



and frequency standards. Under the name of ‘a coherent population trapping resonance’ (CPT resonance), it is used as a reference in compact commercial atomic clocks. The analysis of SRS resonance is usually based on the use of standard equations for the density matrix. At a temperature of cooled atoms on the order of 1  $\mu\text{K}$ , we can neglect the motion of atoms and solve these equations for a stationary atom. In Section 2 of this paper, for the case of helium, we present a solution of these equations and a formula for the SRS resonance. The influence of Doppler broadening, the recoil effect, and the field shift is considered in Section 3. It was also shown there that by using the SRS method it is possible to measure the frequency of the  $2^1S_0-2^3S_1$  transition in the helium atom with an accuracy of  $\sim 1$  kHz.

## 2. Resonance in the form of the SRS line

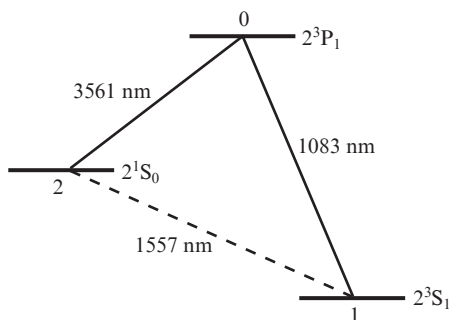
We denote the levels  $2^3P_1$ ,  $2^3S_1$ ,  $2^1S_0$  by numbers 0, 1, 2 (Fig. 2), and the frequencies of the transitions  $2^3P_1-2^3S_1$  ( $\lambda = 1083$  nm) and  $2^3P_1-2^1S_0$  ( $\lambda' = 3561$  nm) by  $\omega_{01}$  and  $\omega_{02}$ , respectively. We consider SRS with the participation of laser fields at frequencies  $\omega$  and  $\omega'$ :

$$E(t) = E \exp(-i\omega t) + E' \exp(-i\omega' t) + \text{c. c.}, \quad (1)$$

at which the atom from level 1 passes to level 2 through intermediate level 0. A resonance approximation is used when the pump field frequency  $\omega$  is close to the transition frequency  $\omega_{01}$ , and the frequency  $\omega'$  is close to the transition frequency  $\omega_{02}$ . In our case, the line widths of the 0–1 and 0–2 transitions are  $\Gamma = \gamma/2$ , where  $\gamma = 10^7$  s $^{-1}$  is the probability of spontaneous emission at the 0–1 transition. For the density matrix elements, we have the equations:

$$\begin{aligned} \dot{\rho}_2 &= -V(t)\rho_{20} - V^*(t)\rho_{02}, \\ \dot{\rho}_1 &= \gamma\rho_0 - U(t)\rho_{10} - U^*(t)\rho_{01}, \\ \dot{\rho}_0 + \gamma\rho_0 &= U(t)\rho_{10} + U^*(t)\rho_{01} + V(t)\rho_{20} + V^*(t)\rho_{02}, \\ \dot{\rho}_{01} + (\Gamma + i\omega_{01})\rho_{01} &= U(t)(\rho_1 - \rho_0) + V(t)\rho_{21}, \\ \dot{\rho}_{02} + (\Gamma + i\omega_{02})\rho_{02} &= V(t)(\rho_2 - \rho_0) + U(t)\rho_{12}, \\ \dot{\rho}_{21} + (\Gamma_{21} + i\omega_{21})\rho_{21} &= -V^*(t)\rho_{01} - \rho_{20}U(t). \end{aligned} \quad (2)$$

Here,  $\rho_{ik} = \rho_{ki}^*$ ;  $\rho_i = \rho_{ii}$  ( $i, k = 0, 1, 2$ );  $\Gamma_{21}$  is the line width of the 2–1 forbidden transition;



**Figure 2.** SRS scheme at the  $2^1S_0-2^3S_1$  transition through the intermediate level  $2^3P_1$ .

$$U(t) = U \exp(-i\omega t); \quad U = dE/(2i\hbar);$$

$$V(t) = V \exp(-i\omega' t); \quad V = d'E'/(2i\hbar);$$

$d$  and  $d'$  are the projections of the dipole moment operators of the transitions to the field directions. We introduce new variables  $r_{01}$ ,  $r_{02}$  and  $r_{21}$  in accordance with the equalities

$$\rho_{01} = r_{01} \exp(-i\omega t), \quad \rho_{02} = r_{02} \exp(-i\omega' t),$$

$$\rho_{21} = r_{21} \exp(-i\omega t + i\omega' t).$$

Taking into account the condition  $\rho_0 + \rho_1 + \rho_2 = 1$  and assuming that  $\rho_2 \ll \rho_0$ , we obtain

$$\begin{aligned} \dot{\rho}_2 &= -2\text{Re}(r_{02}V^*), \\ \dot{\rho}_0 + \gamma\rho_0 &= 2\text{Re}(r_{01}U^*) + 2\text{Re}(r_{02}V^*), \\ \dot{r}_{01} + (\Gamma - i\delta)r_{01} &= U(1 - 2\rho_0) + Vr_{21}, \\ \dot{r}_{02} + (\Gamma - i\delta')r_{02} &= -V\rho_0 + Ur_{21}^*, \\ \dot{r}_{21} + (\Gamma_{21} - i\Omega)r_{21} &= -V^*r_{01} - Ur_{02}^*. \end{aligned} \quad (3)$$

Here,  $\delta = \omega - \omega_{01}$ ;  $\delta' = \omega' - \omega_{02}$ ; and  $\Omega = \omega - \omega' - \omega_{21}$ . Obviously, the value of  $\dot{\rho}_2$  is the probability of a transition from level 1 to level 2 under the action of two fields with frequencies  $\omega$  and  $\omega'$ . Denoting  $\dot{\rho}_2$  by  $W(1-2)$ , we rewrite the first equation of system (3) in the form

$$W(1-2) = -2\text{Re}(r_{02}V^*). \quad (4)$$

The remaining equations determine  $r_{02}$ . In their solution, the fields are considered weak, satisfying the conditions

$$|U|/\Gamma \ll 1, \quad |V|/\Gamma \ll 1. \quad (5)$$

In this case, the derivatives in (3) can be neglected, and in order to find  $r_{02}$ , we should solve the system of equations

$$\begin{aligned} \Gamma\rho_0 &= \text{Re}(r_{01}U^*) + \text{Re}(r_{02}V^*), \\ (\Gamma - i\delta)r_{01} &= Vr_{21} - 2U\rho_0 + U, \\ (\Gamma - i\delta')r_{02} &= -V\rho_0 + Ur_{21}^*, \\ (\Gamma_{21} - i\Omega)r_{21} &= -V^*r_{01} - Ur_{02}^*. \end{aligned} \quad (6)$$

We substitute  $r_{02}$  from the third equation of this system into formula (4), and in the remaining equations we neglect the terms of the lowest order in the field. This gives

$$\begin{aligned} W(1-2) &= \text{Re}\left(\frac{2|V|^2\rho_0}{\Gamma - i\delta'}\right) - \text{Re}\left(\frac{2UV^*r_{21}}{\Gamma - i\delta'}\right), \\ r_{21} &= -\frac{V^*U}{(\Gamma - i\delta)(\Gamma_{21} - i\Omega)}, \\ \rho_0 &= \frac{q}{2\Gamma^2} \frac{\Gamma^2}{\Gamma^2 + \delta^2}. \end{aligned} \quad (7)$$

Here,

$$q = 2|U|^2/\Gamma^2 \quad (8)$$

is a dimensionless saturation parameter for the 0–1 transition, which is considered to be much less than unity. As a result, for the transition probability of an atom from state 1 to state 2 under the influence of a two-frequency field, we have the expression

$$W(1-2) = A \frac{\Gamma^4}{(\Gamma^2 + \delta'^2)(\Gamma^2 + \delta^2)} + A \operatorname{Re} \left[ \frac{\Gamma^3}{(\Gamma - i\delta')(\Gamma - i\delta)(\Gamma_{21} - i\Omega)} \right], \quad (9)$$

$$A = q|V|^2/\Gamma.$$

The probability of a two-photon transition (9) contains two terms, which have a different physical nature. The first one describes two independent transitions: absorption of a photon with the creation of a population at the upper level 0 and one-photon emission. The second term describes SRS (coherent absorption and emission of photons), the form of the line of which has a resonance with a uniform line width of the 2–1 forbidden transition.

When the conditions  $|\delta| \ll \Gamma$  and  $|\delta'| \ll \Gamma$  are fulfilled, we obtain the expression

$$W(1-2) = A \left( 1 + \frac{1 + \Gamma_{21}\Gamma}{\Omega^2 + \Gamma_{21}^2} \right).$$

In our case,  $\Gamma \gg \Gamma_{21}$ ; therefore,

$$W(1-2) = W \frac{\Gamma_{21}^2}{(\omega - \omega' - \omega_{21})^2 + \Gamma_{21}^2}, \quad (10)$$

where

$$W = q|V|^2/\Gamma_{21}.$$

Thus, we have a resonance in the form of the SRS line, when  $\omega - \omega' = \omega_{21}$ .

### 3. Factors affecting the measurement accuracy

*Doppler broadening.* When measuring the transition frequency  $\omega_{21}$ , cold atoms can be considered ‘free’; therefore, the main factor of the line broadening is the Doppler broadening. When formula (10) is used for an atomic gas, we must take into account the Doppler shift of the frequencies  $\omega$  and  $\omega'$  for a moving atom and perform averaging over the velocities with a Maxwellian distribution function. If the waves with frequencies  $\omega$  and  $\omega'$  are unidirectional, then instead of (10) we obtain the expression

$$W(1-2) = W_D \exp[(\omega - \omega' - \omega_{21})^2/\omega_D^2], \quad (11)$$

$$W_D = \sqrt{\pi} q |V|^2/\omega_D,$$

where  $\omega_D = \omega_{21}v_D/c$  is the Doppler width; and  $v_D$  is the thermal velocity of atoms. At a temperature of 1  $\mu\text{K}$ , we have  $\omega_D = 2\pi \cdot 10^3 \text{ s}^{-1}$  (10 kHz). Tuning to the resonance centre with an accuracy of  $0.1\omega_D$  makes it possible to measure the transition frequency  $\omega_{21}$  with an accuracy on the order of 1 kHz.

*Intensity of fields.* According to (8), the saturation parameter for the  $2^3\text{P}_1-2^3\text{S}_0$  transition can be represented in the form  $q = I/I_{\text{sat}}$ , where  $I$  is the radiation intensity at the frequency  $\omega$ ; and

$$I_{\text{sat}} = 16\pi^2 \hbar c \Gamma \lambda^3 = 2 \text{ W cm}^{-2}. \quad (12)$$

We set  $q = 10^{-2}$ , then  $I = 20 \mu\text{W cm}^{-2}$ .

To estimate the radiation intensity at the  $2^3\text{P}_1-2^1\text{S}_0$  transition, we write the expression for the number of atoms that, during the measurement time  $T$ , are in the  $2^1\text{S}_0$  state:

$$N = W_D N_0 T, \quad (13)$$

where  $N_0$  is the number of atoms in the initial state  $2^3\text{S}_1$ . Since both fields are weak, then  $N \ll N_0$ . We set  $N/N_0 = 0.1$ . Then from expressions (11) and (13) we obtain the relation  $|V|^2 = 10\omega_D/T$ . We express the field intensity at the frequency  $\omega'$  in terms of  $|V|^2$ :

$$I' = 160\pi^2 \hbar c \omega_D / (\lambda'^3 \gamma T), \quad (14)$$

where  $\lambda' = 3561 \text{ nm}$  and  $\gamma = 2.7 \times 10^{-2} \text{ s}^{-1}$  [1] is the wavelength of the  $2^1\text{S}_0-2^3\text{P}_1$  transition and the spontaneous emission probability at this transition, respectively. In the experiment [11], the measurement time was several seconds. Assuming  $T = 2 \text{ s}$ , we obtain  $I' \approx 10 \mu\text{W cm}^{-2}$ .

*Field shift.* To estimate the magnitude of the resonance shift  $\Delta\Omega$  as a function of the intensity of the fields, we solved the system of equations (6) with an accuracy greater than that in obtaining (7), by using the smallness of the parameters  $|U|/\Gamma$  and  $|V|/\Gamma$ . For  $|\delta'| \ll \Gamma$ , the shift is

$$\Delta\Omega = q\delta'/2. \quad (15)$$

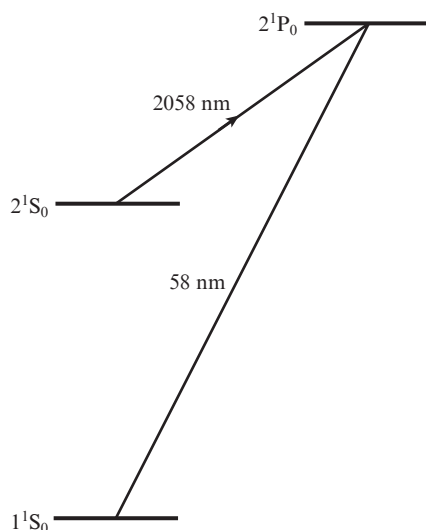
Previously, we set  $q = 10^{-2}$ . The frequency  $\omega'$  can be easily tuned to the centre of the transition line with frequency  $\omega_{02}$  with an accuracy of  $10^{-2}$  from the line width. Setting  $\delta' = 10^{-2}\Gamma = 0.5 \times 10^5 \text{ s}^{-1}$ , we obtain  $\Delta\Omega/2\pi \approx 20 \text{ Hz}$ .

*Recoil effect.* The shift of the SRS resonance due to the recoil effect in the absorption and emission of photons with  $\lambda = 1083 \text{ nm}$  and  $\lambda' = 3561 \text{ nm}$ , respectively, is

$$\Delta\omega = \hbar(k'^2 - k^2)/(2M), \quad (16)$$

where  $k = 2\pi/\lambda$ ;  $k' = 2\pi/\lambda'$ ; and  $M$  is the mass of the  $^4\text{He}$  atom. The value of this shift  $\Delta\omega/2\pi = 39.3 \text{ kHz}$  is calculated with an error of less than 1 kHz, and is taken into account when measuring  $\omega_{21}$ .

*Accuracy of recording the  $2^1\text{S}_0-2^3\text{S}_1$  transition frequency.* In experiment [11], more than  $10^6$  helium atoms in the  $2^3\text{S}_1$  state were obtained at a temperature of 1  $\mu\text{K}$ . Therefore, for estimates we set  $N_0 = 10^6$ . We will register  $N = 0.1N_0 = 10^5$  atoms in the  $2^1\text{S}_0$  state. We are guided by the method of paper [3], based on the detection of spontaneous VUV photons. Transitions involved in the measurement scheme are shown in Fig. 3. Using laser radiation at the  $2^1\text{S}_0-2^1\text{P}_0$  transition of helium ( $\lambda = 2058 \text{ nm}$ ) [13], the atom from the  $2^1\text{S}_0$  state undergoes transition to the  $2^1\text{P}_0$  state, followed by spontaneous emission of a photon at  $\lambda = 58 \text{ nm}$  upon transition to the ground state. We assume that  $0.1N$  atoms participate in this process, i.e., using the vacuum ultraviolet technique,  $N_{\text{ph}} = 10^4$  photons are recorded. For a rough estimate of the detection accuracy, we can assume that the fluctuation of the number of photons is equal to  $\sqrt{N_{\text{ph}}}$ . In the absence of a background, the signal-to-noise ratio is  $N_{\text{ph}}/\sqrt{N_{\text{ph}}} = 10^2$ . This allows us to register the shape of the resonance with an accuracy of  $10^{-2}$ , which is enough to measure the  $2^1\text{S}_0-2^3\text{S}_1$  transition frequency with an accuracy of 1 kHz. The presence of a



**Figure 3.** Transitions involved in the scheme for measuring the number of helium atoms at the  $2^1S_0$  level. Photons are recorded at  $\lambda = 58$  nm.

background leads to a decrease in the signal-to-noise ratio. However, in real experiments the background can be strongly suppressed, since the resonance is recorded using a variety of techniques that allow this to be done [12]: the recording of the resonance with respect to the derivative, the use of frequency modulation, etc.

#### 4. Conclusions

We have shown that the SRS method can be used to measure the frequency of the  $2^1S_0-2^3S_1$  transition of a helium atom with an accuracy of 1 kHz at a laser field intensity of  $\sim 10$  W cm $^{-2}$ . Let us note the features of this transition, which make it promising for various investigations:

1. A small radiation width (8 Hz) allows this transition to be used to create a frequency standard.

2. Precision measurement of the  $2^1S_0-2^3S_1$  transition frequency provides additional information for testing quantum electrodynamics, since a three-particle system of two electrons and a nucleus is considered. It is possible to measure the difference of the radiative corrections for the levels  $2^1S_0$  and  $2^3S_1$  and compare it with the theoretical value.

3. Methods of Doppler-free laser spectroscopy can be used, since the SRS line exhibits a resonance with a uniform line width in the gas [12, 14, 15].

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