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IMAGE RECOGNITION

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Analysis of contour images using optics of spiral beams

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Abstract. **An approach is outlined to the recognition of contour images using computer technology based on coherent optics principles. A mathematical description of the recognition process algorithm and the results of numerical modelling are presented. The developed approach to the recognition of contour images using optics of spiral beams is described and justified.**

Keywords: *spiral light beams, recognition of contour images.*

1. Introduction

The problem of pattern recognition is not new: It arose in the middle of the 20th century due to the development of computer facilities processing various signals. Initially, these were analogue devices and accompanying signals in the form of currents and voltages, and with the development of computers and communication equipment it became possible to transmit large amounts of information in a way more natural for people: presently, sound and video are transmitted in digital format. The growth of the number of signals and the complication of their structure naturally cause the problem of their classification for processing on a computer.

In the present work, the field of research is narrowed down to the problem of 'computer vision', i.e. the branch of the computer pattern recognition theory involving the processing of graphic information obtained from arbitrary recording devices. Within this branch, there exist numerous approaches to image recognition, each of which has its own limits of applicability. These limits must be taken into account when formulating specific requirements of the technical task to the hardware/software systems for image recognition. If one needs to determine whether a red vehicle is currently in the field of view of a particular video camera or not, it is enough to analyse histograms of colour distribution and linear dimensions of a fixed object, which is solved using fairly trivial mathematical algorithms [1]. If the recognition problem is not so trivial (the number of recognisable objects is large and they have a heterogeneous structure), then it is necessary to use more complex procedures.

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Thus, there appears a branch of computer vision, i.e. a contour analysis, within which contours of object edges are selected for the recognition process. This branch is sufficiently well developed; we should note two key approaches to the analysis of contours based on the idea of using correlation functions [2] and neural networks [3]. We propose a different approach to the processing of contours, based on coherent optics principles.

This work continues our research on the possibility of using spiral light beams to recognise contour images. Our previous papers [4, 5] reflect fragmentary but nonetheless important results. In this paper, we present a structured description of the theoretical component of the study, involving coherent optics, mathematical physics, theory of functions of a complex variable, and functional analysis.

Spiral beams as an object of modern coherent optics were first considered in [6, 7] as a solution to the equation of propagation of electromagnetic radiation (the Leontovich–Fock equation) in the paraxial approximation for a laser light source. It was noted that such solutions are a special case of light fields with dislocations of the wavefront described in the seminal work of Berry and Nye [8]. Dislocations are understood to mean points of space in which the complex amplitude of the corresponding field vanishes, or, in other words, points at which the phase is not determined and has a screw structure in the vicinity of these points. Light fields with wavefront dislocations are also commonly called fields with a topological charge. A number of papers have been devoted to the study of such objects of coherent optics, among which one can single out the works of Zel'dovich's school [9]. In our work, spiral beams are fields, all dislocations of which have the same sign (positive or negative).

2. Problems of pattern recognition

The process of object recognition begins with the preliminary processing of the image, namely with the detection of contours or edges and their subsequent parameterisation. Figure 1 shows an example of the contour detection for the image of a ship.

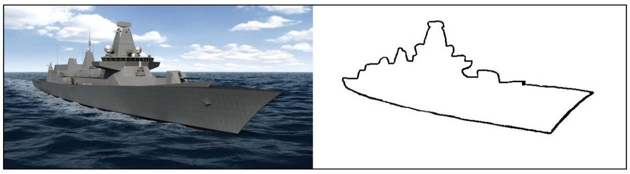


Figure 1. Image of a ship and the detected contour.

It is assumed that contour detection and recognition are different tasks, and therefore to detect the boundaries of objects, it is necessary to use existing methods, for example Sobel filters and Canny edge detector [1]. If more than one contour is obtained during the detection, the result of the recognition of the image as a whole is a set of results of recognition of individual contours on it. Let us describe the process of recognition for the case of a single contour, since the adoption of a general solution to the problem of a set of particular solutions for individual contours is determined by other algorithms that do not directly affect the recognition process.

It is natural to consider the mathematical representation of contours as closed plane curves consisting of an ordered set of points:

$$\zeta(t) = x(t) + \mathrm{i}y(t), \quad t \in [0, T]. \tag{1}$$

Figure 2 shows an airplane image, detected contour and its corresponding parameterisation. In this case, the recognition problem can be restated as follows: Do the two plane curves in question correspond to each other with respect to some similarity criterion?

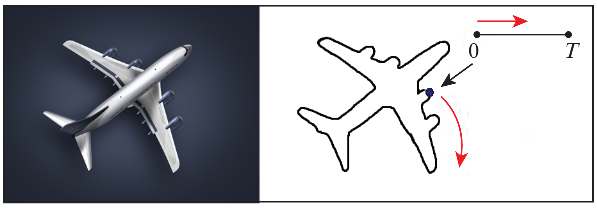
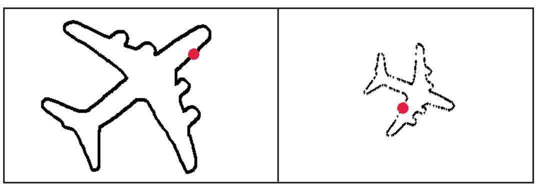


Figure 2. Source image and the parameterised contour.

In his seminal work, Furman [2] mentions four classical problems that are encountered in the context of contour analysis: uncertainty of the choice of the starting point on the contour, uncertainty in determining the mutual rotation and scale, and noises. This situation is clearly illustrated in Fig. 3, where the enlarged point is the starting point on the contour.



a b

Figure 3. (a) Reference from database and (b) input recognisable contour.

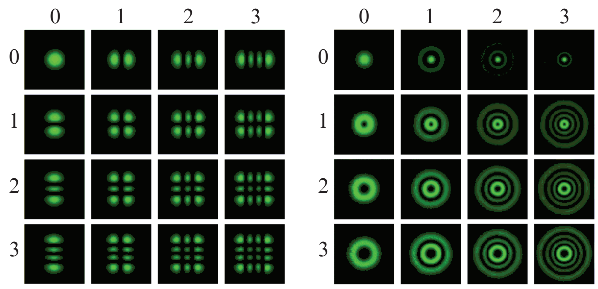
To solve the recognition problem, Furman [2] proposed a routine search for the maximum of a certain likelihood functional of four variables $\Lambda(N, \Delta\varphi, d, |\mu|)$. Why do we have to use complex mathematical objects? The answer is simple: traditional tools for analysing functions of form (1) do not cope with the problem in the formulated conditions of uncertainty. For example, the curve $\zeta(t)$, generally speaking, does not have to be differentiable, which prevents its analysis with the help of Taylor series and the like. In addition, the expansion in a series, even if it exists, for one-dimensional functions depends essentially on the choice of the starting point (see, for example, a similar dependence of the Fourier series expansion coefficients).

3. Optics of spiral light beams

The theory of coherent light fields, called spiral light beams, was developed in [7]. In the general case, the complex amplitudes F of these fields are solutions to the parabolic Leontovich–Fock equation in the paraxial approximation:

$$\frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} + 2\mathrm{i}k\frac{\partial F}{\partial l} = 0, \tag{2}$$

where $(x, y, l) \in \mathbb{R}^3$ is a three-dimensional space (l is the propagation axis of the laser beam). The solutions to this equation are the orthogonal Hermite–Gaussian and Laguerre–Gaussian modes presented in Fig. 4. The above solutions are self-similar, that is, the light fields have the property of structural stability: the intensity distribution during the propagation of beams in space remains unchanged. The only changing property that does not affect the beam structure is the natural scale change (uniform compression or magnification).



a b

Figure 4. Intensity distributions of (a) Hermite–Gaussian and (b) Laguerre–Gaussian modes.

Spiral light beams appeared as a generalisation of classical fundamental modes in the search for other self-similar solutions to equation (2), which could not only be scaled, but also rotated around the beam propagation axis. Adding a new degree of freedom made it possible to obtain a rich family of light fields with finite energy. Thus, it was found that the intensity distributions of these fields can take the form of an arbitrary plane curve (which corresponds to the case of a single contour from Section 2). The expression for the complex amplitude S of a spiral beam in the form of the curve $\zeta(t)$ has the form

$$S(z, z^* \,|\, \zeta(t), t \in [0, T]) = \exp\left(\frac{-zz^*}{\rho^2}\right) f(z) =$$

$$= \exp\left(\frac{-zz^*}{\rho^2}\right) \int_0^T \exp\left\{-\frac{\zeta(t)\zeta^*(t)}{\rho^2} + \frac{2z\zeta^*(t)}{\rho^2} + \frac{1}{\rho^2}\int_0^t [\zeta^*(\tau)\zeta'(\tau) - \zeta(\tau)\zeta'^*(\tau)]\,\mathrm{d}\tau\right\} |\zeta'(\tau)|\,\mathrm{d}t, \tag{3}$$

where $z = x + \mathrm{i}y$ is a complex variable in the image plane; ρ is the parameter of the Gaussian beam; the prime means a derivative; and an asterisk is the complex conjugate.

The expression for the complex amplitude can be visualised in the form of distributions of intensity $I(z,z^*) = |S(z,z^*)|^2$ and phase $\varphi(z,z^*) = \arg S(z,z^*)$ (Fig. 5). Figure 5 clearly shows the relationship between the contour and the spiral beam intensity and phase distributions: the intensity distribution corresponds to the generating curve, and the regularities of dislocations of the wavefront (zeros of the complex amplitude) can be found in the phase distribution after a detailed analysis.

Figure 5. (a) Generating contour, and distributions of (a) intensity and (c) phase of the spiral beam.

4. Mathematical apparatus of spiral beams

One can draw such an analogy: just as the Fourier transform connects the time and frequency spaces of the signal, transform (3) connects the space of contours with the space of spiral light beams. Transform (3) allows one to transfer the problem of contour detection to a more 'comfortable' space for spiral beam analysis. Let us explain the above thesis. First, the space of spiral beams (hereinafter we will identify the concepts of a light beam as a light field and the complex amplitude that defines it as a function) is a subspace of light fields with finite energy $L_2(\mathbb{R}^2)$, i.e., all complex amplitudes are square integrable. Second, all spiral beams contain entire functions that are complex-differentiable an unlimited number of times everywhere in the complex image plane, which allows us to write them in the form of power series:

$$S(z,z^*) = \exp\left(\frac{-zz^*}{\rho^2}\right)\sum_{n=0}^{\infty} c_n z^n = \sum_{n=0}^{\infty} c_n\left[\exp\left(\frac{-zz^*}{\rho^2}\right)z^n\right]$$

$$= \sum_{n=0}^{\infty} c_n \mathcal{L}_{0n}(z,z^*), \tag{4}$$

where $\mathcal{L}_{0n}(z,z^*)$ are orthogonal fundamental Laguerre–Gaussian modes with a zero first index. Third, spiral beams make it possible to introduce a dependence on the choice of the starting point on the contour. Let a be the parameter responsible for the choice of the starting point on the periodic curve $\zeta(t)$, that is, defining it on the interval $[a, a+T]$ rather than on the interval $[0, T]$. Then the complex amplitudes are related by the expression:

$$S(z,z^*|\zeta(t),\, t\in[a,a+T])$$

$$= S(z,z^*|\zeta(t),\, t\in[0,T])\exp[\mathrm{i}\Delta\phi(a)], \tag{5}$$

i.e., the displacement of the starting point affects only the phase of the spiral beam, which at each point z of the plane receives the same additive $\Delta\phi(a)$, and does not change the intensity distribution [7]. Fourth, let the changes in the scale and rotation of the contour in the recognition plane be described by the transformation $\zeta(t) \to \zeta(t)A\exp(\mathrm{i}\alpha)$ (A is the scale factor, and α is the rotation angle), then the complex amplitudes are related by the expression

$$S(z,z^*|\zeta(t)A\exp(\mathrm{i}\alpha)) = S\left(\frac{z}{A\exp(\mathrm{i}\alpha)}, \frac{z^*}{A\exp(-\mathrm{i}\alpha)}|\zeta(t)\right) \tag{6}$$

when the so-called quantisation condition

$$S_{\mathrm{area}} = \frac{1}{2}\pi\rho^2 N_q, \quad N_{\mathrm{q}} = 0,1,2,... \tag{7}$$

is fulfilled, where S_{area} is the oriented area swept out by the curve $\zeta(t)$; N_{q} is some integer, called the quantisation parameter (for more details, see [3, 4, 7]). Fifth, spiral beams are a particular case of Gaussian beams: the exponential $\exp(-zz^*/\rho^2)$, which determines the degree of smoothness of the spiral beam for the curve $\zeta(t)$, enters into the mathematical expression (3).

The process of changing the spiral beam as a function of the parameter ρ is shown in Fig. 6. The figure clearly demonstrates the adjustment of the spiral beam when the parameter of the Gaussian beam is changed. This useful property can be successfully used to counteract the distorting influence of noise, as well as to solve the problem of assigning a recognisable object to a particular class. The upper row in Fig. 6 makes it possible to determine the contour belonging to the aircraft, and the lower one to a specific instance of transport aviation (for comparison, see Fig. 5).

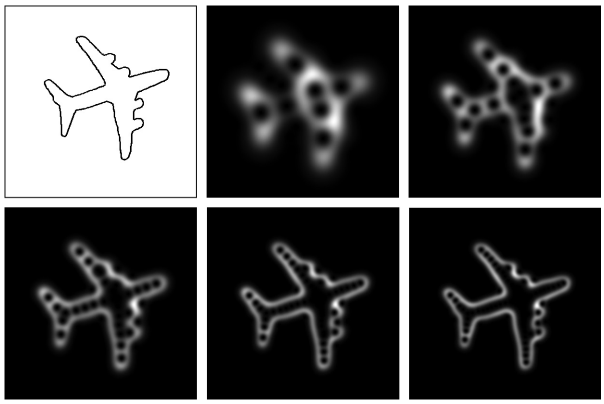


Figure 6. Contour of an aircraft (top left) and the beam intensity distribution with decreasing Gaussian beam parameter ρ.

Thus, the mathematical apparatus of spiral beams allows one to speak of the correctness of the application of the coherent optics principles in the problem of recognising contour images, since it becomes possible to overcome the classical difficulties of the contour analysis described above.

5. Process of contour recognition

Let us introduce some notations, where $\zeta = \zeta(t)$ is the input contour, $\mathbb{K}_{\mathrm{DB}}$ is the reference object database, $\mathbb{K}$ is the set of all contours, $S = S(z,z^*)$ is the spiral beam constructed along the contour ζ, $\mathbb{S}_{\mathrm{DB}}$ is the spiral beam database for the reference objects, and $\mathbb{S}$ is the set of all spiral beams. Thus, the following trivial relations hold:

$$\zeta \in \mathbb{K}, \quad \mathbb{K}_{\text{DB}} \subset \mathbb{K},$$
$$S \in \mathbb{S}, \quad \mathbb{S}_{\text{DB}} \subset \mathbb{S} \subset L_2(\mathbb{R}^2). \tag{8}$$

The classical problem of image recognition consists in setting on the set of all contours $\mathbb{K}$ the metric $\mu_{\mathbb{K}}$, which determines the degree of similarity of the two contours, ζ_1 and ζ_2, with the subsequent calculation of the distance in the given metric from the input contour ζ to the set of reference $\mathbb{K}_{\text{DB}}$ objects. If the computed distance is less than a certain value of ε (ideally $\varepsilon = 0$ corresponds to an exact coincidence), then the fact of recognition is fixed. Existing methods for contour recognition always explicitly (or implicitly) define such metrics, but there is a significant difficulty: it is necessary not only to build a metric, but also to make it maximally invariant to a change in contour characteristics such as the choice of the starting point, scale and rotation, and, if possible, resistance to noise.

In our study, it is proposed to carry out the process of contour recognition by introducing the metric

$$\mu_{\mathbb{S}} : \mathbb{S} \times \mathbb{S} \to [0,1] \tag{9}$$

in the space of spiral beams $\mathbb{S}$ rather than by introducing the metric $\mu_{\mathbb{K}}$ in the space of contours $\mathbb{K}$, because the complex amplitudes have a number of useful properties (see Section 4), which in the above terms can be interpreted as invariants of some promising metric $\mu_{\mathbb{S}}$. Let us construct this metric.

As a basis, we will use the overlap coefficient Π, which is defined as follows:

$$\Pi = \frac{\iint_{\mathbb{R}^2} S_1(z,z^*) S_2^*(z,z^*) \,\mathrm{d}x\mathrm{d}y}{\left(\iint_{\mathbb{R}^2} |S_1(z,z^*)|^2 \mathrm{d}x\mathrm{d}y\right)^{1/2} \left(\iint_{\mathbb{R}^2} |S_2(z,z^*)|^2 \mathrm{d}x\mathrm{d}y\right)^{1/2}}. \tag{10}$$

In essence, this expression is the normalised scalar product of two functions, $S_1(z,z^*)$ and $S_2(z,z^*)$, in the Hilbert space of light fields with finite energy $L_2(\mathbb{R}^2)$. Since in our problem the contours can be rotated relative to each other in the recognition plane, the complex amplitudes will also be 'rotated' in the space of the spiral beams. We extend the classical concept of the overlap coefficient of light fields and introduce the function

$$\Pi(\theta) = \frac{\iint_{\mathbb{R}^2} S_1(z,z^*) S_2^*(z\exp(-\mathrm{i}\theta), z^*\exp(\mathrm{i}\theta)) \,\mathrm{d}x\mathrm{d}y}{\left(\iint_{\mathbb{R}^2} |S_1(z,z^*)|^2 \mathrm{d}x\mathrm{d}y\right)^{1/2} \left(\iint_{\mathbb{R}^2} |S_2(z,z^*)|^2 \mathrm{d}x\mathrm{d}y\right)^{1/2}} \tag{11}$$

by adding the variable θ, which is responsible for the rotation of one spiral beam relative to the other. Since the function $\Pi(\theta)$ is, as before, a normalised complex scalar product, the values of its modulus $|\Pi(\theta)|$ exactly fall into the interval $[0, 1]$.

On the basis of function (11) we give the metric of similarity in the space of spiral beams:

$$\mu_{\mathbb{S}}(S_1, S_2) = 1 - \max_{\theta \in [0,2\pi]} |\Pi(S_1, S_2, \theta)|. \tag{12}$$

Thus, in the present paper it is proposed to identify the metrics $\mu_{\mathbb{K}}$ and $\mu_{\mathbb{S}}$ by using the integral transformation (3) of the contour into a spiral beam.

6. Reduction of the recognition problem to the one-dimensional problem

It should be noted that despite all the advantages, the proposed approach to image recognition has a number of drawbacks. Let us consider them and attempt to eliminate them.

The most important disadvantage follows from the set of advantages that are due to the fact that the recognition process relies on the use of spiral light beams, i.e. objects, generally speaking, of a three-dimensional nature (according to the domain of definition of the corresponding functions). They are given by complex amplitudes – continuous functions in the entire complex plane $\mathbb{C}$. There are a number of issues affecting the realisation of the recognition process on a computer. First, how should one work with continuous functions when designing specialised software for computers? Second, how many spatial (memory) and time resources will the developed recognition algorithms consume? Third, what are the asymptotic estimates of the complexity of such algorithms (since in the general case, two-dimensional algorithms for solving problems are often inferior to one-dimensional ones in the processing rate when the amount of input data is increased)?

We reduce the problem of recognising contours with the help of spiral beams to a one-dimensional one. To this end, we use expansion (4) of the complex amplitude $S(z, z^*)$ in a series of Laguerre–Gauss polynomials. Then we have the following chain of transformations:

$$\zeta(t) \to S(z,z^*) \to \{c_n\}|_{n=0}^{\infty}, \tag{13}$$

that is, to perform the recognition process we will use expansion coefficients rather than the amplitudes, since working with one-dimensional single-index arrays of numbers on a computer is much easier than working with continuous functions. Consider the following sequence of transformations. Denoting in formula (3) inside the integral with respect to dt everything that does not depend on z as $P(t)$, we expand an exponential function under the integral in a Maclaurin series with respect to z and integrate term by term a series converging to an analytic function:

$$S(z,z^*) = \exp\left(\frac{-zz^*}{\rho^2}\right) f(z) = \exp\left(\frac{-zz^*}{\rho^2}\right) \int_0^T \exp\left[\frac{2z\zeta^*(t)}{\rho^2}\right] P(t)\,\mathrm{d}t$$
$$= \exp\left(\frac{-zz^*}{\rho^2}\right) \int_0^T \left\{\sum_{n=0}^{\infty} \frac{1}{n!}\left[\frac{2z\zeta^*(t)}{\rho^2}\right]^n\right\} P(t)\,\mathrm{d}t =$$
$$= \exp\left(\frac{-zz^*}{\rho^2}\right) \sum_{n=0}^{\infty} \int_0^T \left\{\frac{1}{n!}\left[\frac{2z\zeta^*(t)}{\rho^2}\right]^n\right\} P(t)\,\mathrm{d}t.$$

We regroup the expressions by taking everything that does not depend on the integration variable from outside the integral and introducing the Gaussian exponent under the sum sign:

$$S(z,z^*) = \sum_{n=0}^{\infty} \left[\frac{1}{n!} \frac{2^n}{\rho^{2n}} \int_0^T [\zeta^*(t)]^n P(t)\,\mathrm{d}t\right] \left[\exp\left(\frac{-zz^*}{\rho^2}\right) z^n\right]$$
$$= \sum_{n=0}^{\infty} c_n \left[\exp\left(\frac{-zz^*}{\rho^2}\right) z^n\right].$$

Thus, expansion (4) is obtained in an explicit form, and the expression for calculating the expansion coefficients for a spiral beam can be expressed as

$$c_n = \frac{2^n}{n!\rho^{2n}}\int_0^T [\zeta^*(t)]^n \exp\left\{-\frac{\zeta(t)\zeta^*(t)}{\rho^2} + \frac{1}{\rho^2}\int_0^t [\zeta^*(\tau)\,\zeta'(\tau) - \zeta(\tau)\zeta'^*(\tau)]\,\mathrm{d}\tau\right\} |\,\zeta'(t)|\,\mathrm{d}t. \qquad (14)$$

Let us analyse the result obtained. First of all, the integrity of the proposed method for introducing the recognition metric $\mu_{\mathbb{S}}$ is not violated, since the overlap function (11) can be rewritten in terms of the expansion coefficients due to the orthogonality of the basis of Laguerre–Gaussian functions as follows:

$$\Pi(\theta) = \frac{\iint_{\mathbb{R}^2} S_1(z,z^*)\,S_2^*(z\exp(-\mathrm{i}\theta), z^*\exp(\mathrm{i}\theta))\,\mathrm{d}x\mathrm{d}y}{\left(\iint_{\mathbb{R}^2} |\,S_1(z,z^*)|^2\mathrm{d}x\mathrm{d}y\right)^{1/2}\left(\iint_{\mathbb{R}^2} |\,S_2(z,z^*)|^2\mathrm{d}x\mathrm{d}y\right)^{1/2}}$$

$$= \frac{\iint_{\mathbb{R}^2} \sum_n c_n^{(1)}\mathcal{L}_{0n}(z,z^*)\left[\sum_m c_n^{(2)}\exp(\mathrm{i}\theta m)\,\mathcal{L}_{0m}(z,z^*)\right]^* \mathrm{d}x\mathrm{d}y}{\left(\iint_{\mathbb{R}^2}\left|\sum_n c_n^{(1)}\mathcal{L}_{0n}(z,z^*)\right|^2\mathrm{d}x\mathrm{d}y\right)^{1/2}\left(\iint_{\mathbb{R}^2}\left|\sum_m c_m^{(2)}\mathcal{L}_{0n}(z,z^*)\right|^2\mathrm{d}x\mathrm{d}y\right)^{1/2}}$$

$$= \frac{\sum_n\sum_m c_n^{(1)}[c_m^{(2)}\exp(\mathrm{i}\theta m)]^*\iint_{\mathbb{R}^2}\mathcal{L}_{0n}(z,z^*)\,\mathcal{L}^*_{0m}(z,z^*)\,\mathrm{d}x\mathrm{d}y}{\sum_n |c_n^{(1)}|^2\sum_m |c_m^{(2)}|^2}$$

$$= \frac{\sum_n c_n^{(1)}[c_n^{(2)}]^*\exp(-\mathrm{i}\theta n)}{\sum_n |c_n^{(1)}|^2\sum_n |c_n^{(2)}|^2}\,. \qquad (15)$$

Now we can rule out the direct application of complex amplitudes and use the coefficients of the expansion of spiral beams. Recall that the expansion coefficients are a finite array of complex numbers $\{c_n\}|_{n=0}^{N_c}$, ordered by a single index n. The finiteness of the array is determined by the rapid convergence of series (4); this issue was considered in more detail in our work [4]. The typical number of coefficients N_c is determined by the specific recognition problem and the complexity of the objects and lies in the range from 3 to 200 complex numbers.

The process of comparing the input contour ζ_1 with the reference $\zeta_2 \in \mathbb{K}_{DB}$ now looks like this:

1. For the input contour ζ_1, a set of expansion coefficients $\{c_n^{(1)}\}|_{n=0}^{N_c}$ is calculated by formula (14).

2. For the next reference contour, a sample of the stored, pre-calculated coefficients $\{c_n^{(2)}\}|_{n=0}^{N_c}$, is made from the $\mathbb{K}_{DB}$ database.

3. The value of the similarity metric $\mu_{\mathbb{S}}$ is calculated from formulas (12) and (15).

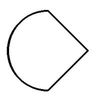
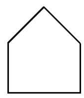
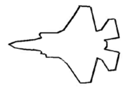
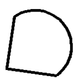
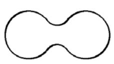
4. A decision is made on recognition if the distance obtained does not exceed a certain threshold value ε.

We draw attention to the fact that the proposed algorithm lacks two-dimensional integrals and geometric dimensions of the image; everything is determined only by the curves ζ_1 and ζ_2. At the same time, the complexity of the calculation increases linearly with increasing database volume of the $\mathbb{K}_{DB}$ reference contours; its p-fold increase leads to an increase in the number of comparisons by a factor of p. The amount of memory used for storing the array of coefficients $\{c_n^{(1)}\}|_{n=0}^{N_c}$ is constant and the same for any contours. The only (and apparently insurmountable) drawback is that the expansion coefficients are calculated by using exponential functions, which require large resources, and the complexity of their calculation is much higher than that for the operation of multiplication.

7. Results of numerical simulation

We present a small sample of the results of numerical simulation in the form of Table 1. The first row of this table shows

Table 1. Results of numerical simulation.

Image	Reference object							
								
	0.491	**0.021** 0.75 -143	0.753	0.126	0.609	0.759	0.669	0.639
	0.817	0.689	0.725	0.692	0.674	0.626	0.630	0.753
	0.761	0.777	0.672	0.756	0.686	**0.029** 0.75 –122	0.579	0.626
	0.761	0.728	0.718	0.727	0.712	**0** 1.00 0	0.602	0.594
	0.532	0.560	0.602	0.579	0.517	0.602	0.712	**0.038** 1.00 –150
	0.738	0.656	0.528	0.632	0.687	0.620	0.645	0.607

the reference objects, and the first column – recognisable images (everywhere the image size is 800×800 pixels). The cells contain values of some metric μ_S for two sets of complex expansion coefficients of spiral beams corresponding to two contours. If the metric value does not exceed the specified threshold of 0.1, then it is assumed that the contours correspond to each other, and the cells additionally contain a scale factor and an angle of mutual rotation.

The time for calculating the data of Table 1 on a personal computer with an Intel Core i7-7700 processor in a single-threaded mode without the use of parallelisation procedures is a fraction of seconds. All found rotation angles and scale factors exactly coincide with the true ones.

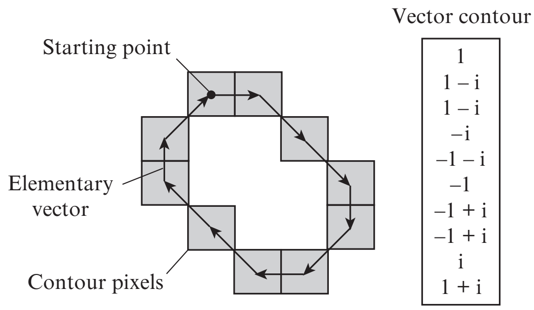


Figure 7. Contour representation with a chain code.

8. Advantages and disadvantages of the method for contour recognition using spiral light beams

The method proposed in the present paper refers to the contour analysis, i.e. an integral part of computer vision studied in monograph [10]. It describes step by step all the stages of graphics processing – from performing elementary algebraic operations on images to morphological analysis. Separately, the authors consider a number of problems that have purely applied value, from optical character recognition in typographic production to the creation of high-precision guidance systems in civil aviation. An important note is made: to date, there is no universal approach to image recognition, which can be successfully used in various applied problems. Moreover, even within a single problem, for example, when recognising aircraft contours, the effectiveness of different approaches can vary for different databases of reference contours.

Nevertheless, we will attempt to compare the proposed approach with its main competitor within the framework of contour analysis – the recognition method using correlation functions. A theoretical description of this method is given in [2, 10], and its implementation in the C# programming language using the OpenCV library [11] is described in [12]. The most reliable way to compare the two approaches is to start recognition procedures for a fixed set of input (recognisable) objects by using a single database of reference contours with measurement of the execution time of the procedure. However, due to the fact that the software developed by us is written in another programming language (C++) and does not use the OpenCV library's results, but processes the images independently (including search and detection of contours), it is not possible to make such a comparison.

In connection with the above said, we confine ourselves to a qualitative comparison of the two recognition methods. For recognition with the use of correlation functions, the contour can be represented in the following form (Fig. 7):

$$\Gamma = \{\gamma_0, \gamma_1, \gamma_2, \dots, \gamma_{k-1}\}, \tag{16}$$

where γ_0 is the starting point on the contour, and γ_i are the elementary displacement vectors.

The correlation function of two contours, Γ_1 and Γ_2, according to [10–12] is the normalised scalar product of complex-valued vectors:

$$\tau(\Gamma_1, \Gamma_2) = \frac{\sum_{i=0}^{k-1} \gamma_{1i}\gamma_{2i}}{\left(\sum_{i=0}^{k-1} \gamma_{1i}^2\right)^{1/2}\left(\sum_{i=0}^{k-1} \gamma_{2i}^2\right)^{1/2}}. \tag{17}$$

This function is essentially dependent on the choice of the starting points on the contours; therefore, to recognise the contour, it is necessary to find the maximum of the correlation function for all possible positions of the starting point on the recognisable contour. This is quite a resource-consuming operation when the contour consists of several hundreds and even thousands of points. At the same time, in the proposed recognition method using spiral light beams this drawback is absent, and the dependence on the choice of the starting point is easily formalised [see expression (5)].

Analysis of the correlation function (17) shows that to compare two contours it is necessary that they consist of the same number of points, which, generally speaking, is not always satisfied. To eliminate this drawback, one resorts to the contours equalisation procedure: throwing out 'redundant' and adding 'hypothetical' points. At the same time, the accuracy of calculating the correlation function deteriorates, since information-significant parts of the contour can be discarded during this procedure. The mathematical apparatus of spiral beams is free from this drawback, since the number of contour points determines only the number of terms in the computation of integrals (3) and (14), rather than the resultant feature of the object, i.e. the vector of the complex expansion coefficients of the spiral beam; therefore, the contour recognition procedure does not require throwing out or adding points to the contour being detected.

Nevertheless, the mathematical apparatus of spiral beams has a significant drawback that is absent in the apparatus of correlation functions: many variables are computed using the exponential calculation operation, which is more resource consuming from the point of view of image processing on a computer than the operation of multiplying complex numbers in calculating the correlation function.

9. Conclusions and direction of new work

We have summed up the theoretical part of the research devoted to the study of the possibilities of using the optics of spiral beams in the problem of pattern recognition. It is shown that spiral beams can be used in the problem of classification and recognition of contours in graphic images. The results are given that allow us to state that the mathematical apparatus of optics of spiral beams enables us to successfully overcome the classical complexity of the contour analysis. The results of numerical modelling are presented, which show that the developed approach to object recognition is applicable to solving model problems. The proposed method is compared with the method for contour recognition using correlation functions.

In the future, we will focus our efforts on the applied component of the study. First, it is necessary to develop specialised software for processing not tens, but hundreds and thousands of contours simultaneously with the use of the algorithms of parallelising computations. This will allow for more extensive numerical modelling, which can clarify how productive the developed approach to contour detection is. Second, it is necessary to pass from model problems to the real application of the described approach to applied problems.

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