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Effect of self-phase modulation and cross-phase modulation on OFDM signals in fibre-optic access networks

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Abstract. **We consider a passive fibre-optic transmission system using orthogonal subcarriers in channels with quadrature phase-shift keying and quadrature-amplitude modulation formats. A technique is proposed for estimating the signal quality index in orthogonal frequency division multiplexing, based on the transformation of phase distortions of channel carriers into distortions of signal levels. The deterioration in the quality index is found to be due to nonlinear phenomena (self-phase modulation and cross-phase modulation) and noise of the optical amplifier. Relations are given that allow an optimum power level of the optical signal to be chosen as a function of the modulation format of the channels.**

Keywords: *passive optical networks, wavelength division multiplexing, orthogonal frequency division multiplexing, self-phase modulation, cross-phase modulation, Q-factor.*

1. Introduction

According to analysts' forecasts, existing access networks based on passive optical network (PON) technology, in which active equipment is located only at transmitting/receiving endpoints, and transmission distances are small (on the order of several tens of kilometres), will not keep pace with the increasing duplex traffic in the near future. Increasing the PON capacity without significant investments on their reconstruction is an urgent task. The application of frequency division multiplexing methods allows this problem to be solved and ensures users that they are provided with duplex broadband services. In the first stage, it is advantageous to make use of frequency division multiplexing in the electrical frequency range, and in the future, with the development of PONs, to employ dense wavelength division multiplexing (DWDM) [1–3].

Modern technical capabilities of digital data processing allow the spectral efficiency to be increased by means of orthogonal frequency division multiplexing (OFDM) technology [4, 5]. In OFDM technology, the transmitted data is first distributed over a plurality of frequency subcarriers orthogonal to each other, the number of which can reach thousands, and then the subcarriers are transmitted in parallel. Despite the fact that with an increase in the number of channels the total frequency band expands, the separately taken frequency bands of channel subcarriers are relatively narrow and nonuniformities of frequency-modulation characteristics of optical and optoelectronic components in the transmission system do not have a significant effect on the transmission quality of channel signals. Subcarrier OFDM channels can be formed using various digital modulation techniques, for example, quadrature phase-shift keying (QPSK) or quadrature amplitude modulation (M-QAM) [6].

Obviously, to increase the capacity of optical transmission systems of OFDM signals, it is necessary to increase the number of channel subcarriers, which leads to an increase in the total optical power in the fibre. An increase in optical power can give rise to such nonlinear phenomena as four-wave mixing, self-phase modulation (SPM), and cross-phase modulation (XPM) [7, 8]. Quite dangerous for frequency division multiplexing systems is four-wave mixing, when, due to nonlinear susceptibility of the third order in an optical fibre, unwanted combinational frequency components appear that fall into the frequency bands of channels. To maintain the required signal quality, it is needed to limit the number of channels or reduce the power in them, to use nonuniform methods of spectral channel allocation, and to apply technical solutions, for example, methods for compensating for four-wave mixing or methods of noise-immune signal coding [9].

The phenomenon of SPM occurs in an optical fibre due to the dependence of the refractive index on the light intensity and causes a change in the optical carrier phase, proportional to the intensity of the signal. In the case of the OFDM-signal transmission, SPM manifests itself as a self-action, i. e., the power level in the subcarrier channel affects its subcarrier frequency, resulting in a frequency deviation of the subcarrier. However, due to a low power in the subcarrier channels, the influence of SPM on the channel quality indices is insignificant compared to the influence of XPM. Unlike SPM, the effect of XPM, like four-wave mixing, arises in frequency-separated multi-channel transmission systems, with the effective refractive index of the fibre at a certain subcarrier frequency being dependent not only on the signal power in the given channel, but also on the powers in other channels.

As applied to the OFDM signal, XPM results in additional phase modulation of frequency subcarriers, the magnitude of which depends on the powers in the other channels at the time of observation. It was shown in Refs [7, 8] that the contribution of XPM depends on the number of channels, if the channels are formed by amplitude-pulse modulation methods, and can become a factor limiting the capacity of the transmission system and causing the reduction of the number of channels or the limitation of the power in them. In this case, the possibility of using phase modulation formats in subcarrier channels in which the information is 'transferred' by the phase rather than by the amplitude of the signal can

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lead to a resultant averaging of the optical power and a weakening of the contribution of XPM to the increase in the number of channels [10]. In addition, to increase the bandwidth, it is efficient to form channel subcarriers not only with phase modulation formats, but also with multilevel, amplitud–phase, ones. However, multilevel channel signals are more susceptible to phase distortions arising in the optical path due to SPM and XPM, since additional phase modulation in the channels can be transformed into amplitude fluctuations during signal transmission along the optical path and lead to a decrease in the signal quality indices on the receiving side.

To compensate for signal power losses in the optical path, use is made of optical amplifiers. At present, erbium-doped fibre amplifiers (EDFA) are commercially available and widely used. The main source of linear noises is the amplified spontaneous emission (ASE) of the optical amplifier, which arises during photodetection when the amplifier's spontaneous noise is mixed with the OFDM signal.

Note that at present, due to an increase in the load on trunk fibre-optic transmission systems by DWDM channels, the distortion of signals due to nonlinear phenomena in the optical fibre is widely studied. In Refs [11–13], models were proposed that allow one to estimate the quality of channel signals in the case of nonlinear distortions, treating the latter as nonlinear noise, and to justify the additive addition of the powers of the nonlinear noise and ASE noise of the optical amplifier, which is confirmed by the results of experimental studies. In the present work, when evaluating the combined influence of SPM, XPM, and ASE noise on the channel signals, the results of these studies are taken into account.

The purpose of this paper is to evaluate the contributions of SPM and XPM to the OFDM signal quality indices using QPSK and M-QAM modulation formats in fibre-optic systems with direct photodetection. The quality of signal transmission is assessed with the help of the Q-factor. The methodology for estimating the Q-factor is based on the calculation of the phase distortions of the channel carriers due to SPM and XPM and their transformation into signal level distortions. The analysis also takes into account the noise of the optical amplifier, gives recommendations for choosing the optimal level of optical power, depending on the modulation format of the channel.

2. Mathematical model

Figure 1 shows a schematic of a fibre-optic OFDM transmission system. The information signals necessary for transmission are converted into an OFDM signal in the OFDM driver,

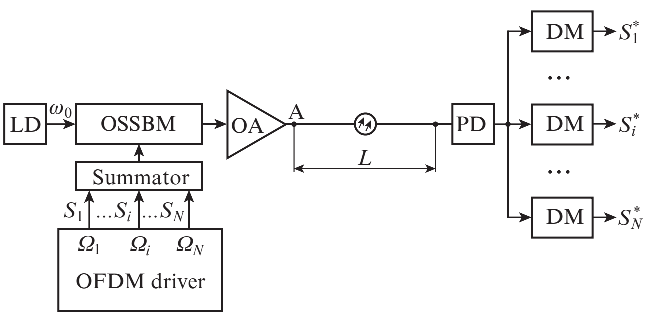

Figure 1. Schematic of a fibre-optic OFDM transmission system (see text).

at the output of which we have N spaced channels with frequencies Ω_i ($i = 1,2,...,N$). Depending on the modulation format, each channel signal can be modulated in amplitude by an information signal sample $S_i(k,t)$ with a duration T_s, and also in phase $\varphi_i(k,t)$, where k characterises the number of values (position) of the modulated signal parameter. For example, if QPSK signals are transmitted in the channels, then $S_i(k,t) =$ const, and $\varphi(k,t) = (2k-1)\pi/4$ ($k = 1,2,3,4$). For the OFDM signal, the frequency spacing between channels is $\Delta\Omega = 2\pi/T_s$. After combining in the summator, a group multichannel signal

$$S(t) = \sum_{i=1}^{N} S_i(k,t)\cos[\Omega_i t + \varphi_i(k,t)] \tag{1}$$

arrives at an optical single-sideband modulator (OSSBM), the other input of which receives light with a frequency ω_0 from a coherent source, for example, from a laser diode (LD). At the OSSBM output, the optical field contains the spectral component E_0 at the frequency ω_0 and the single sideband (SSB) at $\omega_1 - \omega_N$, where $\omega_i = \omega_0 + \Omega_i$ are the optical frequencies of the ith channel subcarriers ($i = 1,2,...,N$). The implementation of optical single-sideband modulation is possible, for example, using a Mach–Zehnder modulator on a $LiNbO_3$ crystal with double electrodes [14,15]. To compensate for losses in the optical path, an optical amplifier (OA) is used at the transmitting end.

Thus, at point A (Fig. 1) an optical OFDM signal is formed, the spectrum of which is shown in Fig. 2 (left-hand plot). The optical signal, propagating over a distance L along the fibre, is detected by a photodiode (PD) and signal samples are extracted using demodulators (DM). To estimate the distortion of signal samples due to the nonlinear phenomena of SPM, XPM, and OA noise, it is necessary to find the Q-factor at the outputs of the demodulators.

If we assume that the OSSBM has a linear modulation characteristic, then at point A, the optical field strength is determined by the carrier with the amplitude E_0 and the group single-sideband signal

$$E_{\rm SSB}(t) = \sum_{i=1}^{N} E_i(t) = \sum_{i=1}^{N} E_i(k,t)\cos[\omega_i t + \varphi_i(k,t)], \tag{2}$$

where $E_i(k,t)$ are the field amplitudes corresponding to multilevel signal samples in the ith channel; and $\varphi_i(k,t)$ are the signal phases in the ith channel.

For further analysis, we transform (2) with respect to some fictitious optical frequency $\omega_{\rm f} = (\omega_1 + \omega_N)/2$:

$$\begin{aligned} E_{\rm SSB}(t) &= \sum_{i=1}^{N} E_i(k,t)\cos[(\omega_i - \omega_{\rm f})t + \omega_{\rm f}t + \varphi_i(k,t)] \\ &= \sum_{i=1}^{N} E_i(k,t)\cos[(\omega_i - \omega_{\rm f})t + \varphi_i(k,t)]\cos(\omega_{\rm f}t) \\ &- \sum_{i=1}^{N} E_i(k,t)\sin[(\omega_i - \omega_{\rm f})t + \varphi_i(k,t)]\sin(\omega_{\rm f}t) \\ &= A(t)\cos(\omega_{\rm f}t) - B(t)\sin(\omega_{\rm f}t), \end{aligned} \tag{3}$$

where

$$A(t) = \sum_{i=1}^{N} E_i(k,t)\cos[(\omega_i - \omega_{\rm f})t + \varphi_i(k,t)], \tag{4}$$

and

$$B(t) = \sum_{i=1}^{N} E_i(k,t)\sin[(\omega_i - \omega_f)t + \varphi_i(k,t)]. \quad (5)$$

As seen from (3), $E_{SSB}(t)$ is an amplitude-modulated oscillation at the fictitious frequency ω_f:

$$E_{SSB}(t) = E_m(t)\cos[\omega_f t + \phi(t)], \quad (6)$$

with an amplitude and a phase

$$E_m(t) = \sqrt{A^2(t) + B^2(t)}, \quad (7)$$

$$\tan\phi(t) = \frac{B(t)}{A(t)}. \quad (8)$$

Thus, the multichannel signal can be represented as an amplitude-modulated oscillation at a fictitious frequency ω_f with an amplitude $E_m(t)$, which is shown schematically in Fig. 2 (right-hand plot). In this case, the analysis is simplified, since instead of a multichannel signal consisting of frequency-separated channel subcarriers, we consider one fictitious amplitude-modulated signal whose power is equal to the power of the group single-sideband signal. Therefore, knowing the amplitude of this fictitious signal, one can find its power. Substituting expressions (4) and (5) into (7) and performing a series of trigonometric transformations, we obtain

$$E_m^2(t) = \sum_{i=1}^{N} E_i^2(k,t) + \sum_{\substack{l=1 \\ l\neq p}}^{N}\sum_{p=1}^{N} E_l(k,t)E_p(k,t) \times \cos[(\omega_l - \omega_p)t + \varphi_l(k,t) - \varphi_p(k,t)]. \quad (9)$$

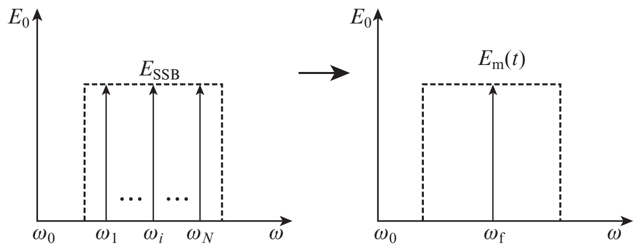

Figure 2. Spectrum of the optical OFDM signal.

Assuming that with optical single-sideband modulation at point A (Fig. 1), 50% of the input optical power (P_{in0}) is concentrated in the optical carrier, and the rest of it is evenly distributed between the subcarrier channels [14], then, given that the power $P \propto E^2(t)$ and also the fact that

$$\sum_{i=1}^{N} E_i^2(k,t) \approx NE_i^2 \approx \frac{P_{in0}}{2}, \quad E_l(k)E_p(k) \leqslant E_i^2,$$

we have

$$P_{SSB}(t) \leqslant \frac{P_{in0}}{2} \times \left\{1 + \frac{1}{N}\sum_{\substack{l=1 \\ l\neq p}}^{N}\sum_{p=1}^{N}\cos[(\omega_l - \omega_p)t + \varphi_l(k,t) - \varphi_p(k,t)]\right\}. \quad (10)$$

Thus, the total power in an optical fibre is given by

$$P_\Sigma(t) \leqslant \frac{P_{in0}}{2} + P_{SSB}(t) \approx P_{in0} \times \left\{1 + \frac{1}{2N}\sum_{\substack{l=1 \\ l\neq p}}^{N}\sum_{p=1}^{N}\cos[(\omega_l - \omega_p)t + \varphi_l(k,t) - \varphi_p(k,t)]\right\}, \quad (11)$$

where P_{in0} is the average value of the optical power at the fibre input (at point A). The second term in (11) shows fluctuations in the optical power near the mean value in time; the magnitude of the fluctuations depends on the frequency difference and the actual phases of the pair subcarriers of the channel signals.

The SPM and XPM phenomena lead to additional phase shifts in the subcarrier channels. The nonlinear phase shift depends on the total optical power in the fibre [7, 8]:

$$\Phi_{nl}(t) = \gamma L_{eff} P_\Sigma(t), \quad (12)$$

where γ and L_{eff} are the nonlinear coefficient and the effective length of the optical fibre, respectively. For standard single mode fibre (SSMF), we have γ = 1.2 W^{-1} km^{-1} and L_{eff} = 12.6 km at L = 20 km.

Substituting (11) into (12) and taking into account the fact that the phase shift in SPM is proportional to the power of the subcarrier channel, we obtain

$$\Phi_{SPM} = \gamma L_{eff}\frac{P_{in0}}{2N}, \quad (13)$$

$$\Phi_{XPM} = \gamma L_{eff}\frac{P_{in0}}{2}\left\{2 - \frac{1}{N} + \frac{1}{N}\sum_{\substack{l=1 \\ l\neq p}}^{N}\sum_{p=1}^{N}\cos[(\omega_l - \omega_p)t + \varphi_l(k,t) - \varphi_p(k,t)]\right\}, \quad (14)$$

$$\Phi_{nl}(t) = \Phi_{SPM} + \Phi_{XPM}. \quad (15)$$

Comparing (13) and (14), we see that the phase shift due to SPM is time-independent and insignificant in comparison with the shift due to XPM, since the optical power per one subcarrier channel is small and equal to $P_{in0}/(2N)$. The phase shift due to XPM has an average value of $\gamma L_{eff} P_{in0}$ and randomly fluctuates depending on the transmitted information symbols. If we do not consider the random character of the transmitted data, the resulting value of the double sum in (14) depends strongly on the number of channels and is $N(N-1)$. In this case, Φ_{XPM} is $\sim N^2$ times greater than Φ_{SPM}. This approach excessively overestimates the effect of XPM on subcarrier channels and is unacceptable for calculations, since it does not take into account the random nature of the information data.

To more accurately estimate the influence of the phase shift due to XPM on the subcarrier channels, computer simulations were performed to calculate the double sum in (14), taking into account the discrete nature of phase changes of multilevel signals in the subcarrier channels. Computer modelling was performed in the MathCAD environment using the built-in 'mean' and 'stdev' functions to calculate the mean and standard deviation of the phase σ_{ph} during a single sample. The simulation results show that the random nature of the change in amplitude–phase positions in information signals in different subcarrier channels leads to the fact that at N

> 32 the third term in (14) is described by the normal phase distribution law.

Figure 3a shows an example of optical carrier phase fluctuations during a single sample when $N = 128$ subcarriers of QPSK channels are transmitted with a frequency spacing of 64 MHz. One can see that XPM leads to the emergence of high-frequency (relative to the channel frequencies) phase fluctuations. Figure 3b shows the phase distribution density as a histogram. The analysis shows that the mean value of the fluctuating phase tends to zero, and its standard deviation tends to $\sigma_{\rm ph} \approx 1/\sqrt{2} \approx 0.707$, which indicates the independence of the standard deviation $\langle \Phi_{\rm XPM} \rangle$ on the number of channels at $N > 32$:

$$\langle \Phi_{\rm XPM} \rangle \approx \gamma L_{\rm eff} \frac{P_{\rm in0}}{2}(2 + \sigma_{\rm ph}) \approx 1.35\gamma L_{\rm eff} P_{\rm in0}. \qquad (16)$$

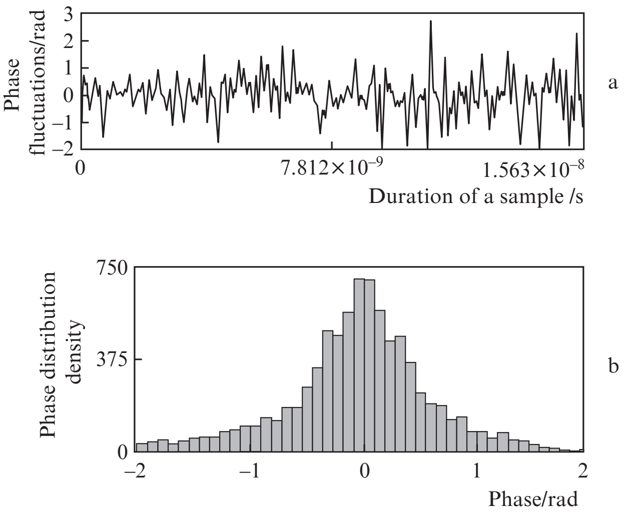

Figure 3. (a) Example of phase fluctuations due to XPM during one sample and (b) phase distribution density when $N = 128$ QPSK-modulated subcarrier channels are transmitted with a frequency spacing of 64 MHz.

Comparing (13) and (16), we see that a significant contribution to the signal distortion is made by XPM, and the contribution of SPM can be neglected even at $N > 4$. For example, for a fibre optic transmission system with an SSMF fibre length of $L = 20$ km, the number channels $N = 64$ and a power $P_{\rm in0} \leqslant 10$ mW, the calculated phase shifts are as follows: $\Phi_{\rm SPM} \leqslant 1.2 \times 10^{-3}$ and $\Phi_{\rm XPM} \leqslant 0.2$. These nonlinear phase shifts are received by all subcarrier channels. Consider the optical field of the ith channel subcarrier, which has reached the PD:

$$E_i(t) = E_i(k,t)\cos[\omega_i t + \varphi_i(k,t) + \Phi_{\rm nl}(t)]$$
$$= E_i(k,t)\{\cos[\omega_i t + \varphi_i(k,t)]\cos[\Phi_{\rm nl}(t)]$$
$$- \sin[\omega_i t + \varphi_i(k,t)]\sin[\Phi_{\rm nl}(t)]\}. \qquad (17)$$

We simplify (17), taking into account that $\Phi_{\rm nl} < 1$,

$$E_i(t) \approx E_i(k,t)\cos[\omega_i t + \varphi_i(k,t)]$$
$$- E_i(k,t)\Phi_{\rm nl}(t)\sin[\omega_i t + \varphi_i(k,t)]. \qquad (18)$$

During detection, the optical carrier mixes with the subcarrier signals and at the PD output we have a photocurrent I proportional to $[E_0\cos(\omega_0 t) + E_{\rm SSB}(t)]^2$ averaged over a time interval T greater than the optical field period:

$$I \propto \lim_{T\to\infty}\frac{1}{T}\int_0^T [E_0\cos(\omega_0 t) + E_{\rm SSB}(t)]^2 {\rm d}t$$
$$= \langle E_0^2\cos^2(\omega_0 t) + 2E_0E_{\rm SSB}(t)\cos(\omega_0 t) + E_{\rm SSB}^2(t)\rangle$$
$$= \langle I_0 \rangle + \langle I_{\rm av} \rangle + \langle I_{\rm n} \rangle, \qquad (19)$$

where $\langle I_0 \rangle$ is the constant component of the photocurrent; $\langle I_{\rm av} \rangle$ is the average photocurrent corresponding to the transmitted OFDM signal $S(t)$; $\langle I_{\rm n} \rangle$ is the average noise current due to the signal–signal beating interference (SSBI). Note that the spectrum of beating interference occupies a low-frequency region and its influence on subcarriers can be eliminated by selecting a guard band between the carrier and the OFDM signal [4].

3. *Q*-factor

To predict the distortions of the signals of the channel subcarriers arising in the optical path due to SPM and XPM, we assume that the photodetection process and subsequent demodulation of the signals do not introduce distortions. In this case, the photocurrent corresponding to the ith channel of the OFDM signal is

$$\langle I_i \rangle \propto \lim_{T\to\infty}\frac{1}{T}\int_0^T 2E_0E_i(t)\cos(\omega_0 t){\rm d}t \propto S_i(k,t)$$
$$\times \{\cos[\Omega_i t + \varphi_i(k,t)] - \Phi_{\rm nl}(t)\sin[\Omega_i t + \varphi_i(k,t)]\}. \qquad (20)$$

The first term in (20) is an undistorted transmitted signal in the ith channel, and the second term is the quadrature distortion of the signal in this channel. In Fig. 4, the IQ diagram corresponding to the QPSK signal in the Cartesian coordinate system shows the resulting quadrature distortion of the signal due to SPM and XPM for two samples of the information signal ($k = 1$ and 4). One can see that SPM and XPM lead

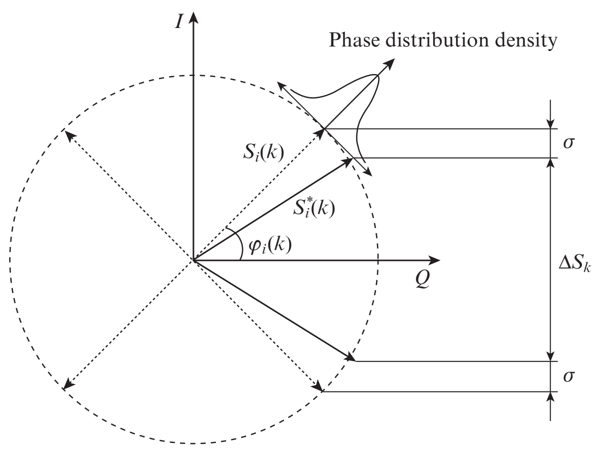

Figure 4. Transmitted signal sample $S_i(k)$ and received signal sample $S_i^*(k)$, distorted due to the influence of SPM and XPM, on the IQ diagram of the QPSK signal.

to the appearance of quadrature noise with a standard deviation σ in the transmission of each signal sample.

It is seen from Fig. 4 that for a QPSK signal the standard deviation of noise σ due to XPM is found through the absolute value of the difference of distorted and undistorted samples: $\sigma = |S_i^*(k) - S_i(k)|\cos[\varphi(k)]$, where $\varphi(k) = \pi/4$. On the other hand, $|S_i^*(k) - S_i(k)|$ is determined after the channel signal demodulation (20); this difference can be found from the right-angled triangle (Fig. 4), formed by a leg $S_i(k)$ and hypotenuse $S_i^*(k)$:

$$[S_i^*(k)]^2 = S_i^2(k) + S_i^2(k)\langle\Phi_{\rm nl}\rangle^2,$$

hence, $|S_i^*(k) - S_i(k)| \approx S_i(k)\langle\Phi_{\rm nl}\rangle^2/2$. Consequently,

$$\sigma = S_i(k)\langle\Phi_{\rm nl}\rangle^2\cos[\varphi(k)]/2. \tag{21}$$

The channel signal quality during reception due to the nonlinear phase shift is determined by the expression [8]

$$Q_{\rm XPM} = \frac{\Delta S_k^{\min}}{2\sigma} = \frac{\Delta S_k^{\min}}{S_i(k)\langle\Phi_{\rm nl}\rangle^2\cos[\varphi(k)]}. \tag{22}$$

When normalising the constellations of samples of a multilevel signal, the minimum distance between the samples of the information signal is $\Delta S_k^{\min} = \sqrt{2}/(\sqrt{M} - 1)$ (M is the signal positionality [2,3]). Then $\Delta S_k^{\min} = \sqrt{2}$ for QPSK, $\sqrt{2}/3$ for 16-QAM, $\sqrt{2}/7$ for 64-QAM and $\sqrt{2}/15$ for 256-QAM.

Taking into account (16), the normalisation of the samples $[S_i(k) \to 1]$ and the worst position of neighbouring samples from the point of view of the quality index of multilevel signals $[\varphi(k) \to 0]$, we obtain

$$Q_{\rm XPM} \geqslant \frac{\Delta S_k^{\min}}{(1.35\gamma L_{\rm eff} P_{\rm in0})^2}. \tag{23}$$

To simplify the analysis, we neglected in (19) the noise component of amplified spontaneous emission, i. e. the OA noise that has reached the PD, at the output of which an additional noise current appears. The quality of the channel signal during reception due to the influence of ASE noise is determined by the expression [2, 3]

$$Q_{\rm ASE} = \frac{\sqrt{P_{\rm in0}}\,\Delta S_k^{\min}}{2N\sqrt{\hbar\omega_i\Delta\Omega F/(2\pi)}}, \tag{24}$$

where F is the OA noise factor (in calculations $F = 4$ dB).

To determine the combined effect of ASE noise and XPM noise on the signal quality, it is necessary to take into account their random nature and independence from each other, i. e., the variance of the resulting noise is equal to the sum of the variances of each noise. Thus, the total Q_Σ-factor can be found from the relation [11–13, 16]

$$1/Q_\Sigma^2 \approx 1/Q_{\rm XPM}^2 + 1/Q_{\rm ASE}^2. \tag{25}$$

4. Results of calculations

The calculations were performed using formulas (22)–(25) for QPSK, 16-QAM, 64-QAM and 256-QAM modulation formats. Figure 5 shows, as an example, the dependences of Q-factors on the total power in the fibre for different number of QPSK-modulated channel subcarriers (N = 64, 128, 256 and 512). This figure corresponds to the case of transmitting an OFDM signal at a wavelength of 1.55 μm over a SSMF with L = 20 km, when the frequency spacing between subcarriers is 64 MHz. Figure 5 also shows the straight lines that limit the Q-factor in power, which correspond to XPM ($Q_{\rm XPM}$) and four-wave mixing ($Q_{\rm FWM}$). Note that the factor $Q_{\rm FWM}$ is calculated by the formulas given in [17]. It can be seen that the limiting factor is four-wave mixing, rather than XPM. This is due to the fact that the frequency spacing between subcarriers is relatively small (up to several GHz) and the transmission distance is 20 km. With increasing power in the channels, when the total optical power in the fibre exceeds 12 dBm, QPSK-signal transmission will become impossible, since for reliable signal transmission (without applying correction coding) with an error probability of 10^{-12}, it is necessary that $Q_\Sigma > 17$ dB [18]. An increase in the number of channels with the total optical power in the fibre to 12 dBm leads to a decrease in the total Q_Σ-factor due to the influence of ASE noise, since at the same noise levels the power per one channel decreases. An increase in the number of channels decreases twofold the total Q_Σ-factor by ~6 dB. Thus, there may be a situation where, in addition to the upper limit, it is necessary to provide a minimum level of the total optical power in the fibre. For example, in Fig. 5, when transmitting N = 512 subcarriers, the minimum optical power should exceed +1.4 dBm.

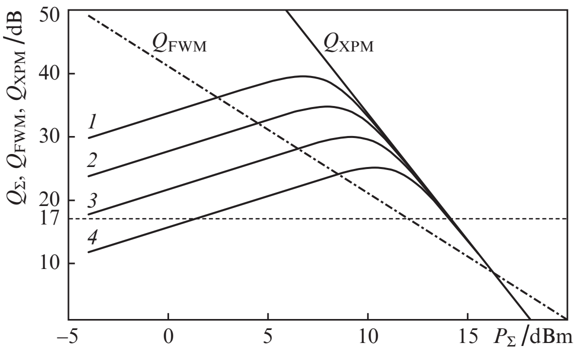

Figure 5. Dependences of Q-factors on the total power P_Σ in a 20-km-long SSMF for QPSK-modulated channel subcarriers with a 64-MHz frequency spacing for N = (*1*) 64, (*2*) 128, (*3*) 256 and (*4*) 512.

The situation changes when amplitude–phase modulation methods are used in subcarrier channels. The analysis shows that with a frequency spacing between subcarriers of not more than 1 GHz and for M-QAM modulation formats, where $M > 16$, the XPM becomes the dominant undesirable phenomenon. Figure 6 shows an example of the dependence of Q-factors on the total power in the fibre for a different number of 64-QAM-modulated channel subcarriers (N = 64, 128, 256, and 512). As in the case of transmission of QPSK signals, the frequency spacing between subcarriers is 64 MHz. Comparing Figs 5 and 6, we see that the M-QAM-modulated signals are about $\sqrt{M} - 1$ times less noise-immune than the QPSK-modulated ones. Moreover, with the number of channels $N > 128$, when $Q_\Sigma < 17$ dB, the channels require the use of forward error correction (FEC) technology [15].

Note that if the optical power in the fibre is maintained at a given level, then the contribution of nonlinear interference due to four-wave mixing does not depend on the number of channels (if $N \gtrsim 64$) [17]. The results of noise calculations due to XPM show a similar pattern. Moreover, it follows from the

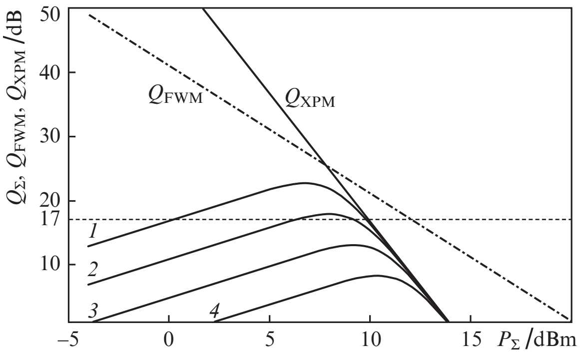

Figure 6. Dependences of Q-factors on the total power P_Σ in a 20-km-long SSMF for 64-QAM-modulated channel subcarriers with a 64-MHz frequency spacing for N = (*1*) 64, (*2*) 128, (*3*) 256 and (*4*) 512.

calculations that the noise due to XPM is slightly dependent on the variation of the frequency spacing between the subcarriers if the maximum frequency spacing does not exceed 1 GHz, in contrast to four-wave mixing, in which small frequency spacings between channels lead to a strong channel interaction [9]. On the other hand, an increase in the frequency spacing between subcarriers causes a significant contribution of ASE noise. Therefore, in designing fibre-optic transmission systems for OFDM signals with specified quality criteria, it is necessary to choose compromise solutions, taking into account not only the frequency spacing between the channels, the modulation format of signals and the OA noise that have reached the PD, but also the effect of XPM and four-wave mixing on signals.

5. Conclusions

We have considered a fibre-optic transmission system for OFDM signals in which subcarrier channels have QPSK and M-QAM modulation formats. A technique for estimating the OFDM-signal quality due to SPM and XPM has been proposed. The analysis shows the following:

– The influence of SPM and XPM on the signal quality does not depend on the number of channel subcarriers in the OFDM signal at a sufficiently large number (more than 32), and also on the frequency spacing between them if the optical power in the fibre is kept constant.

– In practice, the influence of SPM on OFDM-signal subcarrier channels can be neglected because of the low power in these channels.

– To ensure the required noise immunity, it is necessary to limit the total optical power in the fibre. QPSK-modulated signals are less susceptible to XPM, M-QAM-modulated signals are more prone to XPM. At $M = 16$, the contribution from the XPM is commensurate with the contribution from four-wave mixing, and at $M \geqslant 64$ XPM becomes the dominant limiting noise factor.

– The minimum optical power in the subcarrier channels is necessary to overcome the effect of amplified spontaneous emission noise of the optical amplifier. This power depends on the frequency spacing between the subcarrier channels and their number in the OFDM signal.

The results obtained can be used in the design of passive optical networks using OFDM technology and show the need to take into account the nonlinear XPM phenomenon occurring in an optical fibre, especially when transmitting M-QAM signals.

References

1. Vardanyan V.A. *Elektrosvyaz'*, **7**, 61 (2017).
2. Vardanyan V.A. *Optoelectron. Instrum. Data Process.*, **52** (3), 311 (2016).
3. Vardanyan V.A. *Dokl. TUSUR*, **20** (1), 33 (2017).
4. Erkılınç M.S., Pachnicke S., Griesser H., Thomsen B.C., Bayvel P., Killey R.I. *IEEE J. Lightwave Technol.*, **33** (10), 2038 (2015).
5. Armstrong J. *IEEE J. Lightwave Technol.*, **27** (3), 189 (2009).
6. Vardanyan V.A. *Optoelectron. Instrum. Data Process.*, **51** (6), 544 (2015).
7. Shneider T. *Nonlinear Optic in Telecommunications* (Berlin–Heidelberg: Springer-Verlag, 2004).
8. Agrawal G.P. *Lightwave Technology: Telecommunication Systems* (Hoboken, New Jersey: John Wiley & Sons, Inc., 2005).
9. Vardanyan V.A. *Optoelectron. Instrum. Data Process.*, **53** (1), 51 (2017).
10. Listvin V.N., Tretshchikov V.G. *DWDM sistemy* (DWDM systems) (Moscow: Nauka, 2013).
11. Zhitelev A.E., Konyshev V.A., Lukinikh S.N., Nanii O.E., Treshchikov V.N., Ubaydullaev R.R. *Quantum Electron.*, **47** (12), 1135 (2017) [*Kvantovaya Elektron.*, **47** (12), 1135 (2017)].
12. Gurkin N.V., Konyshev V.A., Nanii O.E., Novikov A.G., Tretshchikov V.N., Ubaydullaev R.R. *Quantum Electron.*, **45** (1), 69 (2015) [*Kvantovaya Elektron.*, **45** (1), 69 (2015)].
13. Gurkin N.V., Nanii O.E., Novikov A.G., Plaksin S.O., Treshchikov V.N., Ubaydullaev R.R. *Quantum Electron.*, **43** (6), 550 (2013) [*Kvantovaya Elektron.*, **43** (6), 550 (2013)].
14. Hui R., Zhu B., Huang R., et al. *IEEE J. Lightwave Technol.*, **20** (3), 417 (2002).
15. Gutierrez F.A., Perry P., Smyth F., et al. *IEEE/OSA J. Opt. Commun. Networking*, **7** (4), 248 (2015).
16. Shiva Kumar. *Impact of Nonlinearities on Fiber Optic Communications* (New York–Dordrecht–Heidelberg–London: Springer, 2011).
17. Vardanyan V.A. *Meas. Tech.*, **60** (7), 724 (2017).
18. *ITU-T Recommendation, Ser. G, Supplement* 39 (02/2016); https://www.itu.int/rec/dologin_pub.asp?lang=e&id=T-REC-G.Sup39-201209-S!!PDF-E&type=items.