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Optimisation of parameters of Raman laser pulse compression in a plasma for its implementation using the PEARL laser facility (IAP RAS)

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Abstract. **We consider Raman compression of laser pulses in a plasma under the conditions of an experiment planned at the Institute of Applied Physics of the Russian Academy of Sciences on the PEARL laser facility. The analysis is based on the equations describing, among other things, the effect of plasma dispersion and relativistic nonlinearity, as well as the dynamics of the field near the plasma wave breaking threshold. It is shown that the main limiting factors are excessive frequency modulation of the pump pulse and a too low plasma density in which the plasma wave breaking can occur. To reduce the negative influence of these effects, we suggest using an intense and short (on the order of the plasma period) seed laser pulse. Numerical simulation shows the possibility of a hundredfold increase in the intensity of the compressed pulse in comparison with the intensity of the pump pulse at a length of uniform plasma of 2 cm.**

Keywords: *Raman amplification, compression of laser pulses, plasma wave breaking.*

1. Introduction

In recent decades, the intensities of pulsed laser radiation have significantly increased, mainly due to the use of the method of chirped pulse amplification (CPA) [1]. However, diffraction gratings used in this method to compress an amplified pulse melt under the action of high-power laser radiation, which limits the applicability of the CPA method. To create the next generation of high-intensity lasers, another nonlinear medium will be needed, such as plasma capable of withstanding much higher energy flux densities. In particular, plasma is supposed to be used in the scheme of backward Raman amplification [2, 3] for laser pulse compression. This method is based on a resonant energy transfer between two laser pulses propagating towards each other, which interact through an electrostatic plasma wave. The principal possibility of achieving near-relativistic intensities of unfocused radiation during backward Raman amplification was demonstrated experimentally [4–8].

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The basic physical processes that can affect the Raman amplification include compression of pump pulses with amplitude fluctuations [8–10], violation of three-wave synchronism conditions due to relativistic nonlinearity [11–13], parasitic Raman amplification of plasma noise [3, 14] or a prepulse of a seed pulse [3, 15], scattering by density inhomogeneities [16, 17], additional ionisation of plasma [18, 19], a decrease in the pulse amplitude due to the plasma heating during collisions or as a result of Landau damping [20–24], plasma wave breaking [3, 10, 25], and other processes [26–28]. Most of these harmful effects can be compensated for by choosing optimal parameters of laser pulses and plasma. In addition, the introduction of selective detuning from the three-wave synchronism due to plasma nonuniformity or pulse modulation allows the parasitic effects to be additionally weakened.

The aim of this paper is to determine optimal parameters of laser pulses and plasma to suppress parasitic effects and to achieve maximum efficiency of Raman compression in plasma. The traditional scheme [2] of counterpropagating pulses in a previously prepared plasma is considered. At the same time, the parameters are chosen close to the parameters of the experiment being prepared at the IAP RAS. The analysis is carried out by using the equations describing, among other things, the effect of plasma dispersion and relativistic nonlinearity, as well as the dynamics of the field near the plasma wave breaking threshold.

2. Basic equations

Let us consider the problem of amplification and compression of femtosecond circularly polarised laser pulses during stimulated Raman backscattering in a plasma. To describe this process, we use the quasi-one-dimensional system of equations of relativistic hydrodynamics for the electron concentration n and their momentum p, which is closed by the equations for the scalar and vector potentials of laser pulses:

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial z}\frac{np}{\sqrt{1+p^2+|a_\Sigma|^2}} = 0, \tag{1a}$$

$$\frac{\partial p}{\partial t} = \frac{\partial}{\partial z}(\Phi - \sqrt{1+p^2+|a_\Sigma|^2}), \tag{1b}$$

$$\frac{\partial^2 \Phi}{\partial z^2} = \beta(n - 1 - \delta n), \tag{1c}$$

$$\frac{\partial^2 a_\Sigma}{\partial t^2} - \Delta a_\Sigma = -\frac{\beta n a_\Sigma}{\sqrt{1+p^2+|a_\Sigma|^2}}, \tag{1d}$$

where $\beta = \omega_p^2/\omega^2 = 4\pi n_0 e^2/(m\omega^2) \ll 1$ is the normalised square of the plasma frequency; n_0 is the unperturbed concentration of electrons; ω is the characteristic carrier frequency of the amplified laser pulse; and δn is the perturbation of the ion concentration. The initial system of equations (1) is written in dimensionless variables: p is the longitudinal electron momentum normalised to mc; t is the time normalised to ω^{-1}; z is the longitudinal coordinate normalised to c/ω; n is the electron concentration normalized to n_0; Φ is the scalar potential normalised to mc^2/e; and a_Σ is the amplitude of the vector potential normalised to mc^2/e.

The system of equations (1) is Hamiltonian and is characterised by the following action:

$$S = \iint \left[\frac{|\partial_t a_\Sigma|^2 - |\nabla a_\Sigma|^2}{2\beta} - n\sqrt{1 + p^2 + |a_\Sigma|^2} + \frac{|\partial_z \Phi|^2}{2\beta} + (n - 1 - \delta n)\Phi - n\partial_t \varkappa \right] \mathrm{d}\boldsymbol{r}\mathrm{d}t. \quad (2)$$

Here, $p = \partial_z \varkappa$ and $\partial_x = \partial/\partial x$. Then, we simplify this expression. For counterpropagating laser pulses, the amplitude of the vector potential can be written in the form

$$a_\Sigma = a\exp\left[\mathrm{i}\left(1 + \frac{\sqrt{\beta}}{2}\right)(t - z) + \mathrm{i}\frac{\beta}{2}t\right] + b\exp\left[\mathrm{i}\left(1 - \frac{\sqrt{\beta}}{2}\right)(t + z) + \mathrm{i}\frac{\beta}{2}t\right] + \text{c.c.}, \quad (3)$$

where we clearly divided waves propagating in the positive and negative directions of the z axis. In this case, the plasma response will consist of a smooth (within the laser pulse duration) density profile moving along the pulse propagation direction and of a strongly spatially modulated beat wave. Thus, we must represent the response of the plasma (the concentration of electrons and their longitudinal momentum) as a sum of some average response and terms that rapidly change in space:

$$n = 1 + \delta n + [f\exp(-2\mathrm{i}z) + f_2\exp(-4\mathrm{i}z) + \text{c.c.}], \quad (4a)$$

$$p = q\exp(-2\mathrm{i}z) + q_2\exp(-4\mathrm{i}z) + \text{c.c.} \quad (4b)$$

We assume that the envelopes of plasma (f) and electromagnetic (a, b) waves are smooth on the wavelength scale of the laser pulses (∂_t, $\partial_z \ll 1$). As a result, we find the relationship between Φ and $\varkappa$ with the quantities f and q:

$$\Phi = -\frac{\beta}{4}f\exp(-2\mathrm{i}z) - \frac{\beta}{16}f_2\exp(-4\mathrm{i}z) + \text{c.c.}, \quad (5a)$$

$$\varkappa = -\frac{q}{2\mathrm{i}}\exp(-2\mathrm{i}z) - \frac{q_2}{4\mathrm{i}}\exp(-4\mathrm{i}z) + \text{c.c.} \quad (5b)$$

To obtain truncated equations for the dynamics of the envelopes of the waves, we perform averaging in integrand (2), neglecting rapidly oscillating terms. In this case, we leave only the first nonvanishing corrections to the action, related to the nonuniformity of the plasma density by the plasma dispersion and relativistic nonlinearity. Note that the equations for f_2 and q_2 have the form of oscillator equations with nonresonance force [29], which allows the amplitudes of the second harmonics of the plasma wave to be explicitly expressed through the wave amplitudes at the fundamental frequency:

$$f_2 = \frac{8}{3\beta}(\sqrt{\beta}fq + q^2), \quad q_2 = \frac{\sqrt{\beta}fq + 4q^2}{3\sqrt{\beta}}. \quad (6)$$

Substituting expressions (4) with allowance for (6) into (2), we obtain the truncated action:

$$S = \iint \Bigg\{ \left[\frac{1 + \sqrt{\beta}/2}{\mathrm{i}\beta}(a^*\partial_t a - a\partial_t a^* + a^*\partial_z a - a\partial_z a^*) + \frac{1 - \sqrt{\beta}/2}{\mathrm{i}\beta}(b^*\partial_t b - b\partial_t b^* + b\partial_z b^* - b^*\partial_z b) \right] + \frac{3}{4}|b|^4 - \frac{|\nabla_\perp a|^2 + |\nabla_\perp b|^2}{\beta} - |\partial_z a|^2 - |\partial_z b|^2 - \frac{f^*\partial_t q - f\partial_t q^*}{2\mathrm{i}} - \frac{\beta|f|^2}{4} - fa^*b - f^*ab^* - \delta n(|a|^2 + |b|^2) - (1 + \delta n)|q|^2 - \frac{4}{3\beta}|q|^4 - \frac{|fq|^2}{3} - \frac{4|q|^2}{3\sqrt{\beta}}(fq^* + f^*q) \Bigg\} \mathrm{d}\boldsymbol{r}\mathrm{d}t. \quad (7)$$

Varying the resulting action, we obtain a system of truncated equations:

$$\left(1 + \frac{\sqrt{\beta}}{2}\right)(\partial_t a + \partial_z a) = \frac{\mathrm{i}\beta}{2}[\delta n a + fb\exp(-\mathrm{i}\sqrt{\beta}t)] - \frac{\mathrm{i}}{2}(\Delta_\perp a + \beta\partial_{zz}a) - \beta\nu_{\mathrm{ei}}a, \quad (8a)$$

$$\left(1 - \frac{\sqrt{\beta}}{2}\right)(\partial_t b - \partial_z b) = \frac{\mathrm{i}\beta}{2}[\delta n b + f^*a\exp(\mathrm{i}\sqrt{\beta}t)] - \frac{3\mathrm{i}\beta}{4}|b|^2 b - \frac{\mathrm{i}}{2}(\Delta_\perp b + \beta\partial_{zz}b) - \beta\nu_{\mathrm{ei}}b, \quad (8b)$$

$$\partial_t q = 2\mathrm{i}\left[ab^*\exp(\mathrm{i}\sqrt{\beta}t) + \frac{\beta}{4}f + \frac{|q|^2 f}{3} + \frac{4|q|^2 q}{3\sqrt{\beta}} + \mathrm{i}\nu_{\mathrm{L}}q\right], \quad (8c)$$

$$\partial_t f = 2\mathrm{i}\left[q + \delta n q + \frac{8|q|^2 q}{3\beta} + \frac{|f|^2 q}{3} + \frac{4(2|q|^2 f + q^2 f^*)}{3\sqrt{\beta}}\right], \quad (8d)$$

where ν_{ei} is the frequency of electron–ion collisions; and ν_{L} is the Landau damping decrement.

The system of equations (8) describes the Raman amplification of laser pulses under the influence of plasma dispersion, inhomogeneity of the plasma density, relativistic nonlinearity, and plasma wave breaking in a rarefied plasma. This system of equations will be used in Section 8 for carrying out full-scale numerical simulation. It should be noted that system (8), along with the conservative terms obtained from action (7), contains, as a result of the variation, terms with ν_{ei} and ν_{L} describing the damping of plasma and electromagnetic waves. For the plasma wave, the main mechanism of dissipation is Landau damping, and for electromagnetic waves damping is due to electron–ion collisions.

In our dimensionless variables, the expression for the normalised Landau damping decrement [30] can be written in the form:

$$\nu_{\mathrm{L}} = \frac{\mathrm{e}^{-3/2}}{2}\sqrt{\frac{\pi}{2}}\frac{\sqrt{\beta}}{q_T^{3/2}}\exp[-1/(2q_T)], \quad (9)$$

where $q_T = T_e/(mv_{ph}^2) \equiv 4T_e/(\beta mc^2)$ is the normalised electron temperature; and v_{ph} is the phase velocity of the plasma wave. For the characteristic electron temperature T_e = 300 eV, we obtain $q_T \approx 0.002/\beta$. In this case, the Landau damping decrement is $\nu_L \approx 1200\beta^2\exp(-200\beta) \approx 0.001-0.009$, which is much less than unity. In this case, the normalised frequency of electron–ion collisions [22] is

$$\nu_{ei} = \frac{\nu_0 Z\Lambda\sqrt{\beta}}{\sqrt{q_T}\,(\beta q_T/4 + |b|^2)}, \tag{10}$$

where $\nu_0 = \omega e^2/(mc^3)$; $Z \approx 1$ is the multiplicity of ionisation in a plasma; and Λ is the Coulomb logarithm. Looking ahead (see Section 3), we note that within the framework of the planned experiment, the damping of electromagnetic waves can be neglected, since $\beta\nu_{ei} \approx 10^{-8} \lll 1$ for a plasma with a characteristic length of several centimetres (10^4-10^5 wavelengths of laser radiation).

The system of equations (8) can be simplified in the case of laser pulses with a duration much longer than the plasma wave period. The system can be reduced to the classical form of three-wave equations with additional terms. This simplification seems important for a qualitative understanding of the problem under study. Equations (8c) and (8d) can be transformed to an equation for a nonlinear complex oscillator with an eigenfrequency $\pm\sqrt{\beta}$. Their simplification is possible if one takes into account that the external force ab^* in the quasi-monochromatic approximation acts only at the frequency $+\sqrt{\beta}$. Consequently, the response of the medium at the frequency $-\sqrt{\beta}$ is nonresonant. As a result, we obtain an algebraic relationship between the quantities f and q [29]:

$$q \approx \frac{\sqrt{\beta}}{2}f + \frac{ab^*}{\sqrt{\beta}} + \mathrm{i}\frac{\nu_L}{2}f. \tag{11}$$

Substituting (11) into equation (8d), for the envelope of the plasma wave f in the quasi-monochromatic limit, we obtain the equation

$$\partial_t f = \mathrm{i}\sqrt{\beta}f + \frac{2\mathrm{i}}{\sqrt{\beta}}ab^*\exp(\mathrm{i}\sqrt{\beta}t) + \mathrm{i}\sqrt{\beta}\,\delta n f + 2\mathrm{i}\sqrt{\beta}\,|f|^2 f - \nu_L f. \tag{12}$$

The introduction of new dimensionless variables, $A = a/a_0$, $B = b/a_0$ (a_0 is the maximum amplitude of the pump pulse), and $F = \beta f\exp(-\mathrm{i}\sqrt{\beta}t)/(2\mathrm{i}\gamma)$, as well as new coordinates, $\tau = \gamma(t-z)$ and $\zeta = \gamma z$, allows us to transform the system of equations to the classical form [2] with coefficients for three-wave terms equal to unity:

$$\partial_\zeta A + 2\partial_\tau A = -BF + \mathrm{i}KA - \frac{\mathrm{i}\alpha}{2}\partial_{\tau\tau}A, \tag{13a}$$

$$\partial_\zeta B = AF^* + \mathrm{i}KB - \mathrm{i}\epsilon|B|^2 B - \frac{\mathrm{i}\alpha}{2}\partial_{\tau\tau}B, \tag{13b}$$

$$\partial_\zeta F + \partial_\tau F = AB^* + \mathrm{i}\kappa F + \mathrm{i}\chi|F|^2 F - \nu F. \tag{13c}$$

Here, $\gamma = a_0\sqrt[4]{\beta}$ is the increment of the Raman amplification in the linear stage; $\alpha = a_0\beta^{5/4}$ is the dimensionless coefficient characterising the plasma dispersion; $\epsilon = (3/4)a_0\beta^{3/4}$ is the nondimensional coefficient of the relativistic nonlinearity; $\chi = 8a_0/\beta^{5/4}$ is the dimensionless coefficient of the plasma wave nonlinearity; $\nu = \nu_L/\gamma$ is the dimensionless damping coefficient; and $K = [\beta/(2\gamma)]\delta n$ and $\kappa = (\sqrt{\beta}/\gamma)\delta n \gg K$ are the parameters describing the quasi-stationary inhomogeneity of the plasma density.

3. Main parameters

To carry out further theoretical studies of the impact of various parasitic effects on the Raman amplification and compression of a laser pulse in a plasma, we first determine the basic parameters (electron concentration, pump pulse amplitude, temporal chirp, plasma length, etc.). As noted above, the purpose of this paper is to determine the optimal conditions for the planned experiment at the PEARL facility (IAP RAS). In general, the parameters of this experiment are close to a set of parameters in the previously performed experiments [8, 19], but there are a number of important differences.

The parametric amplification of laser pulses is used in the femtosecond PEARL laser facility. The main parameters of the pulses are the following: the laser wavelength is 910 nm, the pulse energy is 1–10 J, and its duration is ~50 fs. In this case, the pulse can be stretched to a duration of ~1 ns. In the experiment, it is planned to use a gas cell with a length varying from 1 to 6 cm, which should ensure a high uniformity of the plasma density.

In numerical calculations, the pump pulse and the seed pulse were assumed to be Gaussian:

$$a = a_0\exp\left[-\frac{(t+z)^2}{2\tau_{pump}^2} + \mathrm{i}\sigma\gamma^2(t+z)^2\right], \tag{14a}$$

$$b = b_0\exp\left[-\frac{(t-z)^2}{2\tau_{ini}^2}\right]. \tag{14b}$$

In this case, the pump pulse, in contrast to the seed pulse, has an initial time chirp σ.

The absence of a waveguide system limits from below the transverse size of the wave packet $w_\perp$ by the condition that the interaction length L is small compared with the diffraction length

$$L_{diff} = \frac{w_\perp^2}{4\pi\lambda} \geqslant L \gtrsim 6\text{ cm} \Rightarrow w_\perp \geqslant \sqrt{\frac{\lambda L}{4\pi}} \approx 100\ \mu\text{m}. \tag{15}$$

The use of much wider laser beams is undesirable, since this will reduce the amplitude of the pump pulse and thereby increase the impact of parasitic effects. Thus, the transverse sizes of the pump beams and the amplified radiation should be of the order of 100 μm.

On the other hand, the length of the cell determines the optimum duration of the pump pulse, which should be approximately twice as long as the interaction time:

$$\tau_{pump} \geqslant \frac{2L}{c} \approx 70-400\text{ ps}. \tag{16}$$

As indicated above, the transverse size of the seed beam (pulse) should slightly exceed 100 μm, so that the influence of diffraction on the process of stimulated Raman backscattering in the plasma can be neglected. Note that in the planned experiment, the frequencies of the laser pump pulses and the seed pulses are the same. This simplifies the design of the experiment, but imposes additional requirements on the plasma density: it must be selected from the condition that the plasma frequency lies within the width of the spectrum of the seed pulse. In the experiment, the duration of the amplified pulse can be varied in a small range, 50–100 fs, and its energy is $\sim$0.01 J. In dimensionless variables, this corresponds to $b_0 = 0.01$ and $\tau_{\rm ini} = 100-200$. At the given duration of the seed pulse, it is necessary that the electron concentration be sufficiently low, i.e., that condition

$$\sqrt{\beta} \equiv \frac{\omega_{\rm p}}{\omega} \approx \frac{2\pi}{\tau_{\rm ini}\omega} = \frac{1}{30} - \frac{1}{15}$$

be fulfilled.

Consider now the parameters of the pump pulse, which allow much greater freedom of choice. Thus, for a pump pulse having a Gaussian shape (14a), there are, in fact, two main parameters – duration $\tau_{\rm pump}$ and energy $W_{\rm pump}$. The change in the duration of the pump pulse at a constant width of its spectrum entails the appearance of frequency modulation, $i\sigma\gamma^2(t + z)^2$ [see (14a)]. The duration and energy of the pump pulse uniquely determine the amplitude a_0 and chirp σ:

$$a_0 \approx 0.01\sqrt{\frac{W_{\rm pump}/1\,{\rm J}}{\tau_{\rm pump}/10\,{\rm ps}}}, \quad \sigma \approx \frac{\lambda}{2\pi a_0^2 L\sqrt{\beta}\,\tau_{\rm ini}}.$$

The resulting value of the chirp σ, as will be shown below, turns out to be close to optimal. In particular, the use of a pump pulse having a duration of 10 ps gives $a_0 \approx 0.01$ and $\sigma = 0.1-0.3$. The change in the compressor parameters in the femtosecond laser facility makes it possible to further reduce severalfold the chirp. Therefore, its value can be in the range from 0 to 0.3.

For the selected typical experimental parameters and a plasma temperature of 300 eV, the following estimates are obtained for the ranges of the dimensionless parameters:

$$\gamma = 0.002-0.003, \quad \kappa < \omega_{\rm p}/\gamma = 18-25, \quad \alpha \approx 10^{-6}-10^{-5}, \tag{17}$$
$$\epsilon \approx 0.5-1.3\times10^{-4}, \quad \chi \approx 70-400, \quad \alpha \approx 0.5-4.$$

In this case, the duration of the seed pulse in the dimensionless variables is $\gamma\tau_{\rm ini} < 1$.

4. Frequency modulation of the pump pulse

Ideal Raman amplification and, correspondingly, compression of a laser pulse are described by the so-called π-pulse solution [2, 3]. This is a self-similar solution with a decreasing duration of the amplified pulse $\tau_{\rm p}$ and its amplitude and energy linearly increasing in time (Fig. 1):

$$\tau_{\rm p} \propto \frac{1}{t}, \quad |b| \propto t, \quad W_{\rm p} = \int |b|^2 {\rm d}z \propto t. \tag{18}$$

It is essential that this solution depends weakly on the initial conditions. The amplitude of the first pulse increases in proportion to the length of the gain, and its duration is inversely proportional to its amplitude. Thus, in the nonlinear stage, the amplified pulse is not only amplified, but also compressed.

It follows from the Manley–Rowe relations that the energy of the amplified laser pulse is limited by the energy of the pump pulse multiplied by the frequency ratio:

$$W_{\rm p} \leqslant (1 - \sqrt{\beta})\int |a|^2 {\rm d}t.$$

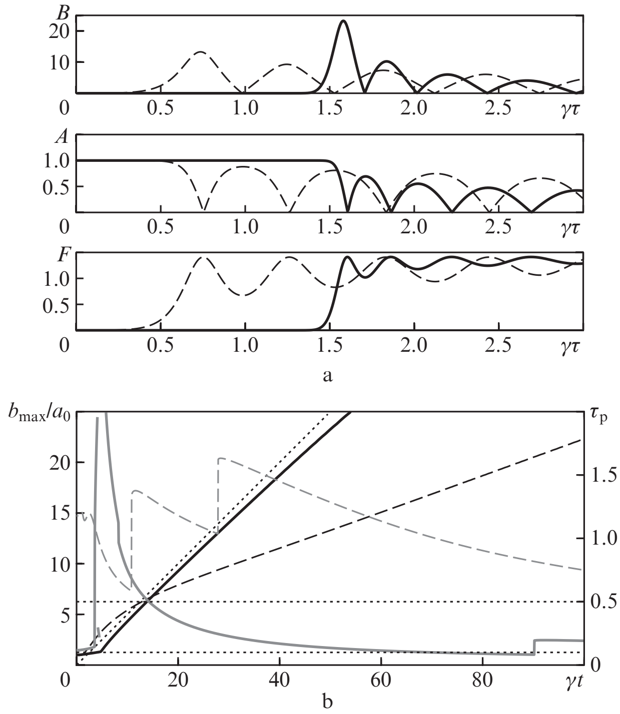

Figure 1. (a) Profiles of the amplified pulse, the pump pulse, and the plasma wave at the output from the plasma region of length $50/\gamma$, and (b) dependences of the maximum amplitude (black lines) and the duration of the amplified pulse (gray lines) on the propagation time for the initial durations of the seed pulse $1/(20\gamma)$ (solid curves) and $1/(2\gamma)$ (dashed curves). The seed pulses had an amplitude equal to the pump pulse amplitude. The dashed curves in Fig. 1b correspond to the initial durations of the amplified pulse and to the linear law of growth of its amplitude, described by the π-pulse solution.

Therefore, in order to achieve a high energy efficiency of compression (the ratio of the energies of the amplified pulse and the pump pulse), it is desirable to select a relatively rarefied plasma.

Note that the π-pulsed solution is realised in pure form only in a homogeneous plasma by using a pump pulse without frequency modulation and a seed pulse in the form of a δ-function (zero duration). For pulses of finite duration, some settling time [estimated as $\gamma^{-1}\ln\int(|b|/a_0)\,{\rm d}(\gamma z)$] is required to reach the π-pulse solution (solid curves in Fig. 1a). This is a so-called linear stage of Raman amplification, when the depletion of the pump is negligible. In Fig. 1b, a 'shelf' in the initial section of the dependence of the maximum amplitude on time corresponds to it. At the stage under consideration, the energy of the amplified pulse increases exponentially, and its duration is many times greater than the duration of the seed pulse. Next, with the amplitude of the amplified pulse reaching values of the order of the pump pulse amplitude ($b \approx a_0$), there occurs a transition of the signal amplification to the nonlinear stage, when the depletion of the pump becomes significant. In this case, the decrease in the pump pulse energy and the amplified pulse duration is determined by the π-pulse solution. In Fig. 1b, the dependence of the linear growth of the amplified laser pulse amplitude on time corresponds this stage.

The above-described well-structured picture of amplification and shortening of laser pulses is realised only for sufficiently short wave packets, i.e. when the duration of the amplified pulse is much longer than the duration of the seed pulse. If the durations become commensurable, then amplification of the precursor (the leading edge of the pulse) is possible. This negative process leads to depletion of the pump until the main peak of the amplified pulse arrives, and thereby slows down the Raman compression. As a result, there are two negative consequences (dashed curves in Fig. 1): the duration of the amplified pulse is limited by the duration of the seed pulse (more precisely, its leading edge), and its amplitude increases much more slowly. Thus, in order to achieve the fastest and most effective Raman compression, a seed pulse with a sharp leading edge and a large amplitude should be selected.

Note that along with the amplification of the useful signal in the linear stage, there is also an exponential increase in the noise of the plasma and the prepulse [3]. This parasitic amplification can completely deplete the pump pulse long before the beginning of amplification of the useful signal, and thereby dramatically decrease the compression efficiency. In this case, the fraction of energy in the reflected wave will be close to 100%, since amplification of the plasma noise results in the backward reflection of the pump pulse with a random phase. One of the standard mechanisms for reducing parasitic noise amplification in a plasma is the introduction of the initial frequency modulation (chirp) into the pump pulse or the realisation of Raman amplification in a linearly inhomogeneous plasma. This mechanism is based on the resonance nature of Raman amplification. The considered three-wave decay of the pump pulse is possible if the difference $\Delta\omega$ of the frequencies of the waves participating in the decay is smaller than the gain increment:

$$\Delta\omega \equiv |\omega_a - \omega_b - \omega_f| \leqslant 2\gamma. \tag{19}$$

The presence of frequency modulation leads to the fact that the instantaneous frequency varies along the pump pulse in accordance with the law

$$\omega_a = \omega_0 + 2\sigma\gamma^2(t + z). \tag{20}$$

This means that within the characteristic time $1/(\sigma\gamma)$, the noise harmonic will leave the three-wave resonance (19) and cease to increase exponentially. If, in addition, its amplitude remains less than a_0,

$$b_{\text{noise}}\Gamma < a_0, \tag{21}$$

where $\Gamma = \exp(1/\sigma)$, then in the process of amplification the harmonic can not appreciably deplete the pump pulse. At the same time, the amplitude of the useful signal, initially stronger ($b_0 \gg b_{\text{noise}}$), must reach the amplitude a_0 and the signal itself must pass to the nonlinear stage, when the conditions of the three-wave resonance noticeably soften. In practice, they are limited by the values of frequency modulation σ = 0.1 – 0.2 [3]. Smaller values of σ can lead to an increase in the noise of the plasma, while larger values can limit the gain of the useful signal (Fig. 2a).

It can be seen from Fig. 2b that an increase in the initial amplitude of the seed pulse leads to the excitation of the π-pulse solution with a larger amplitude. As a result, it reaches the nonlinear stage faster and thereby allows larger values of frequency modulation σ to be used. This is especially clear for sufficiently short pulses. However, even in this case the relatively large pump chirp ($\sigma > 0.6$) makes Raman amplification practically impossible. Therefore, in carrying out the

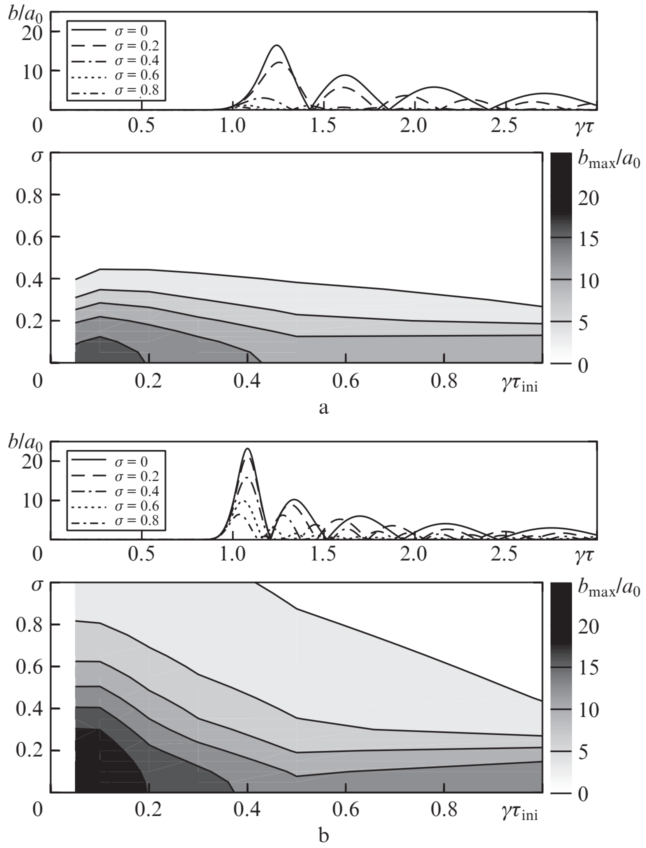

Figure 2. Profiles of the amplified pulse and dependences of its maximum amplitude on the duration of the seed pulse τ_{ini} and the parameter σ for the initial seed pulse amplitude of (a) $0.1a_0$ and (b) a_0. The length of the plasma region is $50/\gamma$.

experiment, large values of the chirp ($\sigma \approx 1$) must be compensated for in some way.

5. Inhomogeneous plasma

The process of stimulated Raman backscattering in a plasma is sensitive to medium inhomogeneities (both regular and fluctuational). In particular, scattering by quasi-random stationary plasma inhomogeneities ($\delta n \neq 0$) is a parasitic factor, which first of all reduces the efficiency of the studied process of amplification and compression of laser pulses [16]. This is due to the fact that the plasma inhomogeneity leads to a shift in the moduli of the wave vectors of the laser pulses and the frequency of the plasma wave:

$$k_a \approx 1 + \frac{\beta}{2}\delta n, \quad k_b \approx -1 - \frac{\beta}{2}\delta n, \quad \omega_f = \sqrt{\beta}\,(1 + \delta n),$$

which violates the conditions of three-wave matching (19) for $\omega_{\text{p}}\delta n \geqslant 2\gamma$, i.e. at

$$\delta n \gtrsim 2a_0/\beta^{1/4}, \text{ or } \kappa \gtrsim 2. \tag{22}$$

Consequently, under the conditions of the planned experiment (see Section 3), estimate (22) will limit the allowable value of plasma density fluctuations by a value of several (up to ten) percent. Fortunately, this estimate is valid only in the linear stage of amplification and for laser pulses with a duration exceeding the period of the plasma wave (Figs 3a and

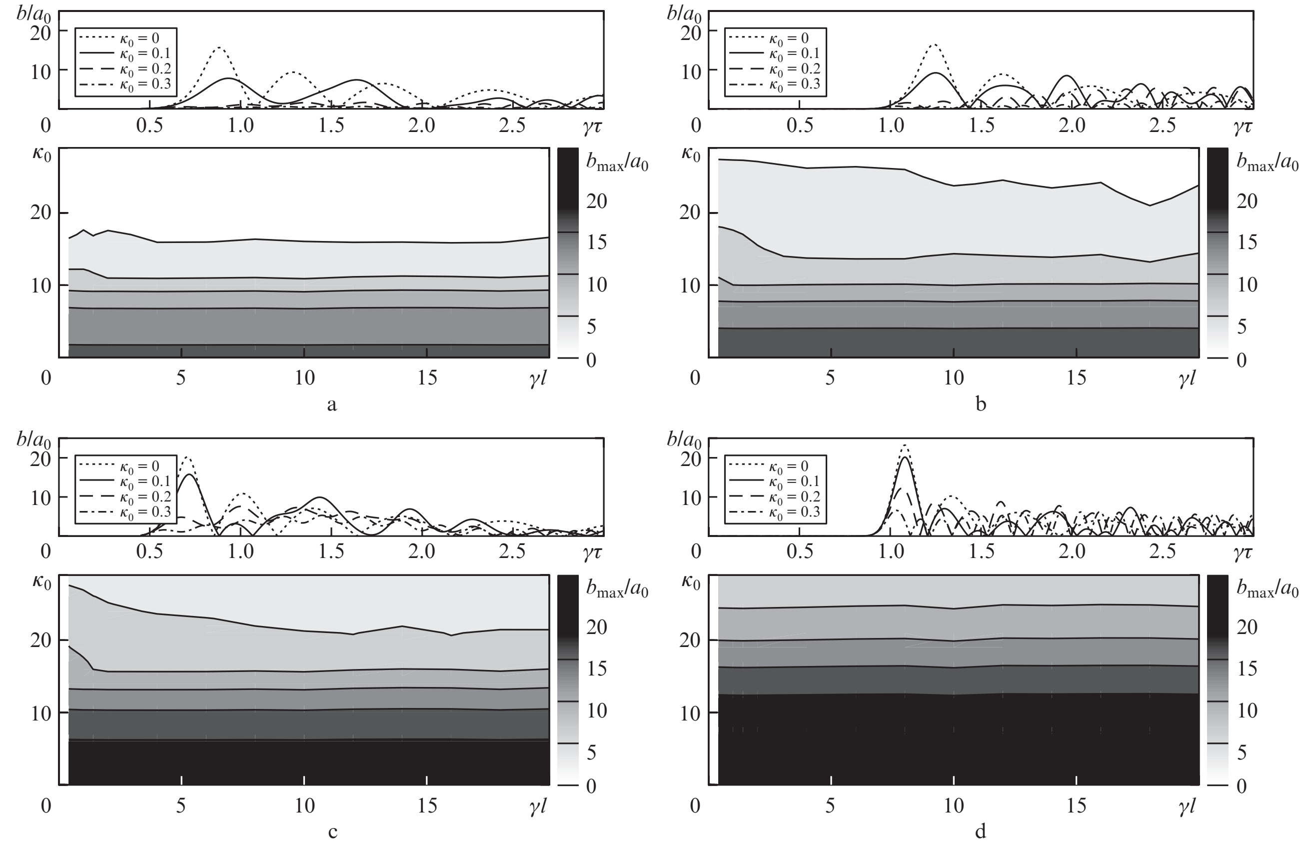

Figure 3. Profiles of the amplified pulse and dependences of its maximum amplitude on the inhomogeneity parameters $\kappa = \kappa_0 \sin(2\pi z/l)$ for the following parameters of the seed pulse: (a) $\tau_{\text{ini}} = 0.5$, $b_0 = 0.1a_0$; (b) $\tau_{\text{ini}} = 0.05$, $b_0 = 0.1a_0$; (c) $\tau_{\text{ini}} = 0.5$, $b_0 = a_0$; and (d) $\tau_{\text{ini}} = 0.05$, $b_0 = a_0$. The pump chirp ($\sigma = 0$) is absent. The length of the plasma region is $50/\gamma$.

3b). In the planned experiment, on the contrary, it is proposed to use a seed pulse with a duration comparable with the period of the plasma wave and an amplitude on the order of the amplitude of the pump pulse.

Indeed, in using a broadband seed laser pulse with a spectral width of the order of or greater thanthe plasma frequency ω_{p}, there is a corresponding resonant spectral component for each instantaneous plasma density. This harmonic will increase exponentially in the process of stimulated Raman backscattering in the plasma. Note that all these harmonics will be in-phase, because they are generated by the same short seed laser pulse. As a result, it is possible to obtain an identical compression ratio for arbitrarily strong density fluctuations, but on a longer path. The latter circumstance leads to the fact that energy efficiency may not be too high. Strictly speaking, this result can be obtained within the framework of the wave equation [17], although the effect is observed (Figs 3c and 3d) even when using quasi-monochromatic equations (13).

It should be noted that the presence of moderate frequency modulation in the pump pulse can lead to the appearance of a spectral plateau at certain linear scales l of plasma density fluctuations. In such sections of the nonlinear medium, the three-wave interaction condition (19) will be better satisfied because of compensation of plasma inhomogeneities due to frequency modulation in the pump pulse. Accordingly, the amplification of plasma noise also increases significantly, and the use of frequency-modulated pumping to limit noise amplification becomes ineffective in this case [14]. Let us obtain an estimate of the parameters of the indicated inhomogeneities of the medium. It is obvious that on such spectral plateaus the first and second derivatives of the frequency difference $\Delta\omega$ are close to zero in coordinate:

$$\Delta\omega \approx 2\sigma\gamma^2(t+z) + \omega_{\text{p}}\delta n(z) \Rightarrow \exists z, l\colon \frac{\partial \Delta\omega}{\partial z} \approx 0 \text{ and } \frac{\partial^2 \Delta\omega}{\partial z^2} \approx 0.$$

In other words, in these sections of the medium, Raman amplification increases and becomes close to ideal. Note that the ideal compensation is possible only for small amplitudes of modulation of the plasma density. For example, for a sinusoidal perturbation of density $\delta n = d\sin(2\pi z/l)$, an ideal compensation is possible only when

$$2\sigma\gamma^2 = \omega_{\text{p}} d\,\frac{2\pi}{l} \Rightarrow d = \frac{\sigma\gamma^2 l}{\pi\omega_{\text{p}}} \lll 1, \text{ or } \gamma l = \frac{\pi\omega_{\text{p}} d}{\sigma\gamma} \gg 1,$$

i.e., for small ($d \lll 1$) or large-scale ($\gamma l \gg 1$) fluctuations in the plasma density with a moderate pump chirp ($\sigma \ll 1$). Fortunately, the plasma in the gas cell must be sufficiently uniform, so that a rather large pump chirp will not lead to the appearance of this type of a plateau in the planned experiment.

In conclusion of this section, let us turn to the most interesting case of using an intense and short seed laser pulse during Raman amplification. Note that a similar regime is planned to be implemented in the experiment. Figure 3d shows the results of numerical simulation for the case under consideration. It can be seen that in a wide range of plasma density fluctuation parameters, the amplification efficiency in this limit is quite high. This is due to two factors. First, as noted above, the use of a broadband seed laser pulse makes it possible to expand the range of allowable amplitudes of plasma density fluctuations. Second, injecting a seed laser pulse with a large initial amplitude ($b_0 \approx a_0$) into the input of a nonlinear medium allows a much faster transition to a nonlinear regime for which the plasma uniformity requirements are further reduced.

6. Nonlinear dispersion of a plasma wave

Another parasitic mechanism that reduces the efficiency of Raman amplification and compression of a laser pulse is associated with the process of plasma wave breaking in a rarefied plasma. Note that this decrease in efficiency with characteristic parameters of the experiment is not so dangerous as the violation of three-wave synchronism due to the nonlinear shift of the frequency of the plasma wave [29], which precedes the process of its breaking. Let us estimate the effect of these two mechanisms on stimulated Raman backscattering in a plasma.

The process of plasma wave breaking, realised in a rarefied plasma, manifests itself, first of all, in limiting the amplitude of plasma oscillations: $|f| < f_{\rm wb} = 1$. Here, the value of f equal to unity corresponds to a decrease in the electron concentration to zero. Note that in a rarefied plasma the quantity $f_{\rm wb} = 1$ turns out to be less than the maximum value $f_\pi = 2\sqrt{2}a_0/\beta^{3/4}$ in the ideal case of the π-pulse regime. This means that the three-wave process under study effectively becomes a two-wave process with a fixed amplitude of the plasma wave f. Consequently, the growth rate of the amplitude of the amplified signal b will decelerate by $f_{\rm wb}/f_\pi$ times, and the compression efficiency of the amplified laser pulse will decrease by the same factor.

A much stronger process that reduces the efficiency of Raman amplification is associated with a nonlinear shift in the frequency of the plasma wave as its amplitude approaches the value of the breaking threshold. Indeed, in the quasi-monochromatic limit, the nonlinear distortion of plasma oscillations is described by the next-to-last term in Eqn (12), which causes a nonlinear shift of the frequency $2\sqrt{\beta}|f^2|$ of the plasma response f. It is this term that violates the three-wave synchronism condition (19) at the amplitude of the plasma wave

$$f = f_{\rm nl} \approx \sqrt{\frac{\gamma}{\omega_{\rm p}}} \equiv \frac{\sqrt{a_0}}{\beta^{1/8}} \ll 1, \qquad (23)$$

much smaller than the amplitude at which the plasma wave breaks. Thus, this mechanism leads to an even more marked decrease in the efficiency of amplification and compression of the laser pulse:

$$\eta_{\rm nl} \approx \frac{f_{\rm nl}}{f_\pi} = \frac{\beta^{5/8}}{\sqrt{8a_0}} \ll \eta_{\rm wb} \approx \frac{f_{\rm nl}}{f_\pi} = \frac{\beta^{3/4}}{\sqrt{8}\,a_0} \leqslant 1. \qquad (24)$$

We note that the positive side of limitation (23) of the stimulated Raman backscattering process is the almost complete absence of parasitic noise amplification in a rarefied plasma. Consequently, depletion of the pump pulse in the absence of its frequency modulation, associated with scattering by plasma noise, will be negligibly small. However, the negative side of this effect is the lack of Raman amplification of the useful signal. To solve this problem, two methods can be proposed: the introduction of 'compensating' frequency modulation into the pump pulse and the use of an initially intense and short seed laser pulse.

Let us explore both possibilities in more detail. First, we consider the case in which the pump pulse has suitable frequency modulation. Obviously, its introduction cannot completely compensate for the nonlinear frequency shift in (23). This is due to the fact that a too large chirp is required in the pump pulse ($\sigma \approx \omega_{\rm p}/\gamma \gg 1$), which stops Raman amplification of the useful signal at the initial stage. However, moderate initial frequency modulation (Fig. 4) allows the violation of the three-wave synchronism to 'be delayed'. As a result, the exponentially growing amplitude of the plasma wave can reach appreciably larger values than $f_{\rm nl}$, and thus the Raman compression efficiency of the amplified laser pulse will not decrease so much. In particular, as shown by the numerical calculations, the introduction of the initial frequency modulation into the pump pulse with $\sigma \approx 0.4$ makes it possible to provide an almost perfect compression at

$$\chi \lesssim 200, \text{ or } \beta \gtrsim 0.08 a_0^{4/5}. \qquad (25)$$

In the experiment, it is proposed to use a pump pulse with an amplitude of $a_0 \approx 0.01$; therefore, the plasma density can be quite small: $\beta \geqslant 0.002$. However, the use of a chirp with a larger value will result in a much stronger decrease in Raman amplification in the linear stage and, therefore, will not allow a high intensity to be reached in the case of a more rarefied plasma.

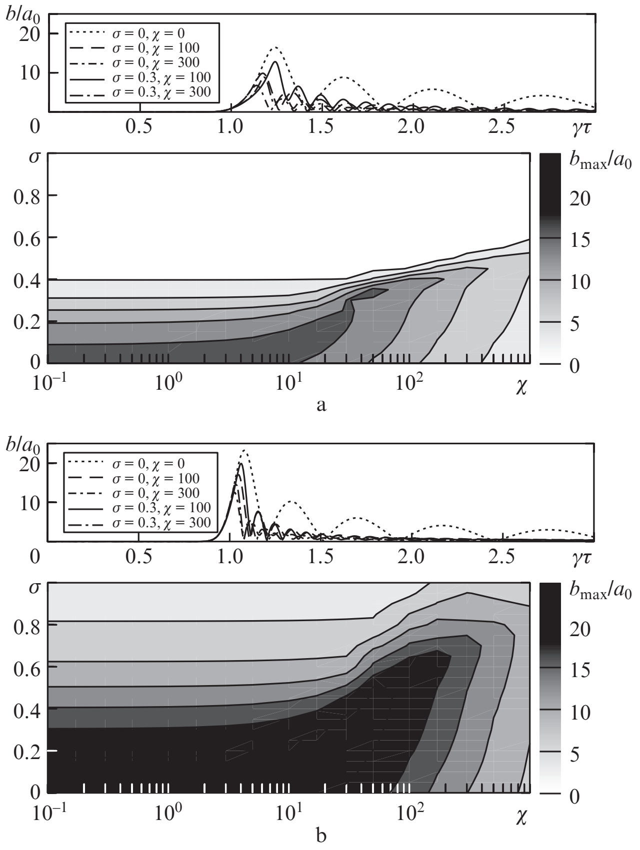

Figure 4. Profiles of the amplified pulse and dependences of its maximum amplitude on the parameters χ and σ for the initial seed pulse amplitude of (a) $0.1a_0$ and (b) a_0. The length of the plasma region is $50/\gamma$.

As noted above, another possibility of overcoming the limitation of the plasma wave amplitude growth (23) is to use an initially intense and short seed laser pulse in order to achieve the nonadiabatic (shock) excitation regime of a plasma wave. In this case, if the duration of the leading edge of the seed pulse is comparable with the period of the plasma wave, and its initial amplitude is sufficient to excite the plasma wave with the amplitude $|f| \approx f_{\rm wb} \approx 1$, then such a plasma wave cannot subsequently change its amplitude (three-wave scattering will be nonresonant for it). Therefore, the plasma wave will cause a quasi-stationary modulation of the medium density at which the pump pulse will be scattered and transformed into an amplified pulse in the quasi-two-wave regime. The results of numerical simulation show that the use of a short seed laser pulse (Fig. 4b) allows us to overcome to a considerable extent the problems of a nonlinear frequency shift and thus achieve an almost perfect efficiency. Such laser pulses will be used later in the experiment.

It should be noted that for a more accurate quantitative description of this process it is necessary to use the kinetic representation or at least the system of four equations (8), since quasi-monochromatic equations (12) are no longer applicable for the regime under consideration. However, as shown by numerical analysis, the results of PIC simulation [31] and modelling within the system of four equations [29] are in good quantitative agreement with the results of quasi-monochromatic calculation. More accurate calculations based on the system of equations (8) for the final optimisation of the experiment are presented in Section 7.

7. Other parasitic effects

Let us discuss other parasitic effects (Landau damping for the plasma wave, linear dispersion of the group velocity in the plasma, relativistic nonlinearity, etc.), which are much less ‘dangerous’ for the planned experiment in comparison with the mechanisms considered above. This is due to the fact that the impact of these effects on the process of stimulated Raman backscattering is either insignificant for the chosen parameters, or they can affect only the final stage of laser pulse amplification and can be compensated for in some way. Next, consider these effects in more detail.

The damping of a plasma wave with a decrement $0 \lesssim \nu_{\rm L} \lesssim 4\gamma$ leads to a delay in the Raman amplification process in the linear stage, since the effective linear increment of the modified decay [21, 22]

$$\gamma_{\rm eff} = \sqrt{\gamma^2 + \nu_{\rm L}^2} - \nu_{\rm L} \approx \gamma^2/(2\nu_{\rm L}) < \gamma$$

becomes smaller, which causes only a delay in the linear stage. At the same time, in the nonlinear stage, when the duration of the amplified laser pulse during Raman compression becomes less than the inverse decrement ($\tau_{\rm p} < 1/\nu_{\rm L}$), the damping of the plasma wave will only lead to a decrease in the gain at the tail of the laser pulse, but will not affect the amplification of the main anterior peak (Fig. 5a). The standard method [21, 22] of eliminating the problem of a delay in the linear stage of laser pulse amplification is the use of a more intense seed laser pulse. This situation is illustrated by Fig. 5b, which shows a noticeable decrease in the length of the path of the exponential growth of the amplitude in the linear stage of Raman amplification.

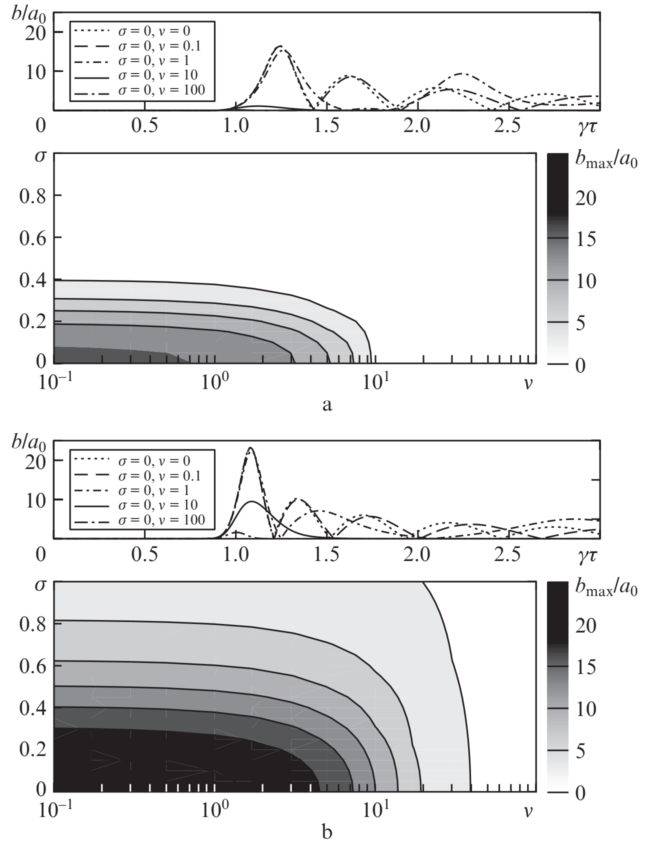

Figure 5. Profiles of the amplified pulse and dependences of its maximum amplitude on the parameters ν and σ for the initial seed pulse amplitude of (a) $0.1a_0$ and (b) a_0. The length of the plasma region is $50/\gamma$.

The influence of the linear group velocity dispersion in the plasma on stimulated Raman backscattering can be noticeable in the final stage of Raman amplification at $(\alpha/2)|\partial_{\tau\tau}B||B|^{-1} \approx \alpha/(2\bar{\tau}_{\rm p}^2) \approx 1$, when the duration of the amplified laser pulse becomes sufficiently small to appear at the considered amplification lengths:

$$\tau_{\rm p} = \frac{\bar{\tau}_{\rm p}}{\gamma} \approx \frac{\beta^{3/8}}{\sqrt{2a_0}}.$$

Estimates show that for the selected experimental parameters the effect of the linear group velocity dispersion on the Raman amplification process will be significant only for the duration of the amplified laser pulse much shorter than the plasma-wave period. However, as shown in Section 4, this duration is unlikely to be realised in the experiment, since already at the input to the nonlinear medium the seed laser pulse has a duration on the order of the plasma-wave period.

A more significant effect on the Raman compression process at the final stage of amplification can be provided by a nonlinear frequency shift due to the relativistic self-action of the amplified laser pulse in the plasma. Note that in the planned experiment it is proposed to use a sufficiently intense pump pulse with an amplitude $a_0 \approx 0.01$. Consequently, even a moderate amplification of the signal (100 times in amplitude) will provide the amplified signal amplitude exceeding the relativistic level. As a result, the influence of relativistic nonlinearity on the process of stimulated Raman scattering will be noticeable. First of all, this leads to a nonlinear frequency shift $(3/4)\beta|b|^2$, which is due to the cubic nonlinear term in (8b). The condition of the three-wave resonance (19) is satisfied only for $(3/4)\beta|b|^2 \lesssim 2\gamma$, which limits Raman amplification by the quantity

$$|b_{\max}| \approx \frac{\sqrt{3a_0}}{\beta^{3/8}} \approx 1-2. \qquad (26)$$

This is in good agreement with the results of numerical simulation, shown in Fig. 6. The achievement of larger amplitudes of the output pulse than those in [13, 21, 22] is possibly due to the small plasma density in the planned experiment.

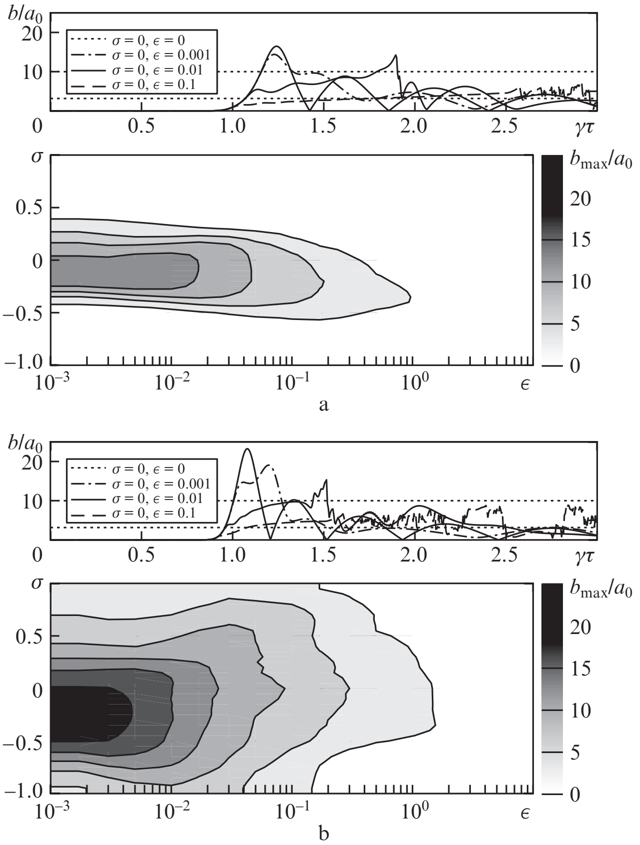

Figure 6. Profiles of the amplified pulse and dependences of its maximum amplitude on the parameters ϵ and σ for the initial seed pulse amplitude of (a) $0.1a_0$ and (b) a_0. The length of the plasma region is $50/\gamma$. The horizontal dashed lines indicate the maximum amplitude (26) for $\epsilon = 0.01$ (top line) and 0.1 (bottom line). The length of the plasma region is $50/\gamma$.

It should be noted that the use of a properly matched frequency modulation of the pump pulse allows one to overcome limitation (26). The linear pump chirp cannot significantly improve the situation, in contrast to the case of a rarefied plasma, which was considered above. This is explained by the fact that the frequency shift due to the relativistic nonlinearity (26) is primarily due to the relatively slow increase in the field amplitude, which in the ideal case is proportional to the interaction time (see Fig. 1). As a result, in order to effectively compensate for the nonlinear frequency shift near the maximum of the amplified pulse, a power (in the simplest case parabolic) chirp in the pump pulse is required [13]. However, in the planned experiment, it is not possible to produce this frequency modulation in the pump pulse.

8. Final optimisation of the system parameters

Let us summarise all the parasitic effects discussed above, which reduce Raman amplification in a plasma, in order to determine the optimal parameters of laser (pump and seed) pulses and plasma that will be used in the experiment at the IAP RAS facility to achieve maximum amplification and compression of the laser pulse. Numerical simulation was carried out on the basis of the complete system of equations (8), since we consider the duration of the seed and output pulses, comparable with the period of the plasma wave. It should be noted that a particular feature of equations (8) is the possibility of an unlimited (explosive) growth of the plasma wave amplitude f due to the fact that only two spatial harmonics are used to describe the dynamics of the plasma response. In order to eliminate this unphysical effect caused by the incorrectness of the hydrodynamic equations in plasma wave breaking, we introduced an artificial limitation on the plasma wave amplitude of the order of unity.

Figure 7 shows the dependence of the efficiency of Raman amplification of the laser pulse, taking into account all parasitic effects considered above, on the parameter β proportional to the plasma density and on the initial pump pulse amplitude a_0. Numerical calculations were performed for a plasma with a length of a homogeneous region of 2 cm and for different durations of the seed laser pulse. Figures 7a and 7b present the results of numerical simulation with a pulse duration of 100 fs, and Figs 7c and 7d – with a duration of 50 fs. Note that the durations used are comparable with the period of the plasma wave. For comparison, Figs 7a and 7c illustrate the case when the difference between the centre frequencies of the amplified and seed pulses is equal to the frequency of the plasma wave, and Figs 7b and 7d demonstrate the case when the centre frequencies coincide. It is seen from Fig. 7 that for such short seed laser pulses, the presence of an optimum shift of the centre frequency from the pump frequency becomes irrelevant (cf. Figs 7a, 7c and Figs 7b, 7d).

Let us now analyse in more detail the efficiency of Raman amplification as a function of the initial amplitude of the pump pulse at a fixed plasma density. The appearance of a local maximum of the amplification efficiency at an amplitude $a_0 \approx 0.02$ is explained by a competition between the relativistic nonlinearity and the three-wave process. In the case of small pump pulse amplitudes, the growth rate of Raman instability is small, which corresponds to a delay in the linear stage of Raman compression. Consequently, there is a decrease in the efficiency of the process, since the length of the medium is fixed, and the length of the nonlinear stage of compression decreases. In the other limiting case, which corresponds to large amplitudes of the pump pulse, the relativistic nonlinearity begins to manifest itself and limits the maximum amplitude of the amplified laser pulse due to violation of three-wave synchronism. Thus, the compression efficiency of the amplified laser pulse decreases inversely proportional to the amplitude of the pump pulse.

In conclusion, let us consider the dependence of the efficiency of Raman amplification of a laser pulse on the plasma density. It follows from Fig. 7 that plasma wave breaking has a noticeable effect on the process of Raman amplification of the pulse at $\beta < 0.05$. We note that this influence is most

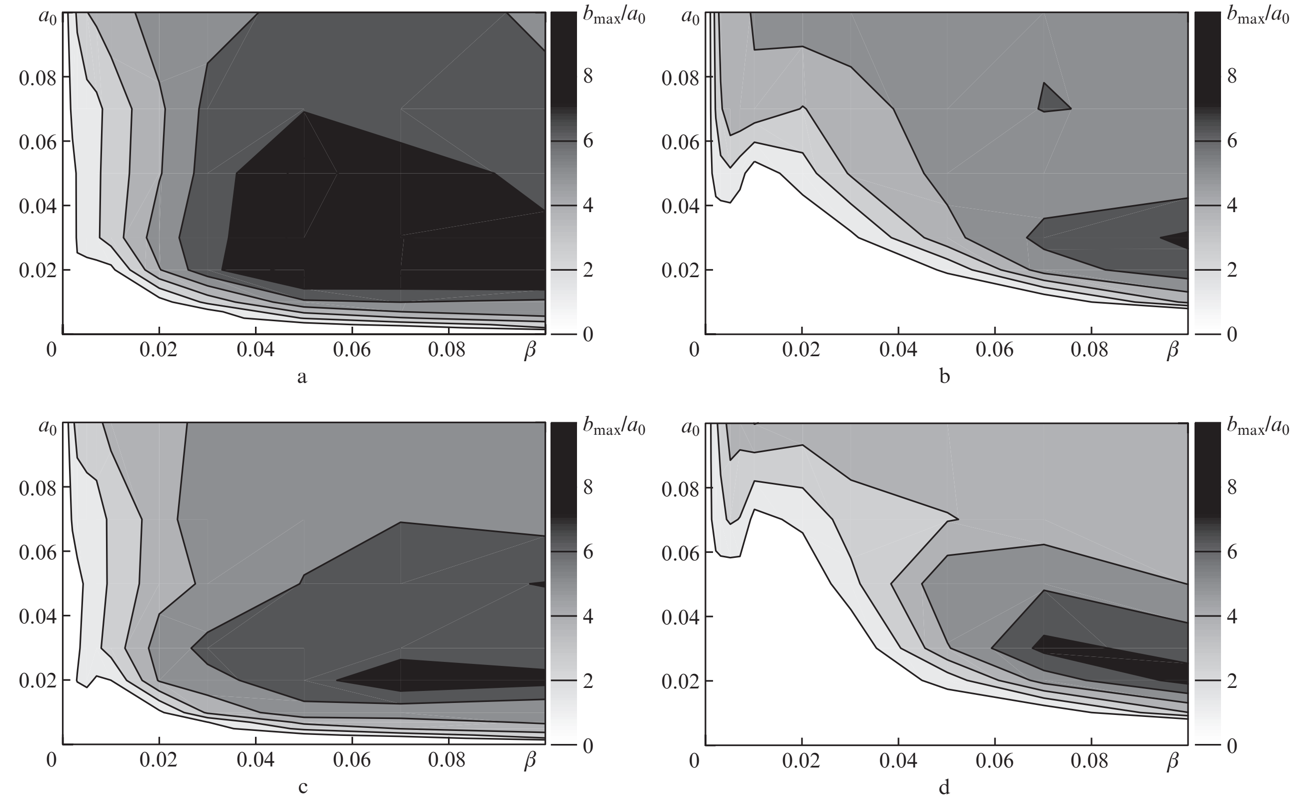

Figure 7. Maximum amplitudes of the amplified pulses b_{max}/a_0 over the plasma region length of 2 cm as functions of the parameters a_0 and β. The duration of the seed pulse is (a, b) 100 and (c, d) 50 fs, and its centre frequency is shifted by the value of the plasma frequency from the centre frequency of the pump pulse (a, c) or equal to the pump frequency (b, d).

noticeable when the centre frequencies of the pump pulse and the seed pulse coincide. This is due to the fact that the amplitude of the resonant harmonic in this case is much smaller and, consequently, the transition to the regime of nonadiabatic excitation of the plasma wave becomes more difficult.

Thus, under the conditions of the planned experiment (see Section 3), a rather high compression efficiency (by a factor of 10 or more) at a moderate length of a homogeneous plasma (2 cm) should be expected. Note that in the experiment the efficiency of compression of the amplified laser pulse can be influenced only by large fluctuations in the plasma density, which, however, are unlikely to occur in the considered gas cell.

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