H_0 [15]. Equating the determinant of this system to zero, we obtain the dispersion equation for SPPs in the structure under consideration [5]:

$$\exp(-2q_2d) = \frac{q_2\varepsilon_1 + q_1\varepsilon_2}{q_2\varepsilon_1 - q_1\varepsilon_2}\,\frac{q_2\varepsilon_3 + q_3\varepsilon_2}{q_2\varepsilon_3 - q_3\varepsilon_2}. \qquad (5)$$

In the presence of absorption in the structure, the polariton branches $\omega(\beta')$ represent a single dispersion dependence without forbidden bands in frequency.

Figure 1 shows the dispersion dependences $\omega(\beta')$, which describe the SPP propagation in the structure under study in the spectral interval $\omega_1/2 < \omega < \omega_\mathrm{p}$. The dashed lines indicate the frequencies $\omega_1 = \omega_\mathrm{p}/\sqrt{1 + \varepsilon_1/\varepsilon_0}$ and $\omega_2 = \omega_\mathrm{p}/\sqrt{1 + \varepsilon_3/\varepsilon_0}$, which, without allowance for absorption in the structure, are asymptotes for low- and high-frequency modes. These frequencies are determined by the plasma frequency of the semiconductor, $\omega_\mathrm{p} \approx 3.42\times 10^{13}$ s^{-1}, the value of $\varepsilon_0 = 13.18$ and the permittivities of the adjacent layers. For given material parameters, $\omega_1 = 2.54\times 10^{13}$ s^{-1} and $\omega_2 = 3.27\times 10^{13}$ s^{-1}. Calculations are performed for a film with a thickness $d = 0.1$ μm with the parameter $\nu/\omega_\mathrm{p} = 0.005, 0.05, 0.02$ and 0.1. Analogous values of this parameter were used in a numerical analysis, for example, in work [16, 17]. Curve (*3*) shows the critical frequency $\omega_\mathrm{cr} = 3.03\times 10^{13}$ s^{-1}, which corresponds to the minimum of the dependence $\beta'(\omega)$ for $\nu = 0.02\omega_\mathrm{p}$. The critical frequency depends on the permittivities of all media, the dissipative losses and the thickness of the waveguide layer.

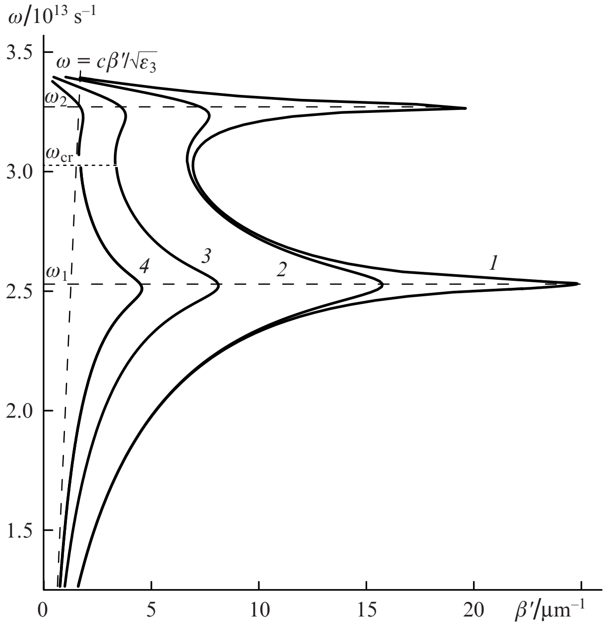

Figure 1. Dispersion curves $\omega(\beta')$ of SPPs in the semiconductor film–dielectric structure for the parameter ν/ω_p = (*1*) 0.005, (*2*) 0.05, (*3*) 0.02 and (*4*) 0.1. The straight line $\omega = c\beta'/\sqrt{\varepsilon_3}$ shows the boundary of the domain of SPP existence in the structure.

Using the dispersion relation (5), let us analyse the group velocity, $v_\mathrm{g} = \mathrm{d}\omega/\mathrm{d}\beta'$ (Fig. 2a), and the phase velocity, $v_\mathrm{ph} = \omega/\beta'$, (Fig. 2b) of SPPs in the investigated frequency range. From the dependences shown in Fig. 2a, it is seen that the group velocity in the region $\omega < \omega_1$ decreases with increasing frequency, its sign being positive. In the intermediate region $\omega_1 < \omega < \omega_2$, there can exist SPPs with both positive and negative group velocities, with the sign changing near the critical frequencies ω_cr that correspond to the minimum of the dependences $\beta'(\omega)$ for each value of the parameter ν. In the frequency range $\omega_2 < \omega < \omega_\mathrm{p}$, the group velocity assumes only negative values.

In the region of negative values of the group velocity, the existence of inverse SPPs becomes possible, for which the direction of the total flux of the transferred energy is opposite to the direction of the phase velocity. In this region, the waveguide structure in question can be considered as an effective 'left-handed' medium. Thus, the condition $v_\mathrm{g}(\omega) < 0$ makes it possible to realise both amplification and generation of SPPs in this structure, operating on the principle of a directional coupler.

It can be seen from the dependences shown in Fig. 2b that near the frequencies ω_1 and ω_2 the phase velocity of SPPs is substantially reduced. With minimal losses, it is possible to

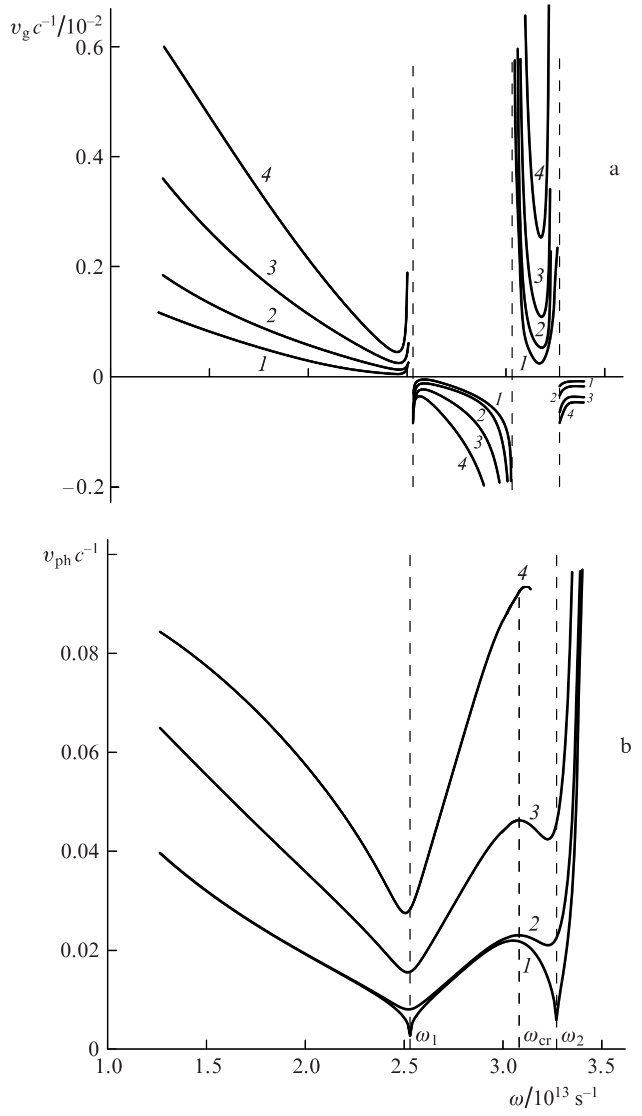

Figure 2. Frequency dependences of (a) group and (b) phase velocities of SPPs for the parameter ν/ω_p = (*1*) 0.005, (*2*) 0.05, (*3*) 0.02 and (*4*) 0.1.

reduce it by more than two orders of magnitude compared to the speed of light in vacuum. Thus, at the frequency ω_1 for $\nu = 0.005\omega_p$, the phase velocity v_{ph} of SPPs can reach values less than $0.01s$ [curve (*1*)].

3. Amplification of SPPs by the drift current

Let us investigate the possibility of SPP-wave amplification in a semiconductor film–dielectric structure when this wave interacts with a flux of charged particles, which is a constant drift current I_0 in a semiconductor film. For their effective interaction and, as a consequence, effective transfer of the energy from the current wave to the SPP wave, it is necessary to ensure the synchronisation of the SPP phase velocity v_{ph} with the charge-carrier drift velocity v_0 [13, 18]. As was shown above, in the spectral regions close to the frequencies ω_1 and ω_2, the SPP phase velocity is reduced in the planar structure to values of the order of or less than $10^{-2}\,c$. On the other hand, also known are a number of materials, which are characterised by high charge-carrier drift velocities reaching $\sim 10^{-3}\,c$. Such materials include high-temperature superconductors [19], graphene [20], and certain types of semiconductor materials [21, 22]. For submicron structures based on gallium arsenide, the dependence of the mobility μ of charge carriers on the temperature T_{ex} and the concentration n [23] has the form:

$$\mu\left(T_{ex}, n\right) \approx \mu_0 \left\{ \left(\frac{T_{ex}}{T_0}\right)^2 \left[1 + \left(\frac{T_{ex}}{5T_0}\right)^6\right] + \frac{n}{n_0} \right\}^{-1/4}, \tag{6}$$

where $\mu_0 = 8 \times 10^3$ cm^2 V^{-1} s^{-1} is the mobility of the undoped sample; $T_0 = 300$ K; and $n_0 = 10^{16}$ cm^{-3}. Thus, for $n = 10^{16}$ cm^{-3} and $T_{ex} = 300$ K, the mobility is $\mu \approx 6600$ cm^2 V^{-1} s^{-1}. In this case, the carrier drift velocities $v_0 = \mu U/L$ (U is the accelerating potential difference and L is the film length) are limited by the threshold value of voltage saturation [23], which in turn is limited by the maximum value of mobility at a given concentration and temperature, and varies in the interval $(1-5) \times 10^5$ m s^{-1}.

Thus, in spite of large drift velocities of the carriers in some materials, it is not possible to satisfy exactly the phase matching condition between the electromagnetic and current waves. Nevertheless, in order to provide conditions for the energy transfer, it is sufficient that the drift current wave velocity and the SPP phase velocity be at least of the same order. The efficiency of such a process is the higher, the smaller the detuning of the phase velocity from the drift velocity.

Due to the interaction of the drift current wave and the SPP wave, there appear a spatial modulation of the amplitude-constant current I_0, i.e., there arises a modulated current wave $I(x)$. Their relationship is established by the hydrodynamic equation for charge carriers [12] and the guidance theorem [24]:

$$\frac{d^2\tilde{I}}{dx^2} + 2i\frac{\omega}{v_0}\frac{d\tilde{I}}{dx} - \frac{1}{v_0^2}(\omega^2 - \omega_q^2)\tilde{I} = i\frac{\omega}{v_0}\frac{I_0}{2U}E_x, \tag{7}$$

$$\frac{dE_x}{dx} + i\frac{\omega}{v_{ph}}E_x = -\frac{\omega^2}{2v_{ph}^2}KI(x). \tag{8}$$

Here, $\tilde{I}(x) = I(x) - I_0$ is the variable component of the current; ω_q is the so-called reduced plasma frequency; and $U = m^* v_0^2/(2e)$. The coefficient $K = |E_x|^2/(2\beta^2 P)$, called the coupling resistance, characterises the degree of interaction between the field of the SPP wave and the flux of charged particles [24, 25], where

$$P \approx \frac{v_g}{8\pi}\left(\frac{d}{d\omega}\omega\,\mathrm{Re}\,\varepsilon_2\right)\int |E|^2 dS$$

is the power carried in the structure by the surface wave, and $|E|^2 = |E_x|^2 + |E_y|^2 + |E_z|^2$ is the square of the absolute value of the electric field strength of the SPP wave. Taking into account relation (1), the expression for the coupling resistance takes the form:

$$K \approx \frac{4\pi}{v_g \beta'^2} \frac{|E_x|^2}{[d(\omega\,\mathrm{Re}\,\varepsilon_2)/d\omega]\int |E_x|^2 dS}$$

$$\approx \frac{4\pi v_{ph}^2}{(\omega^2 + \omega_p^2)v_g} \frac{|E_x|^2}{\int |E_x|^2 dS}. \tag{9}$$

The simultaneous solution of equations (7) and (8) leads to the dispersion relation

$$(\omega - Gv_{\rm ph})\big[(\omega - Gv_0)^2 - \omega_{\rm q}^2\big] = C^3\omega^3, \qquad (10)$$

where $G = G' - {\rm i}G''$ is the complex growth rate of the harmonic disturbance, and the parameter $C = [v_0 K I_0/(4v_{\rm ph}U)]^{1/3}$ is analogous to the Pierce gain parameter used in microwave technology [26]. Equation (10) allows one to determine the SPP gain in the structure under consideration:

$$g = 2\,|\,{\rm Im}\,G\,|. \qquad (11)$$

In the case of interaction of the delayed SPP and the space-charge wave with allowance for the transformations for the coupling resistance [15], the parameter C can be represented in the form

$$C \approx \left(\frac{\eta}{2}\frac{v_{\rm ph}}{|v_{\rm g}|}\frac{\omega_{\rm p}^2}{\omega^2 + \omega_{\rm p}^2}\right)^{1/3}, \qquad (12)$$

where $\eta = |\,E_x|^2/|\,E\,|^2$ is the ratio of the moduli of the longitudinal component and the total electric field of the SPP. The parameter C is determined from the relationship between the plasma frequency of the semiconductor and the frequency of the SPP, as well as by the ratio of the phase and group velocities of the SPP. The analysis shows that throughout the entire spectral range under study the Pierce parameter is virtually independent of the film thickness, except for regions of frequencies close to the characteristic frequencies ω_1 and ω_2. As we approach these spectral regions, we observe a sharp increase in the parameter C (to values of the order of 10^3), to which the gain maxima $g(\omega)$ correspond.

Note that in the theory of a travelling-wave tube, the reduced frequency $\omega_{\rm q}$ is related to the plasma frequency $\omega_{\rm p}$ by the relation $\omega_{\rm q} = r\omega_{\rm p}$, where the coefficient r takes into account the influence of the surrounding walls on the electron beam and is in the range $0 < r < 1$ [27]. For a sufficiently wide electron flux, whose dynamics can be neglected by the influence of the lateral surface, the coefficient r tends to zero. In the general case, the reduced plasma frequency is nonzero and is determined by the concentration of charge carriers in the bulk of the waveguide structure.

Figure 3 shows the frequency dependence of the SPP gain $g(\omega)$ for low-frequency (Fig. 3a) and high-frequency (Fig. 3b) modes, obtained on the basis of the solution of Eqn (10), taking into account expressions (9) and 12). These dependences were calculated for a film with a thickness d = 0.1 μm, a parameter $v = 0.02\omega_{\rm p}$, and a reduced frequency $\omega_{\rm q}/\omega_{\rm p}$ = 0, 0.5 and 1.0. It is seen that only at $\omega_{\rm q} = 0$ the gain is realised in the entire spectral interval [curve (*1*)]. The dip at the frequency ω_1 is due to the fact that at this frequency $v_{\rm g} \to \infty$ and, according to (10), the Pierce parameter $C \to 0$ (despite a significant decrease in the SPP phase velocity). The shape of the curves in the region $\omega_1 < \omega < \omega_2$ is determined by the form of the dispersion dependences in the same region (when the SPP group velocity changes its sign from positive to negative). The analysis shows that the gain [$g(\omega)$ prevails over the losses [$\beta''(\omega)$] for all the considered values of the parameter v at all frequencies except for the interval $\omega_1 < \omega < \omega_2$. An increase in the reduced frequency $\omega_{\rm q}$ leads to the fact that the gain for each fixed SPP frequency takes on smaller values, and the spectral regions of the SPP gain become narrower [curves (*2*) and (*3*)].

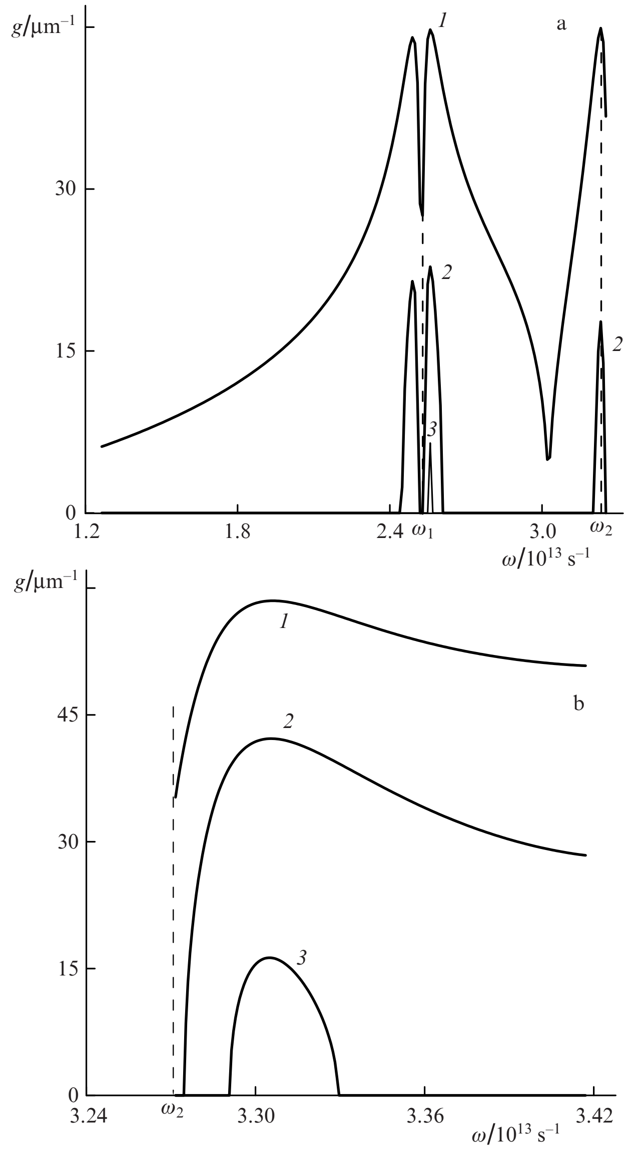

Figure 3. Frequency dependences of the gain $g(\omega)$ for $v = 0.02\omega_{\rm p}$ and the reduced frequency $\omega_{\rm q}/\omega_{\rm p}$ = (*1*) 0, (*2*) 0.5 and (*3*) 1.0.

Note also that the thickness of the semiconductor film significantly affects both the spectral intervals (in which SPPs are amplified) and the attainable values of the gain at a fixed frequency. The frequency dependences $g(\omega)$ shown in Fig. 4 were obtained with the parameter $v = 0.02\omega_{\rm p}$ and the reduced frequency $\omega_{\rm q} = 0$ for film thicknesses d = 0.1, 1.0 and 3.0 μm. It is seen that for d = 0.1 μm, the gain is realised for all the considered SPP frequencies [curve (*1*)], with the maximum possible gain $g \approx 50$ μm^{-1} being reached near the characteristic frequencies ω_1 and ω_2. For large film thicknesses, the dependences $g(\omega)$ will have a similar character [curves (*2*, *3*)]. Note that there is no amplification in the frequency range $\omega_{\rm cr} < \omega < \omega_2$ for thicknesses d = 1.0 and 3.0 μm, and the maximum gains are several times smaller than for d = 0.1 μm.

Thus, in the spectral intervals $\omega < \omega_1$ and $\omega_2 < \omega < \omega_{\rm p}$, a gain, which is much higher than the attenuation coefficient, is attained for the SPP wave. Moreover, in the region $\omega_2 < \omega < \omega_{\rm p}$ the following conditions are satisfied: $|v_{\rm ph}/v_{\rm g}| \gg 1$ and $v_{\rm g} < 0$, which make it possible to realise an SPP generator, which is an analogue of a backward-wave tube, with a Peirce parameter several orders of magnitude higher than that for a microwave counterpart.

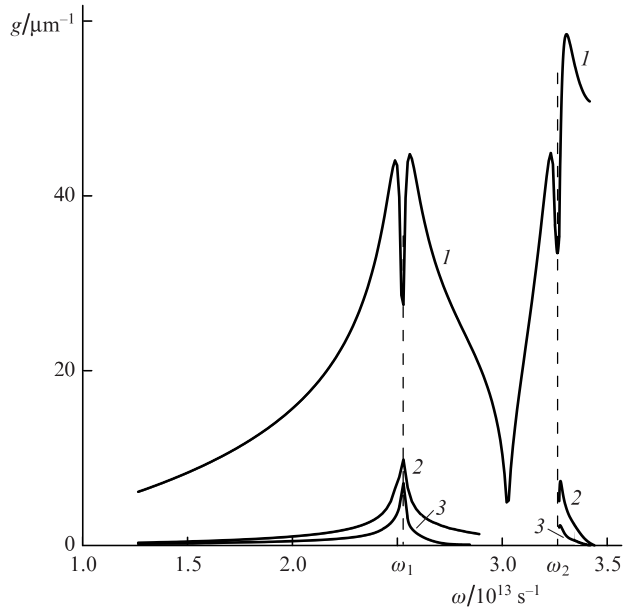

Figure 4. Frequency dependences of the gain $g(\omega)$ for $v = 0.02\omega_p$ at a semiconductor film thickness $d =$ (*1*) 0.1, (*2*) 1.0 and (*3*) 3.0 μm. The reduced frequency is $\omega_q = 0$.

4. Equations of coupled waves and their solutions

Note that in the frequency regions $\omega_1 < \omega < \omega_{cr}$ and $\omega_2 < \omega < \omega_p$, where the existence of backward polariton waves with $v_g < 0$ is possible, the structure under study will play the role of a directional coupler, and positive feedback is established between the backward SPP wave and the direct current wave. Let us now derive the equations describing the coupling of the backward SPP waves with space-charge waves. The consideration will be performed within the undepleted current pump approximation.

We will assume that a drift current wave propagating in a film provides an amplification of SPPs with a sufficiently high gain g. By analogy with (8), the equation describing the effect of the current wave on the backward plasmon polariton wave with $E_x = E$, taking into account the losses and amplification, is written in the form

$$-\frac{\mathrm{d}E^-}{\mathrm{d}x} + \mathrm{i}\frac{\omega}{v_{\mathrm{ph}}}E^- = -\frac{\omega^2}{2v_{\mathrm{ph}}^2}KS_{\mathrm{eff}}\,j(x) + \gamma E^-, \qquad (13)$$

where $\gamma = g - \beta''$; $j(x) = I(x)/S_{\mathrm{eff}}$ is the current density; and

$$S_{\mathrm{eff}} = \left(\int |E|\,\mathrm{d}y\mathrm{d}z\right)^2 \Big/ \int |E|^2\mathrm{d}y\mathrm{d}z$$

is the effective area of the surface mode. The right-hand side of (13) takes into account the effect of both the modulated current wave $j(x)$ and the drift current wave γE^- on SPPs. In this case, the relation between the current wave and the SPP field is given by the equation

$$\frac{\mathrm{d}j}{\mathrm{d}x} + \mathrm{i}\frac{\omega}{v_0}j = \frac{\omega_p^2}{4\pi v_0}E^- + \gamma j. \qquad (14)$$

The system of equations (13) and (14) describes the coupling between the forward wave of the drift current along the x axis and the backward SPP wave, which can be represented in the form

$$j(x) = \sigma E_+(x)\exp(-\mathrm{i}\beta_0 x),\quad E^-(x) = E_-(x)\exp(\mathrm{i}\beta_{\mathrm{ph}}x), \qquad (15)$$

where $\beta_0 = \omega/v_0$; $\beta_{\mathrm{ph}} = \omega/v_{\mathrm{ph}}$; and σ is the conductivity of the medium. Substituting these expressions into Eqns (13) and (14), we arrive at the following system of equations for the amplitudes of the forward (E_+) and backward (E_-) SPP waves in the perturbed waveguide regiona $0 \leqslant x \leqslant L$ [28]:

$$\begin{aligned} \frac{\mathrm{d}E_+}{\mathrm{d}x} &= \frac{v_2}{\sigma}E_-\exp(2\mathrm{i}\Delta\beta x) + \gamma E_+, \\ \frac{\mathrm{d}E_-}{\mathrm{d}x} &= v_1\sigma E_+\exp(-2\mathrm{i}\Delta\beta x) - \gamma E_-, \end{aligned} \qquad (16)$$

where

$$v_1 = \frac{2\pi}{v_g}\frac{\omega^2}{\omega^2 + \omega_p^2}\eta;\quad v_2 = \frac{\omega_p^2}{4\pi v_0};\quad 2\Delta\beta = \beta_{\mathrm{ph}} + \beta_0.$$

We assume that the forward wave with amplitude $E_+(0)$ is fed to the left boundary of the waveguide $x = 0$, and there is no backward wave on the right boundary $x = L$, i.e., $E_-(0) = 0$. In this case, the distributions of the fields of forward and backward waves along the waveguide length have the form

$$E_+(x)\exp(-\mathrm{i}\Delta\beta x) =$$

$$E_+(0)\frac{(\gamma - \mathrm{i}\Delta\beta)\sinh[p(L-x)] + p\cosh[p(L-x)]}{p\cosh(pL) + (\gamma - \mathrm{i}\Delta\beta)\sinh(pL)}, \qquad (17)$$

$$E_-(x)\exp(\mathrm{i}\Delta\beta x) = E_+(0)\frac{\sqrt{v_1 v_2}\sinh[p(L-x)]}{p\cosh(pL) + (\gamma - \mathrm{i}\Delta\beta)\sinh(pL)},$$

where $p = \sqrt{v_1 v_2 + (\gamma - \mathrm{i}\Delta\beta)^2}$. Taking into account the solutions obtained, the expressions for the reflection and transmission coefficients for the forward and backward SPP waves take the form:

$$\begin{aligned} R &= \left|\frac{E_-(0)}{E_+(0)}\right|^2 = \left|\frac{\sqrt{v_1 v_2}\sinh(pL)}{p\cosh(pL) + (\gamma - \mathrm{i}\Delta\beta)\sinh(pL)}\right|^2, \\ T &= \left|\frac{E_+(L)}{E_-(0)}\right|^2 = \left|\frac{p}{p\cosh(pL) + (\gamma - \mathrm{i}\Delta\beta)\sinh(pL)}\right|^2. \end{aligned} \qquad (18)$$

The SPP generation condition can be obtained from the equality of the denominators of expressions (18) to zero:

$$\arctan\left(\frac{\Delta\beta}{\gamma}\right) + \Delta\beta L\left(\frac{v_1 v_2}{\gamma^2 + \Delta\beta^2} - 1\right) = (l + 1/2)\pi, \qquad (19)$$

where l is an integer.

Figure 5 shows the frequency dependences of the transmission and reflection coefficients for the structure under consideration in the regime of coupled-wave propagation (i.e., in the frequency regions where propagation of the SPPs with negative group velocity is possible). The thickness d of the semiconductor film is assumed to be 0.1 μm, the working region length is $L = 1$ and 10 μm, and the parameter v is equal to $0.02\omega_p$. An analysis of these dependencies shows that with the chosen calculation parameters, the reflection (R) and transmission (T) coefficients at the frequency ω_1 tend to infinity [curve (*2*)], i.e., the generation is obtained in the structure due to the distributed feedback. A similar mechanism for 'triggering' generation is well known for distributed-feedback lasers [28].

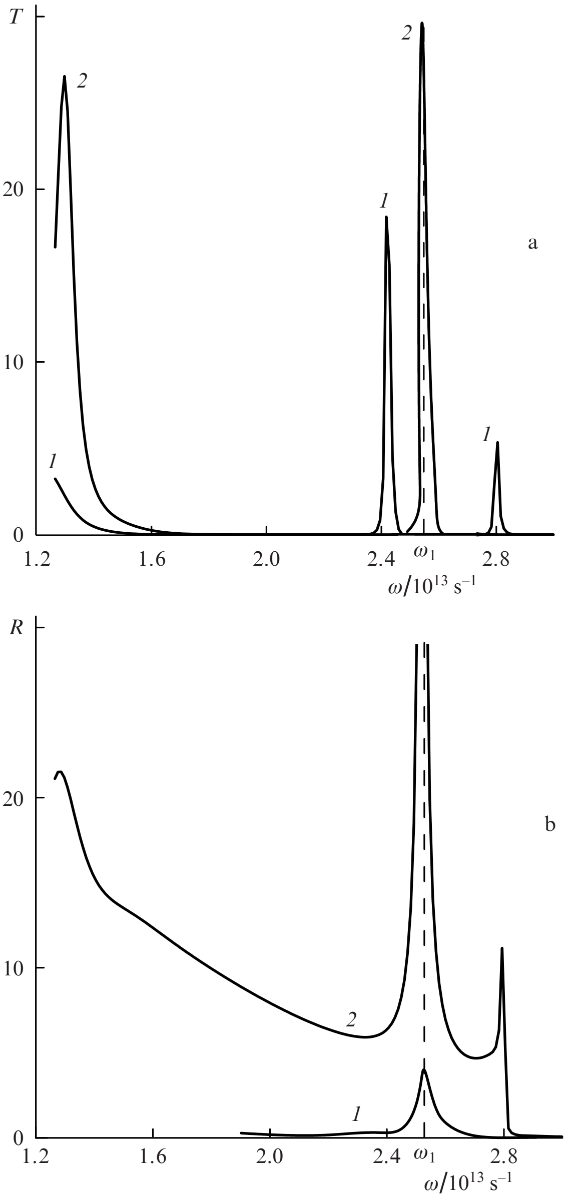

Figure 5. Frequency dependences of the gains on (a) transmission and (b) reflection coefficients for a film with a length L = (*1*) 1 and (*2*) 10 μm.

Note that the structure in question can also operate in the amplification regime both in the reflected signal and in the transmitted signal in the spectral interval $\omega_1 < \omega < \omega_{cr}$.

5. Conclusions

We have shown the possibility of amplifying SPP waves of the THz range when they interact with the drift current under phase-matching conditions and within the undepleted pump approximation. The proposed amplification mechanism does not require the presence of active media and realises direct transfer of the energy of the current wave to the surface electromagnetic wave. With the chosen material parameters of the dielectrics and the semiconductor film in the operating frequency range, the gain can reach values above 10^6 m^{-1}, which is much larger than the damping decrement due to the dissipative losses in the semiconductor.

It is also shown that backward plasmon-polariton waves (waves with negative group velocities) of the THz frequency range can be excited in the spectral intervals $\omega_1 < \omega < \omega_{cr}$ and $\omega_2 < \omega < \omega_p$. Thus, the film structure acts as a slowing-down system, in which positive feedback is simultaneously realised.

A scheme is proposed for generating electromagnetic radiation due to the interaction of the drift current and the surface electromagnetic wave. The scheme is constructed on the same physical principles that underlie the work of the travelling-wave tube well known in microwave technology [12] and a distributed-feedback laser [28]. At the same time, in the proposed generation scheme, a thin semiconductor film rather than a typical diffraction grating acts as a slowing-down system.

It should be noted that in the considered frequency interval $\omega \approx (2-3)\times10^{13}$ s^{-1}, to which the emission wavelengths λ_0 of order of 100 μm correspond, the SPP propagation constant β' exceeds the wavenumber in the free space, $\omega/c = 2\pi/\lambda_0$, approximately by two orders of magnitude, taking on values close to 10 μm^{-1}. Such large values of the propagation constant correspond to the region of SPP localisation in the transverse direction $1/\beta'$ of the order of 100 nm, which indicates the possibility of using the proposed generator in optoelectronics devices. Let us pay attention to the following important circumstance: SPP waves with such a large propagation constant cannot be generated using classical excitation schemes (using optical prisms in Otto or Kretschman geometries) in structures whose longitudinal dimension is much larger than the characteristic wavelength λ_0 of radiation. Excitation of SPP waves by electron beams [29, 30] is not considered here, since such a method is not technological and can hardly be of interest for practical applications in integrated optoelectronics.

Thus, the results obtained in the present study indicate that the SPP waves can arise in the planar structure as a result of generation in the amplifying medium with distributed feedback of the forward and backward waves.

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References

1. Agranovich V.M., Mills D.L. *Surface Polaritons* (Amsterdam–New York–Oxford: North-Holland Publishing Company, 1982).
2. Economou E.N. *Phys. Rev.*, **182**, 539 (1969).
3. Dmitruk N.L., Litovchenko V.G., Strizhevskii V.L. *Poverkhnostnye polyaritony v poluprovodnikakh i dielektrikakh* (Surface Polaritons in Semiconductors and Dielectrics) (Kiev: Naukova Dumka, 1989) p. 25.
4. Martin B.G., Broerman J.G. *Phys. Rev. B*, **24**, 2018 (1981).

5. Berini P., De Leon I. *Nat. Photonics*, **6**, 16 (2012).
6. Moiseev S.G., Korobko D.A., Zolotovskii I.O., Fotiadi A.A. *Ann. Physik*, **529**, 1600167 (2016).
7. Davidovich M.V. *Quantum Electron.*, **47**, 567 (2017) [*Kvantovaya Elektron.*, **47**, 567 (2017)].
8. Leosson K. *J. Nanophotonics*, **6**, 061801 (2012).
9. Maier S.A. *Opt. Commun.*, **258**, 295 (2006).
10. Fedyanin D.Yu., Arsenin A.V., Leiman V.G., Gladun A.D. *Quantum Electron.*, **39**, 745 (2009) [*Kvantovaya Elektron.*, **39**, 745 (2009)].
11. Fedyanin D.Yu., Arsenin A.V. *Opt. Express*, **19**, 12524 (2011).
12. Rabinovich M.I., Trubetskov D.I. *Vvedenie v teoriyu kolebanii i voln* (Introduction to The Theory of Oscillations and Waves) (Kirov: Izd-vo GosUNTs ‘Kolledzh’, 2000) p. 87.
13. Kadochkin A.S., Moiseev S.G., Dadoenkova Y.S., Svetukhin V.V., Zolotovskii I.O. *Opt. Express*, **25**, 27165 (2017).
14. Lao Y.-F., Unil Perera A.G. *J. Appl. Phys.*, **109**, 103528 (2011).
15. Abramov A.S., Zolotovskii I.O., Sementsov D.I. *Opt. Spektrosk.*, **119**, 157 (2015).
16. Sannikov D.G., Sementsov D.I. *Fiz. Tverd. Tela*, **55**, 2212 (2013).
17. Esaev D.G., Matsik S.G., Rinzan M.B., Perera A.G. *J. Appl. Phys.*, **93**, 1879 (2003).
18. Bezruchko B.P., Kuznetsov S.P., Trubetskov D.I. *JETP Lett.*, **29**, 162 (1979) [*Pis’ma Zh. Eksp. Teor. Fiz.*, **29**, 180 (1979)].
19. Basov D.H., Timusk T. *Rev. Mod. Phys.*, **77**, 721 (2005).
20. Buslaev P.I., Iorsh I.V., Shadrivov I.V., Belov P.A., Kivshar Yu.S. *JETP Lett.*, **97**, 535 (2013) [*Pis’ma Zh. Eksp. Teor. Fiz.*, **97**, 619 (2013)].
21. Gad S.A., Boshta M., Moustafa A.M., Abo El-Soud A.M. *Solid State Sci.*, **13**, 23 (2011).
22. Lakhno V.D. *Phys. Usp.*, **39**, 669 (1996) [*Usp. Fiz. Nauk*, **166**, 717 (1996)].
23. Gerkel’ V.A., Kulkova E.Yu., Mokerov V.G., Timofeev M.V., Khrenov G.Yu. *Fiz. Tekh. Popluprovodn.*, **36**, 496 (2002).
24. Trubetskov D.I., Khramov A.E. *Lektsii po SVCh elektronike dlya fizikov* (Lectures on Microwave Electronics for Physicists) (Moscow: Fizmatlit, 2003) Vol. 1, p. 85.
25. Clampitt L.L. (Ed.) *Proc. IEEE (Special Issue on High-Power Microwave Tubes)*, **61** (1973).
26. Pierce J. *Electrons and Waves* (New York: Anchor Books, 1964).
27. Shevchik V.N., Shvedova G.N., Soboleva A.V. *Volnovye i kolebatel’nye yavleniya v elektronnykh potokakh na sverkhvysokikh chastotakh* (Wave and Vibrational Phenomena in Electron Fluxes at Ultrahigh Frequencies) (Saratov: Izd-vo Saratovskogo Univer., 1962).
28. Yariv A. *Quantum Electronics* (New York: Wiley, 1975; Moscow: Sov. Radio, 1980).
29. El’chaninov A.A., Korovin S.D., Rostov V.V., Pegel’ I.V., Mesyats G.A., Yalandin M.I., Ginzburg N.S. *JETP Lett.*, **30**, 719 (2003) [*Pis’ma Zh. Eksp. Teor. Fiz.*, **77**, 314 (2003)].
30. Baryshev V.R., Ginzburg N.S. *Quantum Electron.*, **41**, 776 (2011) [*Kvantovaya Elektron.*, **41**, 776 (2011)].