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A method for measuring coupling coefficients between cores and corrections to mode propagation constants in multicore fibres

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Abstract. **A new experimental method is proposed for reconstructing supermode profiles of a multicore fibre (MCF), small corrections to the propagation constants of its cores and coupling coefficients between the cores without phase-sensitive measurements. The method is based on intensity measurements in the cores at the output of several MCF segments differing in length, using several configurations of controlled light launch into the cores at the fibre input (in the simplest case, by sequentially exciting each core), followed by numerical treatment using an iterative algorithm. The use of the method is demonstrated experimentally for a seven-core fibre. Using numerical simulation, we assess the stability of the method and the effect of the number of measurements on the accuracy of fibre parameter reconstruction.**

Keywords: *multicore fibres, propagation constants, coupling coefficients, supermodes.*

1. Introduction

Multicore fibres (MCFs) have attracted a great deal of attention as promising media for high-speed information transfer systems [1] and as components of various photonic devices and laser systems [2]. There is particular interest in MCFs with a relatively strong optical coupling between their cores, which may lead to efficient radiation transfer between them. Such fibres have great potential for many applications, including high-speed information transfer with coherent space-division multiplexing based on coding using fibre supermodes [3] or modes with an orbital angular momentum [4], quantum optical devices [5], nonlinear switches and saturable absorbers for mode-locked lasers [6], nonlinear endoscopy [7] and even narrow-band filters for astronomical observations [8]. In recent years, ever increasing attention has been paid to the use of passive and active MCFs as key components of high-power laser and amplifier systems [9], where they can be used for the amplification, transmission and control of high-power laser beams, while maintaining a high degree of coherence between their cores.

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In an MCF with strong coupling between its cores, the mode parameters of individual cores and coupling between modes should be controlled with high accuracy to ensure proper predictable interaction, especially if there is strong nonlinearity characteristic of high-power laser systems. It is important to note that, as a result of even small deviations of the fibre structure (shape and arrangement of the fibre cores) and small additional variations in the refractive index, caused by either distinctions between the core preforms or inhomogeneous mechanical stress, the mode propagation constants and coupling coefficients between cores differ from those computed for an ideal structure. Such small differences have a significant effect on light propagation and should be monitored and taken into account in designing and performing experiments for gaining insight into nonlinear processes in MCFs. In designing and fabricating MCFs, one should take measures to reduce the effect of these factors, and a proper experimental method of measuring guidance parameters is needed for monitoring. Moreover, for performing full-scale numerical simulations with realistic parameters of experimentally fabricated MCFs, the actual parameters of their cores and coupling between them, which determine the properties of their guided modes, should be known with high accuracy.

Sufficiently accurate measurements of deviations of the fibre structure from an ideal one during the fibre fabrication process are a challenging problem [10]. For example, microscopic examination of a fibre end face fails to ensure sufficient accuracy in measurements of the geometric shape of cores and gives no way of measuring refractive index variations. Direct measurements of guidance parameters of fibres also face serious difficulties. For fibres with weak core-to-core coupling, some information about coupling coefficients can be gained using optical time domain reflectometry (OTDR) and a long length of fibre [11], but corrections to propagation constants then remain unknown. More complete information about the guidance properties of MCFs can be obtained by measuring the field amplitude and phase at the MCF output, but absolute phase measurements are difficult to perform in experiments.

In this paper, we present a method based on measuring the light intensity at the output of several MCF segments differing in length and then solving an inverse problem for finding the parameters of interest.

2. Description of the method

Consider the propagation of monochromatic light in a multicore fibre with N identical cores, each having only one fundamental mode. The light intensity is taken to be sufficiently low for nonlinear effects to be neglected. Coupling between the cores is taken to be weak enough for the associated changes in mode profile in each core to be neglected and, at the same time, strong enough to ensure efficient energy transfer between the cores if light propagates over a distance exceeding the characteristic length scale for nonuniformities of parameters along the length of the fibre.

The amplitude of a linearly polarised electric field in each core can then be represented in the form $E_i(z,t) = A_i(z) \times F_i(x,y)\exp(\mathrm{i}wt - \mathrm{i}\beta_0 z)$, where $i = 1, 2, \ldots, N$ is the number of the core; $F_i(x,y)$ is the fundamental mode profile in the ith core; β_0 is the propagation constant; and $A_i(z)$ is a slowly varying amplitude. Let the propagation constants of different cores, $\beta_1, \beta_2, \ldots, \beta_N$, in general differ slightly. Then, we take $\beta_0 = (\beta_1 + \beta_2 + \ldots + \beta_N)/N$, and the difference between the phase shift in the core, $\exp(-\mathrm{i}\beta_i z)$, and $\exp(-\mathrm{i}\beta_0 z)$ will be taken into account in the slowly varying amplitude $A_i(z)$. The mode profile $F(x,y)$ is normalised so that the optical power is $|A_i(z)|^2$. Light propagation is then described by the equations

$$\frac{\mathrm{d}A_i(z)}{\mathrm{d}z} = \mathrm{i}\Delta\beta_i A_i(z) + \sum_{j\neq i} \mathrm{i}c_{ij}A_j, \tag{1}$$

where $\Delta\beta_j = \beta_i - \beta_0$ are corrections to the propagation constants and c_{ij} are the coupling coefficients between the cores [12]. If the fibre material does not absorb, we have $c_{ij} = c_{ji}$ and both β_i and c_{ij} are real. Let $\boldsymbol{A}(z)$ be a column vector of the slowly varying amplitudes $[A_1(z), A_2(z), \ldots, A_N(z)]$. The above equation can then be written in matrix form [13, 14]:

$$\frac{\mathrm{d}A}{\mathrm{d}z} = \mathrm{i}CA, \quad C = \begin{pmatrix} \Delta\beta_1 & \cdots & c_{1N} \\ \vdots & \ddots & \vdots \\ c_{N1} & \cdots & \Delta\beta_N \end{pmatrix}. \tag{2}$$

Thus, light propagation in this regime is fully described by matrix C, whose elements are corrections to propagation constants and coupling coefficients, which are independent of the z coordinate. This equation is markedly easier to solve if we find a set of orthonormal eigenvectors ($\boldsymbol{L}_1, \boldsymbol{L}_2, \ldots, \boldsymbol{L}_N$) and the corresponding eigenvalues ($l_1, l_2, \ldots, l_N$) of matrix C. Then we have $C\boldsymbol{L}_i = l_i\boldsymbol{L}_i$ and

$$(\boldsymbol{L}_i \cdot \boldsymbol{L}_j) = \sum_{k=1}^{N} L_{ik}L_{jk} = \delta_{ij}$$

($\delta_{ij} = 1$ at $i = j$ and $\delta_{ij} = 0$ at $i \neq j$). For matrix C, such vectors always exist, and $l_1, \ldots, l_N$ are real numbers because matrix C is Hermitian. These vectors will then correspond to the eigenmodes (normal modes) of the coupled cores, also referred to as supermodes, and $l_1, \ldots, l_N$ are equal to their propagation constants relative to β_0. Any $\boldsymbol{A}(z)$ vector can then be expanded in terms of the eigenvectors: $\boldsymbol{A}(z) = p_1(z)\boldsymbol{L}_1 + p_2(z)\boldsymbol{L}_2 + \ldots + p_N(z)\boldsymbol{L}_N$, where

$$p_i(z) = (\boldsymbol{A}(z)\cdot\boldsymbol{L}_i) = \sum_{j=1}^{N} A_j(z)L_{ij}.$$

The expansion coefficients satisfy the equation $\mathrm{d}p_i/\mathrm{d}z = \mathrm{i}l_ip_i$, so $p_i(z) = p_i(0)\exp(\mathrm{i}l_iz)$ and

$$\boldsymbol{A}(z) = \sum_{i=1}^{N} p_i(0)\exp(\mathrm{i}l_iz)\boldsymbol{L}_i. \tag{3}$$

Thus, if matrix C is known, the solution $\boldsymbol{A}(z)$ is easy to find. The inverse is also true: if a sufficient number of pairs of the amplitudes at the MCF input [$\boldsymbol{A}(0)$] and output [$\boldsymbol{A}(\mathcal{L})$, where $\mathcal{L}$ is the fibre length] are known, we can reconstruct matrix C. However, measuring the phases $\arg(A_i(\mathcal{L}))$ and $\arg(A_i(0))$ requires much effort, so it is of particular interest to reconstruct matrix C if one can measure only the intensity at the input of each core [$|A_1(0)|^2, |A_2(0)|^2, \ldots, |A_N(0)|^2$] and that at their output [$|A_1(\mathcal{L})|^2, |A_2(\mathcal{L})|^2, \ldots, |A_N(\mathcal{L})|^2$]. Since there is no information about the phase, a larger number of measurements are needed than in the case of a known phase.

Experimentally, the simplest case is when light is launched into only one core at the fibre input, i.e. $|A_i(0)|^2 = 0$ for all $i \neq k$ and $|A_k(0)|^2 = I$. For convenience, we take $I = 1$. At the output of the fibre (of length $\mathcal{L}$), we will then observe some intensity distribution over its cores: $|A_1(\mathcal{L})|^2, |A_2(\mathcal{L})|^2, \ldots, |A_N(\mathcal{L})|^2$. After M series of measurements at different fibre lengths, $\mathcal{L}_1, \ldots, \mathcal{L}_M$, we will have a data set, $I_{ijk}^{\mathrm{exp}} = |A_i(\mathcal{L}_j)|^2$, for light launched into the kth core at the fibre input. Note that it is reasonable to choose the lengths $\mathcal{L}_1, \ldots, \mathcal{L}_M$ so that they correspond to a uniform coverage of at least one period of energy transfer between adjacent cores. Next, using this data set we find a matrix $\tilde{C}$ such that the discrepancy

$$\varDelta = \sum_{i,j,k} |I_{ijk}^{\mathrm{exp}} - I_{ijk}^{\mathrm{calc}}|^2$$

(where $I_{ijk}^{\mathrm{calc}} = |A_i(L_j)|^2$) is minimised with the $A_i(L_j)$ amplitudes calculated by formula (3) for light launched only into the kth core at the fibre input and matrix C equal to $\tilde{C}$. Thus, if we can find matrix C such that the discrepancy $\varDelta$ is near zero, matrix C can be thought to approach the sought matrix $\tilde{C}$ and describe light propagation in the fibre.

Matrix C can be found numerically using an iterative algorithm. In effect, the problem reduces to finding a minimum of a function of several variables, which can be done using a large number of distinct approaches. Most classic approaches are, however, inapplicable if there are a large number of unknown variables, and much effort is needed to compute the function. In the case under consideration, the number of independent real variables is $N(N + 1)/2$ and rises sharply as the number of cores, N, increases. Note that, to compute the discrepancy $\varDelta$, it is necessary to find the eigenvectors of an $N \times N$ matrix, which requires a considerable computational time. In view of this, we employed the stochastic gradient descent method [15].

The algorithm begins with matrix $\tilde{C}$, obtained from the refractive index profile for an ideal geometric structure of the fibre, and iteratively improves the approximation. In each iteration, each matrix element is changed at random within some range Δc and then $\varDelta$ is computed. The symmetric matrix elements are changed in the same way, so that the condition $c_{ij} = c_{ji}$ is satisfied. In each iteration, some number T of such changes are made in the same matrix and then the approximation that ensures the lowest Δ value is chosen. If this value is better than the preceding one, matrix $\tilde{C}$ is replaced by the changed matrix that ensures the lowest Δ value. Otherwise, the random changes made are thought to be too large to continue the search for a maximum, so the maximum change Δc is multiplied by $r < 1$ and the algorithm continues to work. This allows us, on the one hand, to reduce the computational cost at the beginning of operation, where the current approximation is rather far from the optimal one, and, on the other, to eventually find the optimal approximation with sufficient accuracy. Moreover, increasing the number of iterations, we increase the number of changes, T, in each step, which serves the same purpose. The initial value of Δc was taken to be $\max|c_{ij}|/2$, with $r = 0.995$. The initial value of T was 20, and T was linearly increased to 50 in the iterations. It is worth noting that, if a reasonable approximation of $\tilde{C}$ cannot be found from the refractive index profile or in another way, the algorithm can operate with zero initial approximation, but the initial value of Δc should then be taken in the same order as the expected maximum coupling coefficients.

Note that a method based on the principle of measuring the output intensity distribution over the MCF cores for different light launch configurations at the fibre input and then adjusting parameters of the system was proposed by Mosley et al. [16]. However, their method is based on measuring the output intensity only at a single fibre length and has serious limitations: all the coupling coefficients are taken to be identical, only coupling with nearest neighbours is taken into account, and the fibre length should be considerably shorter than the beat length. As shown below, measurements at only one fibre length drastically degrade the accuracy of fibre parameter reconstruction.

Note also a known method that allows one to reconstruct mode profiles and group velocities in multimode fibres and is based on spatially and spectrally resolved output field measurements [17]. This method requires a broadband light source and is incapable of reconstructing propagation constant corrections: only corrections to the group velocities of modes can be reconstructed.

3. Experimental verification of the method

To experimentally verify the performance capability of the method, we used a seven-core fibre with rather closely spaced cores. The fibre was fabricated at the Fiber Optics Research Center, Russian Academy of Sciences, by a technique similar to that described by Egorova et al. [18]. The structure of the fibre is shown in Fig. 1a. Laser light (wavelength $\lambda = 1550$ nm) was sequentially launched into all the fibre cores. In each case, we measured the light intensity at the fibre output using an infrared camera (Ophir Spiricon Pyrocam IV). The fibre was then cut off and measurements were repeated.

Measurements were performed at different ($M = 20$) fibre lengths and then the above algorithm was applied. To verify the results, we compared the intensity profiles measured along the fibre in each core and those calculated using a reconstructed C matrix. Figure 2 presents examples of such profiles and the profiles calculated using the initial approximation of matrix C (obtained from the refractive index profile).

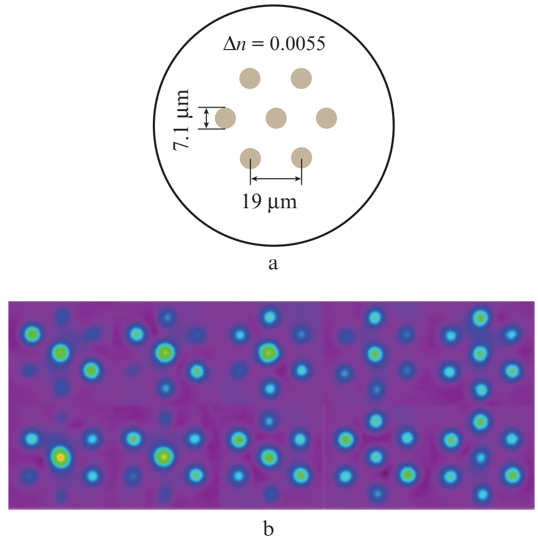

Figure 1. (a) Structure of the fibre and (b) examples of intensity distributions visualised using an IR camera; Δn is the core–cladding index difference.

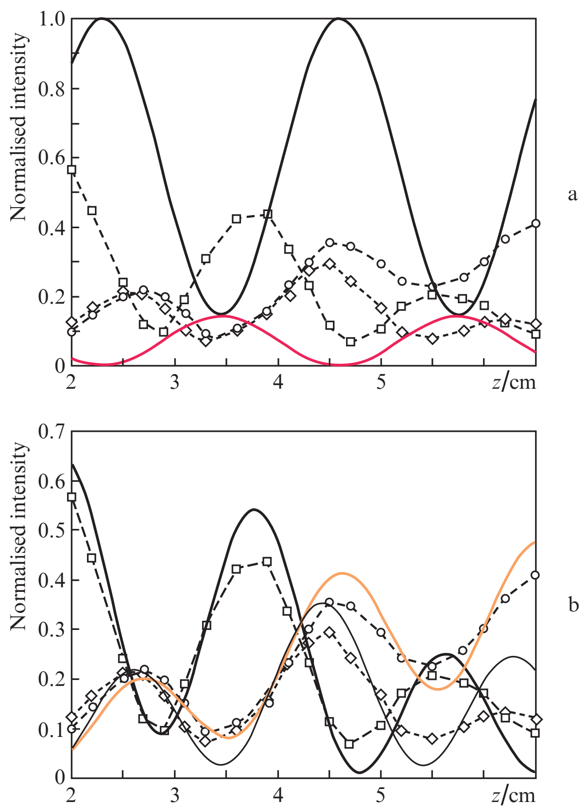

Figure 2. Measured (dashed lines) and reconstructed (solid lines) light intensities in cores vs. MCF length for several cores: data obtained using (a) the initial approximation of matrix C and (b) the best approximation found.

It is seen that the curves obtained using the final C matrix (Fig. 2b) reproduce all the features of the measured curves. At the same time, the C matrix derived from an ‘ideal’ refractive index profile (Fig. 2a) yields a different curve. Thus, the described method allows one to construct an approximation of the matrix describing light propagation in a given fibre. Note that the measured intensity does not completely coincide with the intensity calculated using the iterative algorithm. This may be due to various factors, e.g. to additional changes in the structure of the fibre, coupling coefficients along the fibre, polarisation dynamics left out of account, nonmonochromaticity of propagating light and uncertainties in experimental intensity measurements.

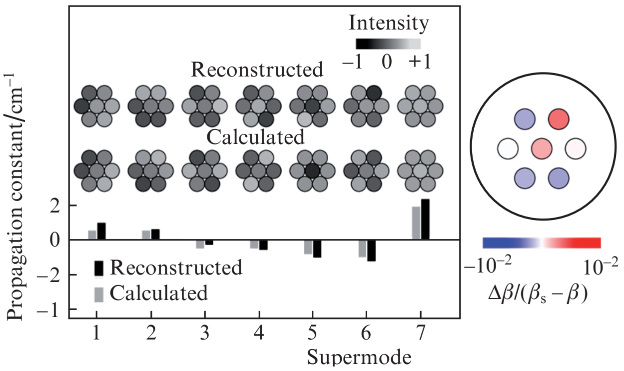

Figure 3. (Colour online) (a) Profiles and propagation constants of supermodes and (b) corrections to the propagation constants for each core.

Figure 3a shows reconstructed profiles of fibre supermodes and their propagation constants (relative to β_0). It is seen that, on the whole, the supermode profiles are similar to those of an 'ideal' fibre with the same parameters, but the existing distinctions determine a very different type of light propagation. Also shown in Fig. 3b are corrections to the propagation constants $\beta_1,\beta_2,\ldots,\beta_N$ for each fibre core, normalised to the difference between the propagation constant of silica ($\beta_s = n\omega/c$) and that of one core: $\beta_s - \beta$.

The algorithm for optimising matrix $\tilde{C}$ was implemented in C^{++}. About 15 000 steps were required for experimental data. The computational time was ~1 min on a personal computer. Thus, this method requires no large computational resources. Figure 4 shows the discrepancy Δ against the number of steps.

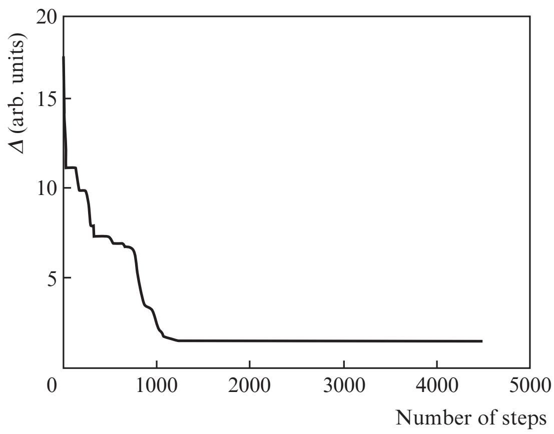

Figure 4. Discrepancy Δ against the number of steps.

4. Stability of the method

Since the proposed method of solving the inverse problem for reconstructing corrections to propagation constants and coupling coefficients is based on searching for a minimum of a function of many variables, it is important to recognise in which cases the best solution found is indeed similar to the sought matrix C and in which cases the algorithm found a local minimum far from the sought matrix C. Since input $I_i(\mathcal{L}_j)$ data can be inaccurate for various reasons (including inaccurate light launching into one core and noise in the detector array of the camera), the parameter Δ is not always zero, even if the $\tilde{C}$ solution found approaches the sought matrix C.

To assess the accuracy of the method, we numerically simulated light propagation in a nonideal fibre with known parameters and then the parameters were reconstructed from known intensities at different points, $I_i(\mathcal{L}_j)$. Next, we compared the reconstructed matrix $\tilde{C}$ with the input matrix C that was used to calculate the $I_i(\mathcal{L}_j)$ intensities.

As a model for a nonideal fibre, we chose a seven-core fibre with identical cores, each displaced from a symmetric position by a random vector with a magnitude within pd, where d is the spacing between the cores and the parameter p characterizes fibre nonideality and takes values in the range 0–0.1. At the highest value $p = 0.1$, because of the strong exponential dependence on distance the coupling coefficients between adjacent cores may differ by a factor of 2 from those for the 'undistorted' core positions. The coupling coefficients for such fibre can be calculated numerically. Next, we calculated the deviation of matrix C from an ideal one using the formula

$$E_0 = \sum_{i,j} \frac{|c_{ij} - c_{0ij}|}{\max|c_{0ij}|}$$

and then calculated the intensity in each core for different light launch configurations at $2N$ points and employed the algorithm for finding matrix $\tilde{C}$. After carrying out the algorithm, we calculated the deviation of the reconstructed matrix $\tilde{C}$ from the sought one:

$$E = \sum_{i,j} \frac{|\tilde{c}_{ij} - c_{0ij}|}{\max|c_{0ij}|}.$$

If $E \ll E_0$, matrix C can be thought to be successfully reconstructed; otherwise, there is no reconstruction. For each p value, calculations were performed ten times. The results are presented in Fig. 5.

It is seen that, with increasing fibre nonideality, the algorithm more often finds wrong solutions and even impairs the quality of approximations (even though the discrepancy Δ decreases considerably). At the same time, even with nonidealities where the maximum coupling coefficients differ from the initial value by a factor of 2, the algorithm finds the proper solution in half of the cases. It should be noted that, if the algorithm fails to find the proper solution, this is easy to find out because the deviation E is large and the $I_i(\mathcal{L}_j)$ curves differ drastically from the measured ones. We can then restart the algorithm, e.g. with another initial approximation or with a larger initial step, in order to avoid getting into a local minimum.

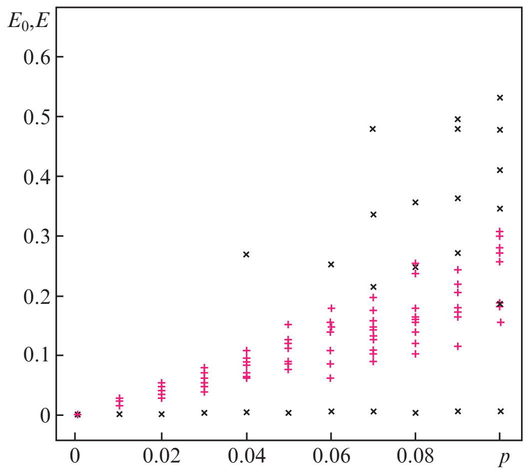

Figure 5. Initial values E_0 (+) and final values E (×) as functions of parameter p.

Since the coupling coefficients between adjacent cores are exponential functions of distance, they far exceed the other coupling coefficients, so the latter have little or no effect on the nature of light propagation in the fibre. At the same time, since the method does not deliberately separate out the coupling coefficients between adjacent cores, absolute errors in reconstructed coupling coefficients between nonadjacent cores may turn out to be greater than the coefficients. However, as above they play no significant role in light propagation through the fibre.

To examine the effect of the number of initial measurements on the accuracy of the method, we performed analogous numerical simulation. The parameter M was varied from 1 to $2N$ and the parameter p was taken to be 0.03. In the course of the operation of the algorithm, the current discrepancy Δ was determined using only the first m measurements (with minimum $\mathcal{L}_j$ values). After the end of the operation of the algorithm, the discrepancy Δ was computed using all $2N$ measurements. The results are presented in Fig. 6.

It is seen that, on the whole, the discrepancy Δ decreases as the number of measurements rises, stabilising on average at

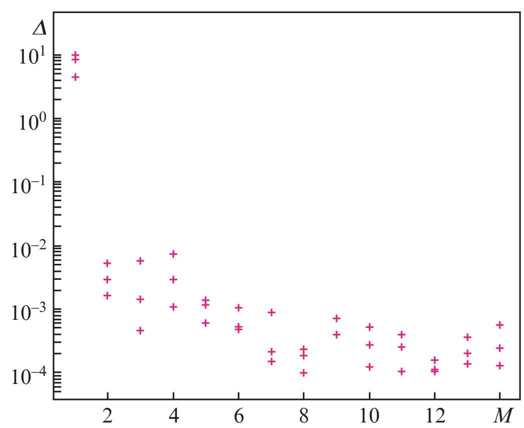

Figure 6. Discrepancy Δ as a function of the number of measurements, M.

$M > 5$. It should be emphasised that, if measurements were made at only one fibre length (which was in effect proposed by Mosley et al. [16]), the discrepancy was several orders of magnitude higher and the reconstructed parameters poorly described light propagation in the fibre.

5. Conclusions

A method has been developed for reconstructing supermode profiles of a multicore fibre, small corrections to the propagation constants of its cores and coupling coefficients between the cores. The method is based on intensity measurements and requires no phase measurements. Light intensity in the cores at the multicore fibre output is measured at several fibre lengths and several configurations of controlled light launch into the cores (e.g., by sequentially launching light into each core). An iterative algorithm has been developed which allows the coupling coefficients between all cores and corrections to propagation constants in each core, i.e. all the parameters that completely define the guidance properties of a multicore fibre, to be reconstructed using such measurements. In contrast to previously proposed methods, the method described here places no severe limitations on scatter in coupling coefficients and allows one to reconstruct their values for all cores (rather than for only adjacent ones), which may be important for multicore fibres with complex structures. The use of the method has been demonstrated experimentally for a seven-core fibre. Using numerical simulation, we have assessed the stability of the method and the effect of the number of measurements on the accuracy of fibre parameter reconstruction.

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