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Frequency of self-modulation oscillations in a solid-state coupled-cavity ring laser

I.I. Zolotoverkh, E.G. Lariontsev

Abstract. **Self-modulation oscillations of the intensity of a solid-state coupled-cavity ring laser are theoretically investigated. The results obtained analytically show that external optical coupling allows the frequency of self-modulation oscillations to be varied in a wide range (from a few kilohertz to hundreds of megahertz). Using coupled cavities, it is possible to significantly weaken the influence of the coupling of counterpropagating waves through backscattering on the self-modulation frequency and to increase the scale factor that determines the dependence of the self-modulation frequency on the rotation velocity.**

Keywords: *solid-state ring laser, coupled cavities, self-modulation oscillations, laser gyroscope, scale factor.*

1. Introduction

In studies related to the applications of laser gyroscopes (LGs), considerable attention is paid to the possibility of increasing their scale factor and sensitivity. It was shown theoretically and experimentally in Refs [1–5] that the LG scale factor can be increased by using an anomalous dispersion of light in a medium placed inside the laser cavity. At a critical value of the anomalous dispersion, a pole appears in the expression for the scale factor, and it is shown in [1, 2] that in this case it is possible to increase the scale factor by 10^5 times. Using a passive ring cavity filled with rubidium vapour, Smith et al. [3] increased the scale factor by 2.4 times. An analysis carried out in [4] showed that linear gas media with anomalous dispersion are not promising for use in LGs based on He–Ne lasers. For semiconductor ring lasers, the possibility of using anomalous dispersion to increase the scale factor was discussed in [5].

Another method of increasing the scale factor, based on the use of coupled cavities, was investigated theoretically in Refs [6–8]. Instead of resonances in an intracavity medium with an anomalous dispersion, use was made of resonances of an additional cavity coupled with the main laser cavity through a partially transmitting coupling mirror. The authors of [6–8] showed that in a coupled-cavity ring He–Ne laser, it is possible to control an intracavity dispersion and create an anomalous dispersion that allows a large increase in the scale factor. Unfortunately, as far as we know, these conditions have not yet been implemented experimentally. Smith et al. [9] studied theoretically and experimentally the possibility of increasing the scale factor for passive coupled cavities.

Previous studies have shown that antiphase self-modulation oscillations of the intensities of counterpropagating waves with a frequency that depends on the angular rotation velocity are excited in solid-state ring lasers (SRLs), in particular in miniature ring chip lasers, due to the competition of counterpropagating waves. This generation regime was called the self-modulation regime of the first kind (SMR1). The first experimental and theoretical studies of this regime in diode-pumped annular chip lasers were performed in Refs [10–13]. Using SRLs operating in the self-modulation regime, it is possible in principle to create one of the versions of an active LG, which differs from the conventional method of measuring the rotational velocity. In a conventional LG, a beat signal, which arises during interference of counterpropagating waves as a result of their mixing outside the cavity, is processed, and in the variant using SMR1 it is necessary to measure the frequency of intensity self-modulation of one of the counterpropagating waves emerging from the laser cavity. SMR1 is the main regime of operation for miniature monolithic ring chip lasers, but the possibilities of using such sensors for navigation applications are limited by the small value of the scale factor, due to the small size of the chip laser.

In [14], the self-modulation oscillations of the intensity in a coupled-cavity SRL were investigated experimentally and simulated numerically. These studies have shown that the use of external optical coupling opens up new possibilities for controlling the frequency of self-modulation oscillations. The aim of this paper is an analytical study of self-modulation oscillations and an analysis of the control capabilities of the self-modulation frequency by external optical coupling.

2. System of equations

Figure 1a shows a schematic of a coupled-cavity ring laser. Inside the main ring cavity containing an active element, two counterpropagating waves with complex amplitudes $E_{1,2}$ propagate. The radiation emitted from the main cavity through a partially transmitting mirror M excites the optical fields in the external ring cavity and returns again to the main cavity through the same mirror.

In [14] we obtained a system of differential and difference equations relating to a laser with two coupled cavities. The excitation of counterpropagating waves with complex amplitudes $E_{c1,c2}$ in the external cavity is described by the difference equation

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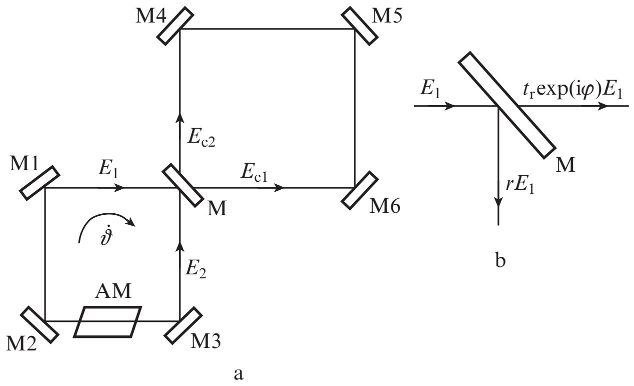

Figure 1. Schematic of (a) a coupled-cavity ring laser and (b) a wave on the coupling mirror M:
(M31, M2, M3) mirrors of the main cavity; (M) coupling mirror; (M4, M5, M6) mirrors of the additional cavity; (AM) active medium; E_1, rE_1 and $t_r\exp(i\varphi)E_1$ are the amplitudes of the incident, reflected and transmitted waves, respectively.

$$E_{c1,c2}(t) = r_c\exp(-i\omega_n T_c \pm i\Omega_c T_c/2)$$
$$\times[rE_{c1,c2}(t-T_c) + t_r\exp(i\varphi)E_{1,2}(t-T_c)]. \quad (1)$$

Here, r and $t_r = \sqrt{1-r^2}$ are the amplitude coefficients of reflection and transmission for the coupling mirror M (Fig. 1b); the factor $\exp(i\varphi)$ takes into account that the transmitted wave with respect to the reflected wave acquires a phase shift equal to φ; r_c is the effective coefficient, which is equal to the product of the reflection coefficients of all the mirrors of the additional cavity, with the exception of the coupling mirror (this factor also takes into account the field attenuation due to all other losses in the additional cavity); Ω_c is the difference of eigenfrequencies for counterpropagating waves in the additional cavity; T_c is the round-trip time of the light inside the additional cavity; and ω_n is the eigenfrequency of the main cavity for the generated counterpropagating waves in the absence of rotation.

We will assume that the frequency of self-modulation oscillations ω_m is small in comparison with the interval between neighbouring axial modes $1/T_c$ of the additional cavity:

$$\omega_m T_c \ll 1. \quad (2)$$

Then we can pass from difference equations (1) to differential equations, using the relation

$$E_{c1,c2}(t-T_c) = E_{c1,c2}(t) - T_c\dot{E}_{c1,c2}(t). \quad (3)$$

In addition, we will consider the case of a low-Q additional cavity, assuming the effective reflection coefficient r_c to be small:

$$r_c \ll 1. \quad (4)$$

In this case, we can neglect the term with the factor $rE_{c1,c2}$ in comparison with the term with the factor $t_r\exp(i\varphi)E_{1,2}$ in the right-hand side of Eqn (1).

In a laser with a low-Q additional cavity there is no accumulation of an intracavity field due to multiple round trips, and the resonance properties of the additional cavity are weakly manifested. This cavity basically plays the role of an optical delay line for the time of a single pass through it (T_c).

As a result, the system of equations obtained in [14] relating to a laser with two coupled cavities takes the form

$$\dot{E}_{1,2}(1+\alpha_{1,2}T_c) = -\frac{\omega}{2Q}E_{1,2} \pm i\frac{\Omega}{2}E_{1,2}$$
$$+\frac{i}{2}\tilde{m}_{1,2}E_{2,1} + \frac{\sigma l}{2T}(N_0E_{1,2} + N_{\pm}E_{2,1}) + \alpha_{1,2}E_{1,2}, \quad (5)$$

$$T_1\dot{N}_0 = N_{th}(1+\eta) - N_0 - N_0 a(|E_1|^2 + |E_2|^2)$$
$$-N_+aE_1E_2^* - N_-aE_1^*E_2, \quad (6)$$

where

$$\alpha_{1,2} = \frac{r_c}{T}(1-r^2)\exp[i(2\varphi - \omega_n T_c \pm \Omega_c T_c/2)]. \quad (7)$$

The following notations are used in equations (5) and (6): ω/Q is the bandwidth of the main cavity (the losses inside the cavity are assumed equal for counterpropagating waves); ω is the optical frequency; Q is the Q-factor of the cavity; Ω is the difference of eigenfrequencies for counterpropagating waves in the main cavity; σ is the laser transition cross section; l is the length of the active medium; T is the round-trip time of the light inside the main cavity; T_1 is the longitudinal relaxation time; $a = T_1c\sigma/(8\hbar\omega\pi)$ is the saturation parameter; and c is the speed of light. The pump rate is equal to $N_{th}(1+\eta)/T_1$, where N_{th} is the threshold population inversion, and η is the excess of the pump power over the threshold value. The linear coupling of counterpropagating waves is determined phenomenologically by the introduced complex coupling coefficients

$$\tilde{m}_1 = m_1\exp(i\vartheta_1), \quad \tilde{m}_2 = m_2\exp(-i\vartheta_2), \quad (8)$$

where $m_{1,2}$ are the moduli of the coupling coefficients, and $\vartheta_{1,2}$ are their phases.

The population inversion is expanded in a series of spatial harmonics with allowance for the zeroth (N_0) and second ($N_{\pm}$) harmonics:

$$N(z,t) = N_0(t) + N_+(t)\exp(i2kz) + N_-(t)\exp(-i2kz), \quad (9)$$

where k is the wave number. Because of the interference of the counterpropagating waves, the light intensity inside the cavity changes periodically along the cavity axis z and, as a result of saturation of the population inversion by the intracavity field, lattices are formed in the active medium, the amplitudes of which are determined by the harmonics $N_{\pm}$.

Equations (5) and (6) are written for the case of generation at the gain line centre. In these equations, the optical frequency ω is set equal to ω_n. The sensitivity to rotation arises from the Sagnac effect: in the main and additional cavities, there appears a difference of the eigenfrequencies for the counterpropagating waves [15, 16]

$$\Omega = \frac{8\pi S\dot{\vartheta}}{n\lambda L}, \quad (10a)$$

$$\Omega_c = \frac{8\pi S_c\dot{\vartheta}}{\lambda L_c}, \quad (10b)$$

where λ is the laser wavelength. It is assumed in (10) that the main cavity is filled with an optical medium having a refractive index n, and there is no medium in the additional cavity; S, S_c are the projections of the area vectors of the main and additional cavities on the rotation axis; and L, L_c are the perimeters of ring cavities.

3. Self-modulation oscillations of intensity

3.1. Derivation of basic formulas

It is possible to find an approximate analytical solution of the system of equations (5) and (6) by assuming that the self-modulation oscillation frequency ω_m is large in comparison with the fundamental relaxation frequency $\omega_r = \sqrt{(\omega/Q)\eta/T_1}$. We will use the method of successive approximations with respect to a small parameter

$$\varepsilon = \omega_r/\omega_m \ll 1. \tag{11}$$

In the zeroth approximation, we neglect the modulation of the population inversion with the frequency of self-modulation oscillations and take into account only the constants of the spatial harmonics N_0 and $N_\pm$; we write the harmonics $N_\pm$ in the form

$$N_\pm = N_r \pm iN_i. \tag{12}$$

In this approximation, Eqns (5) and (6) for the complex field amplitudes $E_{1,2}$ are a system of two first-order differential equations with constant coefficients.

For simplicity, we assume that the complex coupling coefficients $\tilde{m}_{1,2}$ are complex-conjugate (symmetric coupling with the same moduli of coupling coefficients, $m_1 = m_2 = m$, and phases $\vartheta_1 = \vartheta_2 = 0$). In accordance with equations (5), the complex amplitude E_1 can be represented in the form

$$E_1 = \frac{2\dot{E}_2 + (\omega/Q + i\Omega - \sigma l N_0/T)E_2 - 2\beta(E_2 - T_c\dot{E}_2)}{\sigma l N_+/T + im}, \tag{13}$$

where

$$\beta = \beta_r + i\beta_i$$

$$\beta_r = r_c(1-r^2)\frac{1}{T}\cos\Phi\cos(\Omega_c T_c/2);$$
$$\beta_i = r_c(1-r^2)\frac{1}{T}\cos\Phi\sin(\Omega_c T_c/2); \tag{14}$$

$$\Phi = 2\varphi - \omega_n T_c.$$

The parameter Φ determines the phase of the external optical coupling. It depends on the phase shift φ between the waves reflected and transmitted by the coupling mirror, and also on the phase shift $\omega_n T_c$ in the additional cavity. In the general case, for an arbitrary phase Φ, the analysis turns out to be very cumbersome. Next, we consider two particular cases: $\Phi = 2\pi p$, i.e. in-phase optical coupling and $\Phi = 2\pi p + \pi$, i.e. antiphase coupling (p is an integer).

It follows from (14) that in the case of in-phase optical coupling

$$\beta_r = \beta_r^s = r_c(1-r^2)\frac{1}{T}\cos(\Omega_c T_c/2),$$
$$\beta_i = \beta_i^s = r_c(1-r^2)\frac{1}{T}\sin(\Omega_c T_c/2), \tag{15}$$

and in the case of antiphase coupling

$$\beta_r = \beta_r^a = -r_c(1-r^2)\frac{1}{T}\cos(\Omega_c T_c/2),$$
$$\beta_i = \beta_i^a = -r_c(1-r^2)\frac{1}{T}\sin(\Omega_c T_c/2) \tag{16}$$

Substituting (13) into equation (5), we obtain for the complex amplitude E_2 a second-order differential equation with constant coefficients. The characteristic frequencies $\omega_{1,2}$, found from this equation, must be real quantities. This condition is satisfied if

$$\frac{\sigma l}{T}N_0 = \frac{\omega}{Q} - 2\beta_r + \frac{\beta_i T_c(\Omega - 2\beta_i)}{1 + \beta_r T_c}, \tag{17}$$

$$N_r = 0. \tag{18}$$

The frequencies $\omega_{1,2}$ are determined by the expressions:

$$\omega_{1,2} = \pm\frac{1}{2}\sqrt{\frac{m^2 - \sigma l N_i^2/T}{(1+\beta_r T_c)^2 + (\beta_i T_c)^2} + \frac{(\Omega - 2\beta_i)^2}{(1+\beta_r T_c)^2}}. \tag{19}$$

Expressions for the field amplitudes of the counterpropagating waves in the self-modulation regime can be written in the form

$$E_{1,2} = A_{1,2}\exp(i\omega_1 t) + B_{1,2}\exp(i\omega_2 t). \tag{20}$$

Substituting (20) into Eqns (6), we obtain a system of algebraic equations for the unknown constants $A_{1,2}$, $B_{1,2}$, and N_i. Taking into account the fact that the solution of a similar system of equations is described in detail in [17, 18], we omit the intermediate calculations and give only the results obtained:

$$N_i = 0, \tag{21}$$

$$|A_2|^2 = \frac{[\omega_1(1+\beta_r T_c) + (\Omega - 2\beta_i)/2]^2}{4\omega_1^2(1+\beta_r T_c)^2}\eta, \tag{22}$$

$$|B_2|^2 = \frac{\omega_1(1+\beta_r T_c) - (\Omega - 2\beta_i)/2}{\omega_1(1+\beta_r T_c) + (\Omega - 2\beta_i)/2}|A_2|^2. \tag{23}$$

The self-modulation frequency ω_m is equal to the beat frequency of two spectral components:

$$\omega_m = \omega_1 - \omega_2 = \sqrt{\frac{m^2}{(1+\beta_r T_c)^2 + (\beta_i T_c)^2} + \frac{(\Omega - 2\beta_i)^2}{(1+\beta_r T_c)^2}}. \tag{24}$$

In formulas (17), (19), (22)–(24), the quantities β_r and β_i should be set equal to β_r^s and β_i^s in the case of in-phase optical coupling and to β_r^a and β_i^a in the case of antiphase coupling.

We will not perform a detailed analysis of the amplitude characteristics of self-modulation oscillations in this paper; the main attention is paid to the analysis of frequency characteristics. Note that the obtained results are approximate. In formula (24), in the first approximation with respect to the parameter ε [see (11)], there is a displacement of the self-modulation frequency, caused by the interaction of self-modulation oscillations with relaxation oscillations [19, 20]. Corrections in the first approximation will be considered below in the numerical solution of equations (5) and (6).

3.2. In-phase optical coupling

In the case of in-phase coupling, as follows from (17), losses in the main cavity decrease, which leads to an increase in the average intensity of the SRL output and the amplitude of the self-modulation intensity oscillations.

Let us consider the effect of external optical coupling on the frequency of self-modulation oscillations in the case of in-phase coupling. First, we confine ourselves to the case when the additional cavity is insensitive to rotation [the projection S_c of the area vector on the rotation axis in (10b) is zero or small]. In this case, $\Omega_c = 0$, and formula (24) is substantially simplified. With in-phase coupling of cavities, this formula yields the expression

$$\omega_m = \omega_m^s = \frac{\sqrt{m^2 + \Omega^2}}{1 + r_c(1 - r^2)T_c/T}. \qquad (25)$$

In the absence of the additional cavity ($r_c = 0$), the self-modulation frequency is determined by the known formula $\omega_m = \sqrt{m^2 + \Omega^2}$. It follows from (25) that for the in-phase coupling of cavities, the frequency ω_m is less than its value $\sqrt{m^2 + \Omega^2}$ in the absence of external optical coupling.

Figure 2 shows the dependence of the self-modulation frequencies $f_m = \omega_m/2\pi$ on the perimeter L_c of the additional cavity for different parameters of external optical coupling. The dependence in Fig. 2a is calculated under the following assumptions. It was assumed that the main cavity is a monolithic ring cavity on a YAG:Nd crystal (a cavity of a ring chip laser). The perimeter of the main cavity is $L = 5$ cm, the reflection coefficient of the coupling mirror is $r = 0.97$, and the bandwidth of the main cavity is $\omega/Q = 4.5 \times 10^8$ s^{-1}. The coupling coefficients were assumed to be the same: $m_1 = m_2 = m = 1.3 \times 10^6$ s^{-1}. In this case, the frequency of self-modulation oscillations in the absence of optical nonreciprocity ($\Omega = 0$) in a chip laser without an additional cavity is 206 kHz. Excess of pumping above the threshold is $\eta = 0.1$. The effective reflection coefficient r_c for the additional cavity was assumed to be 0.25. The solid curve in Fig. 2a is calculated from formula (25) for $\Omega = 0$, and the points show the results obtained on the basis of a numerical solution of the system of equations (5) and (6) for the given parameters.

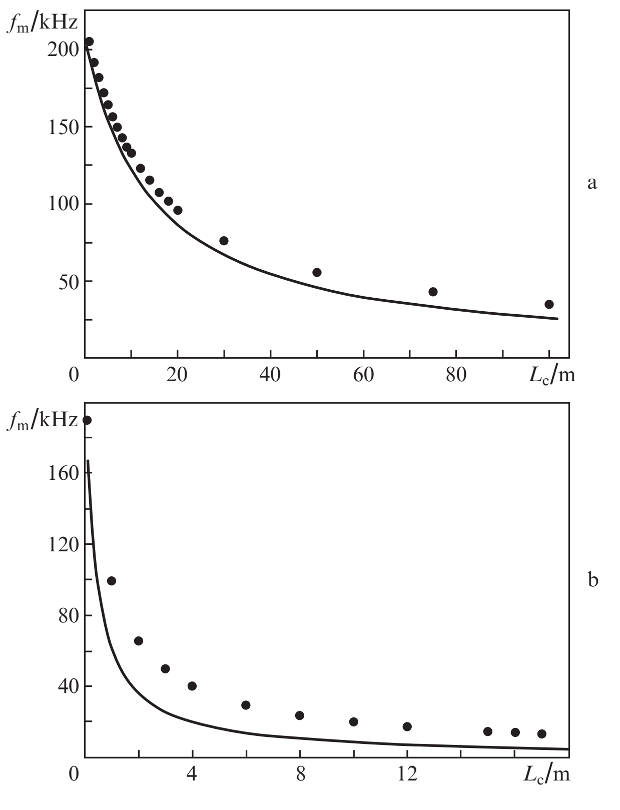

Figure 2. Dependences of the self-modulation frequency f_m on the additional cavity perimeter L_c for the amplitude reflection coefficients of the coupling mirror r = (a) 0.97 and (b) 0.7.

The dependence of the self-modulation frequency f_m on the perimeter L_c of the additional cavity, shown in Fig. 2b, was obtained for an amplitude reflection coefficient of the coupling mirror, $r = 0.7$, which is larger than that for Fig. 2a. The remaining parameters for Fig. 2b have the following values: $r_c = 0.23$, $L = 10$ cm, $m = 1.3 \times 10^6$ s^{-1}, and $\eta = 0.03$.

It is seen from Fig. 2 that the effect of external optical coupling increases significantly with increasing transmission coefficient of the coupling mirror, $1 - r^2$. The results in Fig. 2 show that with in-phase optical coupling makes it possible to substantially reduce the self-modulation frequency.

Let us now consider the case when the additional cavity is sensitive to rotation. Assuming that the splitting Ω_c of the eigenfrequencies of the additional cavity due to rotation is small in comparison with the intermode interval $1/T_c$ ($\Omega_c T_c \ll 1$), we obtain from (24) the following simplified formula:

$$\omega_m = \omega_m^s = \frac{\sqrt{m^2 + [\Omega - r_c(1 - r^2)\Omega_c T_c/T]^2}}{1 + r_c(1 - r^2)T_c/T}. \qquad (26)$$

Let us further consider the case when the frequency nonreciprocity Ω is created in the main ring chip cavity by means of a constant magnetic field [21], satisfying the condition $\Omega^2 \gg m^2$. Then from (26) we have

$$\omega_m^s = \frac{\Omega - \Omega_c r_c(1 - r^2)T_c/T}{1 + r_c(1 - r^2)T_c/T}. \qquad (27)$$

Figure 3a shows the dependence of the self-modulation frequency f_m on the frequency nonreciprocity of the additional cavity $\Omega_c/2\pi$. The straight line is calculated from formula (27) for the amplitude reflection coefficient of the coupling mirror $r = 0.993$. The remaining parameters are as follows: $L = 5$ cm, $r_c = 0.3$, $\Omega/2\pi = 400$ kHz, $L_c = 100$ m, and $m = 1.3 \times 10^6$ s^{-1}. The points represent the results obtained by solving numerically equations (5) and (6) for these parameters and for $\eta = 0.05$.

The dependence of the self-modulation frequency f_m on the frequency nonreciprocity of the additional cavity $\Omega_c/2\pi$, shown in Fig. 3 (b), was obtained for a larger (than in Fig. 3a) amplitude reflection coefficient of the coupling mirror – $r = 0.7$. The remaining parameters are as follows: $L = 10$ cm, $r_c = 0.23$, $\Omega/2\pi = 400$ kHz, $L_c = 17$ m, and $m = 1.3 \times 10^6$ s^{-1}. The straight line is calculated by formula (27), and the points show the data found by solving numerically equations (5) and (6) for these parameters and for $\eta = 0.03$.

Note that when obtaining the results presented in Fig. 3, it was assumed that the nonreciprocity Ω_c occurs only in the additional cavity when rotation is due to the Sagnac effect. The dimensions of the main cavity were considered small in comparison with those of the additional cavity ($S/L \ll S_c/L_c$). In this case, in accordance with formulas (10a) and

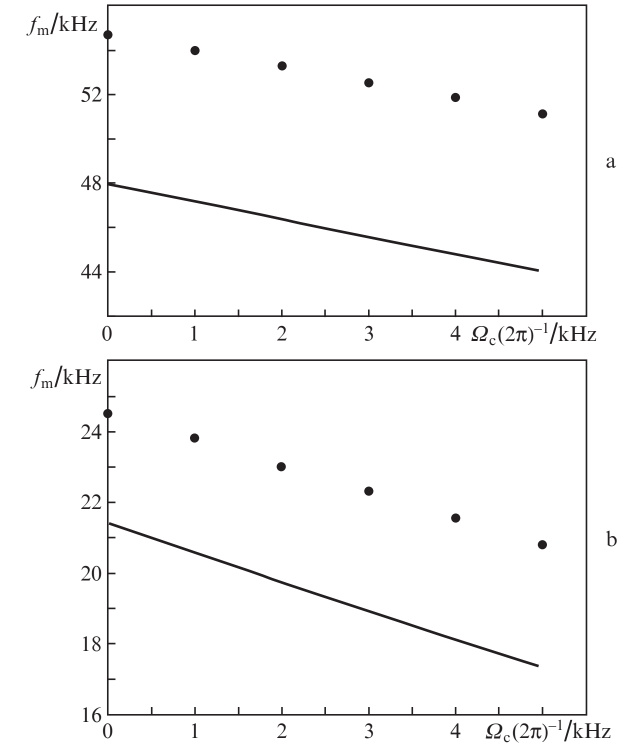

Figure 3. Dependences of the self-modulation frequency f_m on the frequency nonreciprocity in the additional cavity $\Omega_c/2\pi$ for the amplitude reflection coefficients of the coupling mirror r = (a) 0.993 and (b) 0.7.

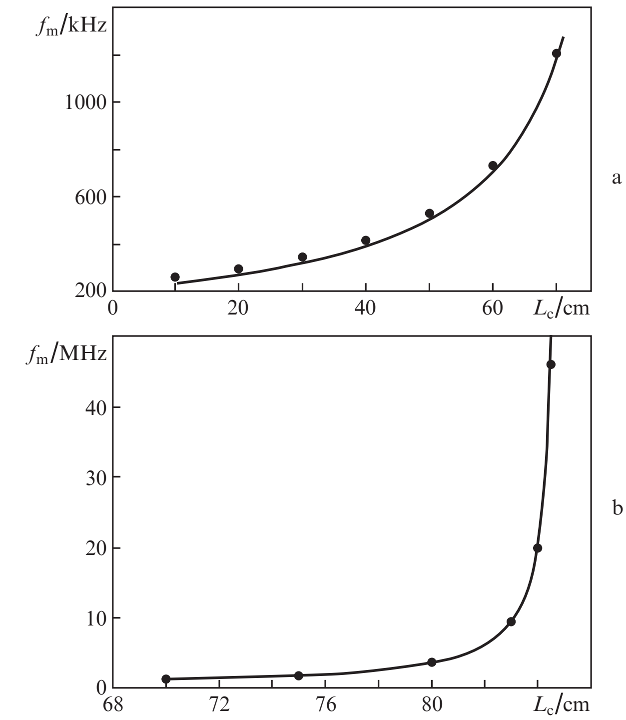

Figure 4. Dependence of f_m on the additional cavity perimeter L_c.

(10b), the effect of rotation on the nonreciprocity Ω can be neglected.

3.3. Antiphase optical coupling

Antiphase optical coupling increases losses in the main cavity, which leads to a decrease in the amplitude of self-modulation oscillations. Let us analyse the effect of this coupling on the frequency of self-modulation oscillations. First we consider the case when the additional cavity is insensitive to rotation [the projection S_c of the area vector on the rotation axis in (10b) is zero or small]. In this case, from (24) we obtain the expression

$$\omega_m = \omega_m^a = \frac{\sqrt{m^2 + \Omega^2}}{1 - r_c(1 - r^2)T_c/T}. \quad (28)$$

It follows that in the case of antiphase optical coupling of the cavities, the frequency of self-modulation oscillations is always greater than its value $\sqrt{m^2 + \Omega^2}$ in the absence of external optical coupling.

In accordance with Eqn (28), the frequency of self-modulation oscillations increases monotonically with increasing the ratio of the cavity perimeters $L_c/L = T_c/T$. At the critical value of this relation

$$(L_c/L)_{crit} = [r_c(1 - r^2)]^{-1}. \quad (29)$$

The denominator in (28) vanishes, which leads to a sharp increase in ω_m.

In Fig. 4, the solid curve shows the dependence of f_m on the perimeter L_c of the additional cavity, calculated from Eqn (28), and the points represent the data found by solving numerically equations (5) and (6). The results in Fig. 4 were obtained for L = 10 cm, r = 0.7, r_c = 0.23 and η = 0.1. With these parameters, the critical length L_{crit} of the additional cavity perimeter, in accordance with (28), is equal to 84.9 cm.

When L_c varies from 10 to 70 cm (Fig. 4a), the self-modulation frequency f_m gradually increases from 200 to 1300 kHz. When L_c approaches the critical value L_{crit} (Fig. 4b), the frequency f_m increases sharply, reaching 50 MHz or higher. With the lengths of the additional cavity perimeter exceeding the critical value ($L_c > L_{crit}$), there are no self-modulation oscillations in the case of antiphase optical coupling (in this region they exist only for in-phase optical coupling).

Let us now consider the case when the additional cavity is sensitive to rotation. We will assume that the splitting Ω_c of the eigenfrequencies of the additional cavity due to rotation is small in comparison with the intermode interval $1/T_c$, i.e., the inequality $\Omega_c T_c \ll 1$ is met. Suppose also that in the main ring chip-resonator the constant magnetic field [19] produces the frequency nonreciprocity Ω, which satisfies the condition $\Omega^2 \gg m^2$. Under these conditions, from (24) we obtain the expression

$$\omega_m = \omega_m^a = \frac{\Omega - r_c(1 - r^2)\Omega_c T_c/T}{1 - r_c(1 - r^2)T_c/T}. \quad (30)$$

It follows that the dependence of the self-modulation frequency on the angular velocity of rotation $\dot{\vartheta}$ is determined by the scale factor

$$K = \frac{r_c(1 - r^2)T_c/T}{1 - r_c(1 - r^2)T_c/T} K_c \dot{\vartheta}, \quad (31)$$

where K_c is the scale factor for the additional cavity, which, in accordance with (10b), is

$$K_c = 8\pi S_c/(\lambda L_c). \quad (32)$$

Formula (31) is written under the condition that the dimensions of the main cavity are small in comparison with those of the additional cavity ($S/L \ll S_c/L_c$). In this case, the influence of rotation on the nonreciprocity Ω in (30) can be neglected.

It is seen from (31) that when the perimeter length of the additional cavity is close to the critical one ($L_c \approx L_{crit}$), the scale factor K_c sharply increases. At the main cavity perimeter $L = 10$ cm, the amplitude reflection coefficient of the coupling mirror $r = 0.7$ and the effective reflection coefficient of the additional cavity $r_c = 0.23$, we find that for $L_c = 83.5$ cm the scale factor is $K = 60K_c$.

Above we considered self-modulation oscillations for two values of the phase $\Phi = 2\varphi - \omega_n T_c$ of external optical coupling. To realise these generation regimes, it is necessary to adjust the optical length of the additional cavity. The frequency of the laser light and the cavity perimeter should be stable. The instability of these parameters is an additional source of error in measuring the velocity of rotation. The issues relating to this error and stability of the laser frequency necessary for the realisation of the considered generation regimes require additional investigation.

A possible scheme for a SRL with coupled ring cavities was demonstrated in [14], where a nonplanar monolithic ring cavity was used as the main cavity, in which the polarisation of the generated light is circular. When the light passes through the coupling mirror, the polarisation is transformed into elliptical, close to linear. In the model considered above, the polarisation of the light in the coupled ring cavities is assumed to be the same (the scalar model of the SRL). In the general case, when one of the cavities (or both) is not planar, it is necessary to develop a vector model of a SRL with coupled ring cavities.

4. Conclusions

We have investigated theoretically the effect of external optical coupling on the self-modulation oscillations of the intensity of a coupled-cavity SRL. The obtained results have shown that external optical coupling allows the frequency of self-modulation oscillations, f_m, to be varied in a wide range (from several kilohertz to hundreds of megahertz). The in-phase optical coupling makes it possible to weaken the influence of coupling of counterpropagating waves through backscattering and to substantially reduce f_m, while the antiphase optical coupling, on the contrary, enhances the influence of the coupling of counterpropagating waves and leads to an increase in f_m.

External optical coupling allows the scale factor to be controlled. With the in-phase coupling, the scale factor K, which determines the dependence of the self-modulation frequency on the rotational velocity, can be increased to the value K_c given by the dimensions of the additional cavity (10b). With the antiphase coupling, the scale factor K can be increased by two orders of magnitude in comparison with the value of K_c.

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