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***LASER BEAMS***

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# Generation of nonuniformly polarised vortex Bessel beams by an interference polariser

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***Abstract.*** **A new optical system is proposed for the generation of azimuthally and radially polarised vortex Bessel beams. The system is based on the transformation of conical wavefronts passing through an interference polarisation plate. The polarisation characteristics during the beam formation are controlled by changing the curvature of the wavefront of the beam illuminating a diffractive axicon. Radially and azimuthally polarised vortex Bessel beams are experimentally obtained using a binary phase axicon.**

***Keywords:*** *vortex Bessel beams, nonuniform polarisation, interference polariser, diffractive axicon.*

## 1. Introduction

Azimuthally and radially polarised laser beams have today a variety of applications [1, 2], in particular for free-space communication [3] and materials processing [4, 5]. Vortex beams with different polarisations are also used in various applications [6]. Among vortex beams, Bessel beams are important for practice [7]. Ornigotti and Aiello [8] considered theoretical principles of nonparaxial propagation and transformation of nonuniformly polarised Bessel beams. The methods for the formation of Bessel laser beams with nonuniform polarisation differ both in the way the initial Bessel beam is generated and in the properties of the optical systems producing the required polarisation state. Liquid-crystal light modulators are a universal tool for solving the problem of forming such beams, but their low resolution affects the quality of the experimental results obtained [9].

Gaofeng et al. [10] experimentally demonstrated the self-recovery property of not only the amplitude distribution but also of the polarisation state of the Bessel–Gauss beams obtained using a spatial light modulator. The polarisation state of these beams was formed by means of a sector plate (a radial polarisation generator). The transformation of circular polarisation into nonuniform polarisation using segmented polarisation plates [11] results in the appearance of a vortex phase in the beam. Unfortunately, the use of the sector plate [12] leads to a deterioration in the beam quality.

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Higher quality beams can be obtained if the initial Bessel beam is produced by a refractive axicon, and the further formation of the polarisation state occurs in a biaxial birefringent crystal [13]. This method allowed both radial and azimuthal polarisations to be obtained experimentally in the same optical system by rotating the direction of the polarisation axis of the incident beam by 90°. A simpler solution is the use of a multilayer structure (one-dimensional photonic crystal) to form the polarisation of the Bessel beam [14], with nonlinear optical materials making it possible to control the properties of the crystal by additional irradiation with an auxiliary laser source. In such a variant of the optical system proposed in [14], the polarisation state is transformed simply by controlling a nonlinear photonic crystal without any changes in the optical system. However, this principle has not yet been experimentally implemented.

A simplified version of the optical system, implemented in practice [15], contains a diffractive axicon instead of a refractive one and a Stoletov's pile instead of a multilayer structure (Stoletov's pile also makes it possible to produce radial polarisation). The use of the diffractive axicon increases the quality of the beam in addition to the advantages that diffraction optics provides. However, the diameter of the beam after its passage through Stoletov's pile greatly increases because of the required large divergence angle, and collimation of the beam becomes a complex task requiring the use of high-aperture large-diameter optics. In addition, the energy efficiency of Stoletov's pile to achieve a sufficient polarisation contrast is small. Any control of the polarisation state of the beam in such a system is impossible.

In this paper, the optical system described in [16] is taken as the basis, which uses the transformation of conical beams in a multilayer interference structure. Such a system was effectively used to produce a two-ring radially polarised beam [17]. The suitability of this system for the formation of radially polarised Bessel beams has been experimentally shown in [18], but the control of the polarisation state in it is impossible because of the large angles of incidence of light on the multilayer structure in order to obtain radial polarisation. Further increase in the angles for obtaining azimuthal polarisation is not possible due to technological limitations in the manufacture of diffractive optical elements and a strong decrease in energy efficiency with increasing angles of incidence.

In this paper, it is proposed to form nonuniformly polarised vortex Bessel beams with a controlled polarisation state by changing the divergence of the illuminating beam incident on the diffractive axicon. It has been shown theoretically and experimentally that when the operating angles of the light incidence on an interference polariser decrease, radial and azimuthal polarisations can be obtained with the same diffractive axicon only by replacing the lenses in the illuminating beam.

## 2. Modelling of the optical system

The basis of the proposed approach is the passage of a conic beam with circular polarisation through a polarisation plate [16]. It is known [17, 19] that circular polarisation can be represented as a superposition of radial and azimuthal polarisations with a vortex phase:

$$\mathbf{e}_x \pm \mathrm{i}\mathbf{e}_y = (\cos\phi\,\mathbf{e}_r - \sin\phi\,\mathbf{e}_\phi) \pm \mathrm{i}(\sin\phi\,\mathbf{e}_r + \cos\phi\,\mathbf{e}_\phi)$$

$$= \exp(\pm\mathrm{i}\phi)(\mathbf{e}_r + \mathrm{i}\mathbf{e}_\phi). \quad (1)$$

As a result of the action of the polarisation plate, one of the polarisations, radial or azimuthal, will be separated depending on the angle of incidence, and the vortex phase will remain in the beam in both cases. A conic beam is proposed to be formed with the help of an axicon, which gives a zero order Bessel beam with a maximum on the beam axis. After passing through the polarising plate, the beam becomes nonuniformly polarised, but the presence of the vortex phase leads to the formation of a nonzero intensity on the optical axis with circular polarisation. This fact will be further demonstrated by numerical modelling. Thus, a small central region with uniform polarisation remains in the beam, but its energy contribution is negligibly small.

The principle of operation of the interference polariser determines the high angular selectivity in the transformation of the polarised state of the transmitted light. The conical wavefront has radial symmetry and ensures constant angles of light incidence on the polariser, and as a consequence, the polarisation transformation over the entire beam area is the same for all rays and radially symmetric. A conventional axicon (both refractive and diffractive) forms a conical wavefront upon illumination by a plane wavefront. If it is needed to change the propagation angles of the rays of the conical wavefront, then the exact solution is to replace the forming axicon by an axicon with another numerical aperture, while maintaining the illumination by the plane wavefront. However, both diffractive and refractive axicons are rather single purpose and expensive elements, and it is often not possible to have a wide range of axicons with exact numerical aperture values. In this paper, it is proposed to correct the angles of convergence of the beams formed by an axicon by replacing a plane illuminating wave with a spherical wave. If the beam illuminating the axicon is spherical (whether divergent or convergent), then the wavefront of the transmitted beam will no longer be purely conical and will have a quadratic 'phase additive'. This will lead to the fact that the propagation angles of the rays will vary along the beam area as a function of the radius, and consequently the angles of incidence of the light on the interference polariser will also change. The main issue in the development of the proposed optical system is the estimation of the effect of this quadratic 'phase additive' in a beam with a conical wavefront on the 'purity' of the polarisation transformation. The purpose of the numerical simulation presented below was to estimate the variation of the angles over the beam cross section as a function of various parameters in order to be able to compare it with the characteristics of the interference polariser and to select the parameters of the elements of the optical system necessary to achieve the required quality of the generated beams.

The numerical aperture of the diffractive axicon is determined by the formula:

$$\mathrm{NA} = \lambda/d, \quad (2)$$

where $\lambda$ is the wavelength of the light illuminating the axicon, and $d$ is the period of the axicon rings. Using the numerical aperture (2), we can determine the angle of convergence of the conical wavefront:

$$\theta = \arcsin(\lambda/d). \quad (3)$$

In particular, when a diffractive axicon with a period of 2 μm is illuminated by the laser light with a wavelength of 0.633 μm (helium–neon laser), we obtain a conical wavefront converging at an angle $\theta \approx 18.45°$. In the ray approximation (if one does not take into account the diffraction), all the rays will propagate at this angle to the optical axis; in this case, a zero-order Bessel beam is formed. Diffraction phenomena can be taken into account if we calculate numerically the angular spectrum of the light transmitted through the diffractive axicon. In this paper, such numerical simulation was performed by the plane wave expansion method [20] using a fast calculation algorithm for axisymmetric beams [21]. The results of modelling the formation of a zero-order Bessel beam and the angular spectrum for a convergent diffraction order of the light transmitted through a binary diffractive axicon with a period of 2 μm are shown in Fig. 1.

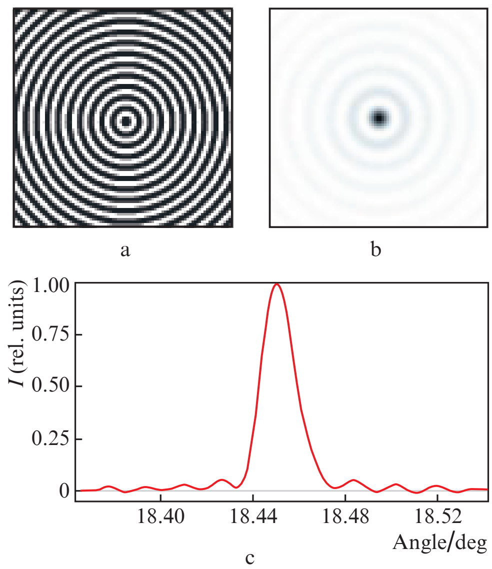
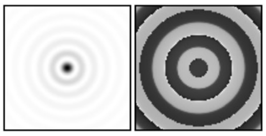
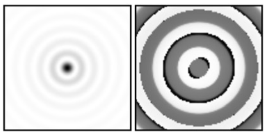
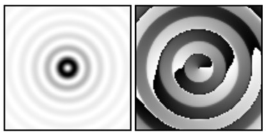
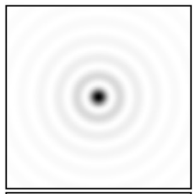
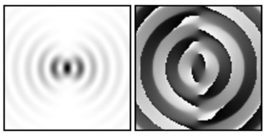
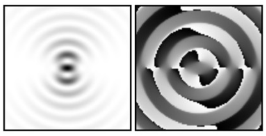
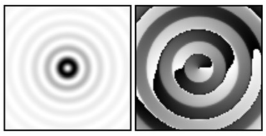
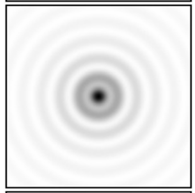
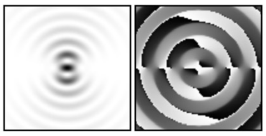
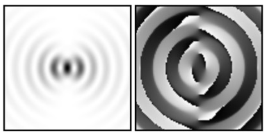
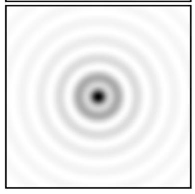



**Figure 1.** Results of modelling the formation of a zero-order Bessel beam: (a) central part of a binary diffractive axicon with a period of 2 μm (white colour corresponds to a phase equal to π, black colour – to a zero phase value); (b) intensity distribution at a distance $z$ = 5 mm from the axicon (negative image of size 10 × 10 μm); (c) angular spectrum of the light transmitted through the axicon.

The polarisation transformation of a zero-order Bessel beam with circular polarisation as it passes through a polarisation plate is shown in Table 1. It follows from the table that in the case of circular polarisation, the transverse components are a zero-order Bessel beam. In this case, the longitudinal component (its contribution to the total intensity is negligible,

**Table 1.** Polarisation transformation of the Bessel beam (componentwise intensity and phase distributions).

| | *x*-component (intensity and phase) | *y*-component (intensity and phase) | *z*-component (intensity and phase) | Total intensity |
|---|---|---|---|---|
| Initial zero-order circularly polarised Bessel beam |  |  |  |  |
| Radially polarised vortex Bessel beam |  |  |  |  |
| Azimuthally polarised vortex Bessel beam |  |  | |  |

since the numerical aperture of the axicon is small) corresponds to a first-order vortex Bessel beam. For cylindrical polarisations, the transverse components essentially differ in structure from the initial zero-order Bessel beam and are a hybrid of a cosine–sinusoidal beam (inherent to cylindrical polarisations) and vortex distributions. It is seen that the transverse components of vortex Bessel beams with radial and azimuthal polarisations are orthogonal everywhere except in the central region, where they have a nonzero value of intensity and circular polarisation.

It is known [17, 22] that the binary diffractive axicon has two intensity peaks in the spectrum, corresponding to converging and diverging wavefronts. Note that, by varying the ratio of the widths of the rings [17, 23] or the central zone [24], it is possible to redistribute the energy into one or two equal peaks.

Figure 1c shows the angular spectrum for the Bessel beam shown in Fig. 1b, i.e., for a convergent conical wavefront. The position of the maximum of the angular spectrum of the Bessel beam depends on the numerical aperture (2), and its width depends on the radius of the aperture of the optical element. The width of the angular spectrum of the beam is about $0.01°$ at a 0.5 level. As the size of the element's aperture increases, the angular spectrum will narrow and, in the limit, will tend to the delta function with an angle of convergence determined by (3).

In order to shift the maximum of the spatial frequencies to the zone of large convergence angles, it is possible to reduce the period of the axicon $d$. In particular, to obtain $\theta = 20°$ and $25°$ it is necessary to produce an axicon with a period $d = 1.85$ and 1.5 μm, respectively. However, as already noted, the fabrication of a set of axicons with different periods to ensure accurate angular values is very costly. In addition, axicons with such small periods require an alignment comparable with the period of accuracy, which complicates the process of their replacement to redesign the optical system. The spatial light modulators (SLMs) could help to avoid the replacement, but it is not possible to experimentally implement axicons with periods in the required range using a standard SLM, since its cell size is on the order of a few micrometers.

Let us consider the proposed method of adjusting the angles of convergence of beams formed by an axicon by replacing a plane illuminating wave with a spherical wave [25]. There are two variants of this adjustment, i.e. either decreasing the convergence angles for an axicon with a higher numerical aperture illuminated by a diverging wavefront, or increasing the angles for an axicon with a lower numerical aperture illuminated by a converging wavefront. Obviously, the second variant is preferable, because it is technologically easier (the period of the axicon rings is larger), and in addition, this variant benefits from the distance along which the Bessel beam is retained [26]:

$$z_{\max} \approx R/\mathrm{NA}, \tag{4}$$

where $R$ is the radius of the axicon.

It follows from expression (4) that the higher the numerical aperture of the optical element, the shorter the working distance. In particular, for an axicon with a period of 2 μm and a radius $R = 8$ mm, the maximum distance $z_{\max}$ is about 25 mm. To lengthen this distance, one can increase the size of the element, but in this case it is also necessary to expand the illuminating beam, which finally leads to a decrease in the useful energy. Therefore, it is preferable to use axicons with a smaller numerical aperture.

Illumination of an axicon by a converging spherical beam will theoretically result in a broadening of the angular spectrum and a large-scale change in the Bessel beam at various distances from the optical element. This behaviour is characteristic of the beams formed by lensacons [27, 28] and fracxicons (fractional axicons) [29, 30].

Figure 2 shows the results of modelling the formation of a zero-order Bessel beam under illumination of a binary diffraction axicon with a radius $R = 8$ mm and a period of 2 μm by a converging spherical beam with a focal length of 80 mm. Comparison of the number of rings in Fig. 1b and Fig. 2a indicates an increase in the numerical aperture of the Bessel beam produced due to such illumination. The width of the angular spectrum of the beam in Fig. 2b is about $0.7°$ in the 0.5 level.

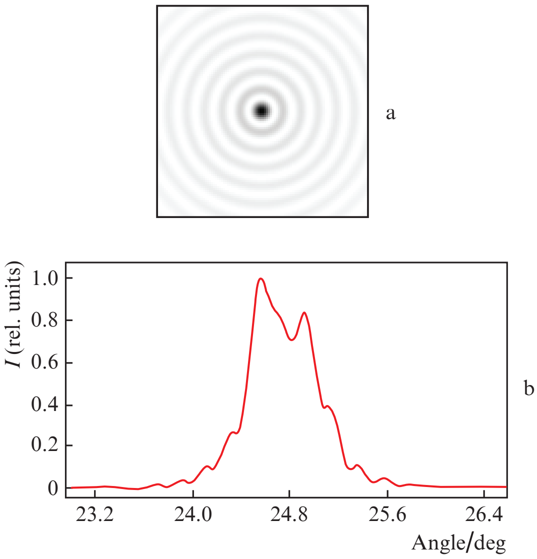



**Figure 2.** Results of modelling the formation of a zero-order Bessel beam under illumination by a converging spherical beam: (a) intensity distribution at a distance $z = 5$ mm from the axicon (negative image of size $10 \times 10$ μm) and (b) angular spectrum of the light transmitted through the axicon.

Thus, the illumination of an axicon by a converging spherical beam makes it possible to increase the numerical aperture of the beam being formed. However, the angular spectrum of the transmitted light also expands. However, as will be shown below, this expansion agrees with the operating range of the interference polariser angles, which allows one to preserve the 'purity' of the polarisation of the nonuniformly polarised Bessel beam being formed. Note also that in the case under study, the Bessel beam decreases dramatically with increasing distance from the optical element; this leads to a decrease in the distance necessary for the Bessel beam properties to be retained in comparison with a separate axicon [29, 30].

## 3. Experimental study

It is known from the results of [18] that the relative angular difference between the positions of the transmission maxima of p- and s-polarised light for an interference polariser is about 30%. Taking into account the results of the simulation, the transmission maximum of the interference polariser for p-polarisation was chosen at an incidence angle of 20°. In this case, the transmission maximum for s-polarised light should correspond to an angle of incidence of about 25°. The experiment was supposed to use a binary diffraction axicon with a period of 2 μm. In this case, p-polarised light was generated by illuminating the axicon with a plane wave (in this case, radially polarised light is formed at the output), and s-polarised light was produced by illuminating the axicon with a spherical wave having the same parameters, as those in the simulation (Fig. 2). The results of modelling for such an axicon illuminated by a plane wave (see Fig. 1) give a slightly different position of the maximum of the angular spectrum, namely, $\theta \approx 18.45°$. However, because of the small width of the spectrum (see Fig. 1c), the value of this angle need not be 20°, but may be different within the bandwidth of the interference polariser. As shown below, the shift of the maximum of the angular spectrum of the axicon toward smaller angles makes it possible to improve the 'purity' of the resulting polarisation state and to equalise the transmission for p- and s-polarised light.

An interference polariser with a working angle of 20° for a wavelength of 632.8 nm was designed and manufactured by OAO IZOVAK (Republic of Belarus, Minsk). It consists of 43 alternating $Nb_2O_5/SiO_2$ layers of different thickness, deposited on a quartz substrate with a diameter of 25.4 mm and a thickness of 3 mm. Polarisation characteristics of the element transmission were investigated with a J.A. WollamV-VASE spectral ellipsometer in the range of angles from 0 to 40° with a step of 0.2°. The wavelength of the probing light was set at 632.8 nm, the spectral width was no more than 0.5 nm. For greater accuracy, before each measurement, the light power of the spectral block of the ellipsometer (baseline) was determined, and the measurement results were averaged in time over 10 points. The dependences of the transmission of p- and s-polarised light on the angle of incidence are shown in Fig. 3. From the measured characteristics it follows that the interference polariser provides a ratio of transmission coefficients of the radial and azimuthal component $T_p/T_s$ of at least 80 : 1 to 90 : 1 for a working angle of about 20°. The maximum transmission for $T_p$ is 72%, and the transmission for $T_s$ is 62%. The bandwidth of the interference polariser is ~4° in the 0.5 level for both p- and s-polarised light. Thus, taking into account the modelled width of the angular spectra of the light transmitted through the axicon, we can draw a conclusion that there is some freedom in choosing the position of the maxima of the angular emission spectrum. In this case, since the transmission for s-polarised light is smaller, it is desirable to have an exact coincidence of the maxima of the angular spectrum of the light transmitted through the axicon and the transmission of the polariser, and for p-polarised light it is possible to equalise the transmission of the components by shifting the maximum of the angular spectrum of the incident light. As can be seen from Fig. 3, the shift of the maximum to smaller values by about 1.5° allows both the transmission of the components to be equalised and the parasitic transmission of the other polarisation to be reduced, thereby improving the 'purity' of the resulting radial polarisation. It is these values of the angles for the angular spectrum of the axicon that we strived for in performing the experiment.

To generate a zero-order Bessel beam, we employed a phase diffractive axicon with a period of 2 μm and a diameter

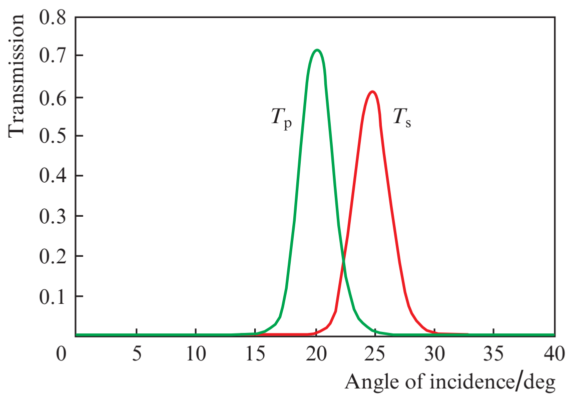



**Figure 3.** Transmission of the radial $T_p$ and azimuth $T_s$ components by an interference polariser.

of 20 mm, made on a quartz substrate with the help of a CLWS-200S circular laser writing station. An electronic photograph of the fragment of the diffractive axicon microrelief is shown in Fig. 4.

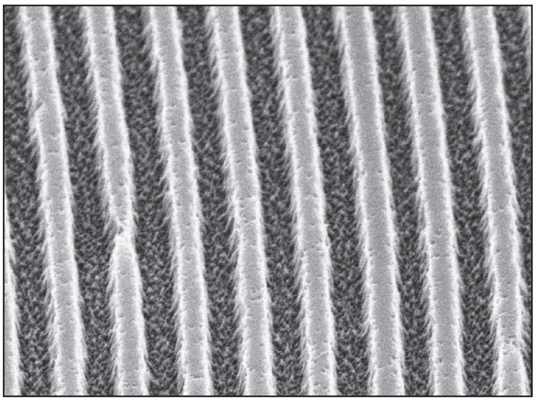


**Figure 4.** Electronic photograph of a fragment of the microrelief of the diffractve axicon.

Radially and azimuthally polarised vortex Bessel beams were produced using an optical setup, the schematic of which is shown in Fig. 5. The setup consisted of a linearly polarised helium–neon laser, a spatial filter (beam expander), a quarter-wave plate (($\lambda/4$), a diffractive axicon, an interference polariser (IP), a 40$^{\times}$ lens, a film analyser (A), and a DCM310 video matrix (CCD). The beam expander consisted of a 20$^{\times}$ lens, a 15-μm point diaphragm, and a collimating lens (L) with a focal length of 200 mm. The radius of the illuminating beam at the 0.5 level after the expander was about 10 mm.

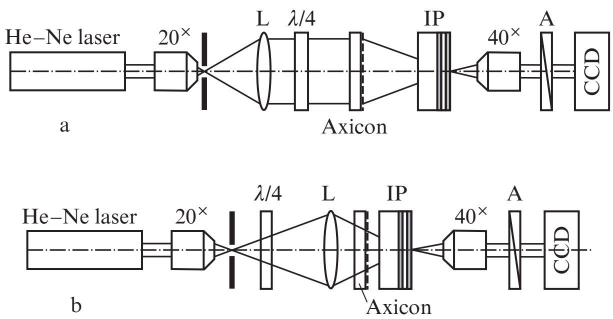



**Figure 5.** Schematics of the experimental setup for generating (a) radially and (b) azimuthally polarised vortex Bessel beams.

Figure 6 shows the intensity distributions obtained for different positions of the transmission axis of the analyser. The azimuth angle of 0° corresponds to the vertical orientation of the analyser axis. It follows from the data obtained that the generated Bessel vortex beam is radially polarised and the intensity distributions are in good agreement with the simulation results (Table 1, second row).

To form an azimuthally polarised vortex Bessel beam, in accordance with the foregoing it is necessary to illuminate the diffractive axicon by a spherical beam with a focus of 80 mm. To this end, a collimating lens with a focal length of 200 mm was replaced by a lens with a focal length of 50 mm, located

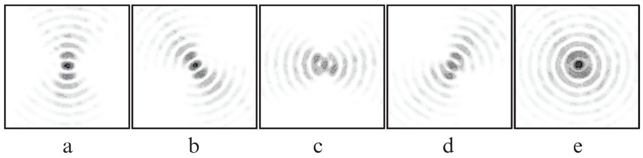



**Figure 6.** Generated vortex Bessel beam with radial polarisation at different positions of the analyser: (a) 0°, (b) 45°, (c) 90°, (d) 135°, and (e) without the analyser.

at a distance of 150 mm from the point diaphragm. The diffractive axicon was placed close to this lens, with the focus of the converging spherical beam being obtained about 80 mm from it. The radius of the illuminating beam decreases to approximately 8 mm due to the approach of the lens to the point diaphragm. Figure 7 shows the intensity distributions obtained for different positions of the analyser axis. From the obtained data it follows that the generated Bessel vortex beam azimuthally polarised and the intensity distributions are in good agreement with the simulation results (Table 1, third row).

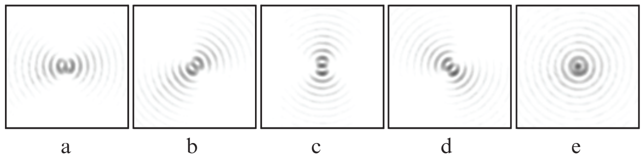



**Figure 7.** Generated vortex Bessel beam with azimuthal polarisation at different positions of the analyser: (a) 0°, (b) 45°, (c) 90°, (d) 135°, and (e) without the analyser.

## 4. Conclusions

An adjustable optical system is proposed for the formation of both radially and azimuthally polarised vortex Bessel beams, which includes an illuminating optical system, a binary-phase axicon and an interference polariser. The adjustment of the system is based on the shift of the maximum of the angular spectrum of the light transmitted through the diffractive axicon illuminated by a spherical wave instead of a plane one. The formation of radial and azimuthal polarisations was carried out by an interference polariser at average angles of incidence 18.45° and 25°, respectively. Images of nonuniformly polarised beams were obtained with a high-aperture microscope. Radially and azimuthally polarised vortex Bessel beams can be used for optical communication and materials processing.

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