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# Calculation of single-pass gain for laser ceramics with losses

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***Abstract.*** **Rate equations describing the single-pass gain in an active medium with losses are analytically solved. The found relations illustrate the dependences of the amplification efficiency of Nd:YAG ceramics on the pump power density and specific losses. It is concluded that specific losses can be estimated from comparative measurements of unsaturated and saturated gains.**

***Keywords:*** *laser ceramics, optical amplifiers, specific losses, amplification efficiency.*

## 1. Introduction

Significant progress in the synthesis of oxide laser ceramics has made it possible to produce large samples of Nd:YAG ceramics (up to 50 mm in diameter) with comparatively low losses and residual porosity [1, 2] based on Russian pressing and sintering technologies [3, 4]. According to the estimates of Bagayev et al. [5], specific losses in some samples of Russian ceramics are $\sim 10^{-2}$ cm$^{-1}$, while the slope lasing efficiency is about 80% of the efficiency for the best world samples [5, 6].

Large-size laser ceramics is used not only as active elements of high-power diode-pumped lasers but also in power amplifiers and final amplification stages of high-power laser systems [7, 8]. In this connection, evaluation of the influence of specific losses in ceramics on the amplification efficiency and the maximum possible energy output is of considerable scientific and practical interest and is the main aim of the present work.

## 2. Model

Let us consider a one-dimensional model of an optical amplifier with length $L$, in which a monochromatic light wave propagates along the $z$ axis. We assume that the pump power density and the photon number density in the optical amplifier are uniform along transverse axes ($x$, $y$); then, the rate equations of a quasi-three-level amplification scheme in the general case will have the form [9–11]

$$\frac{\mathrm{d}n_2}{\mathrm{d}t} = \eta Q - \frac{n_2}{\tau} - c(\sigma_{\mathrm{e}} n_2 - \sigma_{\mathrm{a}} n_1)\Phi, \quad (1)$$

$$\frac{\mathrm{d}\Phi}{\mathrm{d}z} = (\sigma_{\mathrm{e}} n_2 - \sigma_{\mathrm{a}} n_1 - \alpha)\Phi, \quad 0 < z < L, \quad (2)$$

$$n_2 + n_1 = n. \quad (3)$$

Here, $n_1$ and $n_2$ (in cm$^{-3}$) are the populations of the ground and metastable states, respectively; $\eta Q$ is the specific rate of population of the metastable level with lifetime $\tau$ (in s); $Q$ (in photon cm$^{-3}$ s$^{-1}$) is the pump power density, i.e., the number of pump photons absorbed by the amplifying medium in unit volume per unit time; $\eta$ is the luminescence quantum yield; $c$ (in cm s$^{-1}$) is the speed of light; $\sigma_{\mathrm{e}}$ and $\sigma_{\mathrm{a}}$ (in cm$^2$) are the cross sections of stimulated transitions between the metastable and ground states (2 → 1 and 1 → 2) at wavelength $\lambda$ of the amplified light field; $\Phi$ (in cm$^{-3}$) is the photon number density in the beam; $\alpha$ (in cm$^{-1}$) are the total (absorption and scattering) specific losses in the optical material; and $n$ (in cm$^{-3}$) is the concentration of active centres (for example, of rare-earth ions). Condition (3) suggests that the lifetimes of all the other excited states, except for the metastable state, are considerably shorter than $\tau$, i.e., the population of these states can be neglected. This approximation is valid for a large number of activator ions, including $Yb^{3+}$, $Er^{3+}$, $Tm^{3+}$, $Ho^{3+}$, and $Cr^{2+}$, and was discussed in detail in [11–13].

In the case of steady-state amplification, $\mathrm{d}n_2/\mathrm{d}t = 0$, and we have from Eqn (1) the expression

$$n_2 = \frac{\tau(\eta Q + c\sigma_{\mathrm{a}} n\Phi)}{1 + c\tau(\sigma_{\mathrm{e}} + \sigma_{\mathrm{a}})\Phi}. \quad (4)$$

With allowance for (4), the solution to Eqn(2) has the form

$$\ln\frac{\Phi(z)}{\Phi(0)} - \frac{\tau q(z)\sigma}{\alpha}\ln\left\{\frac{\tau q(z)\sigma - \alpha[1 + c\tau\sigma\Phi(z)]}{\tau q(z)\sigma - \alpha[1 + c\tau\sigma\Phi(0)]}\right\} = \tau\sigma\int_0^z q(z)\,\mathrm{d}z - \alpha z, \quad 0 < z < L, \quad (5)$$

where $\sigma = \sigma_{\mathrm{e}} + \sigma_{\mathrm{a}}$ and $q(z) = \eta Q(z) - \sigma_{\mathrm{a}} n/(\tau\sigma)$. Thus, Eqn (5) implicitly determines the dependence of the photon number density in the amplifier on the active medium parameters and the pump power distribution.

As an illustration, let us consider the stored energy extraction efficiency

$$\varkappa = c\frac{\Phi(L) - \Phi(0)}{\eta\int_0^L Q(z)\,\mathrm{d}z},$$

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which corresponds to the fraction of the pump power spent on the light beam amplification in Nd:YAG ceramics at different $\Phi(0)$, $Q$, and $\alpha$. For wavelength $\lambda = 1064$ nm (transition $^4F_{3/2} \rightarrow {}^4I_{11/2}$), we may take $\sigma_a = 0$, $\sigma = 2.5 \times 10^{-19}$ cm$^2$, $\tau = 2.5 \times 10^{-4}$ s, $\eta = 1$ [11–16], and, in addition, $L = 10$ cm, which corresponds to the typical dimensions of slabs for final amplifiers [7, 8]. The calculation results are shown in Figs 1–4. We assumed that the $Q$ distribution is uniform everywhere, while $\Phi(L)$ was numerically calculated taking into account (5) with given $\alpha$, $Q$, and $\Phi(0)$.

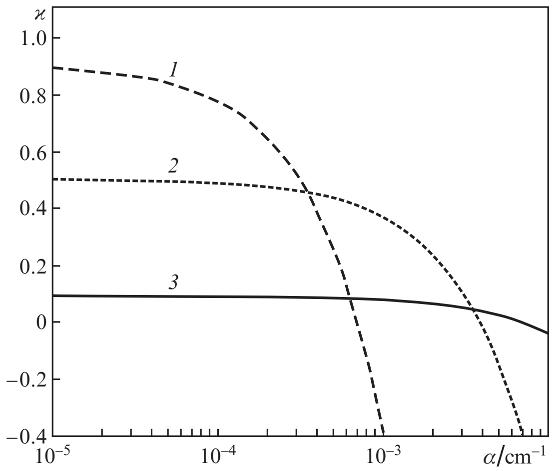


**Figure 1.** Dependences of the stored energy extraction efficiency on specific losses $\alpha$ for Nd:YAG ceramics ($L = 10$ cm) at $Q = 1.23 \times 10^{20}$ photon cm$^{-3}$ s$^{-1}$ (30 W cm$^{-3}$) and $c\Phi(0) =$ (*1*) $1.6 \times 10^{23}$ (32 kW cm$^{-2}$), (*2*) $1.6 \times 10^{22}$ (3.2 kW cm$^{-2}$), and (*3*) $1.6 \times 10^{21}$ photon cm$^{-3}$ s$^{-1}$ (0.32 kW cm$^{-2}$). The pump wavelength is $\lambda = 808$ nm; $\sigma\tau = 6.25 \times 10^{-23}$ cm$^2$ s.

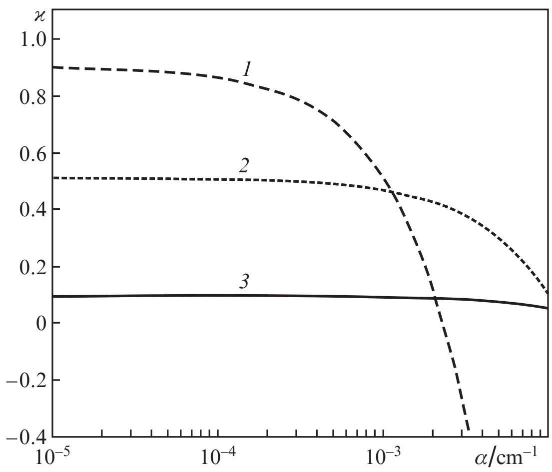


**Figure 2.** The same as in Fig. 1 but at $Q = 4.1 \times 10^{20}$ photon cm$^{-3}$ s$^{-1}$ (100 W cm$^{-3}$).

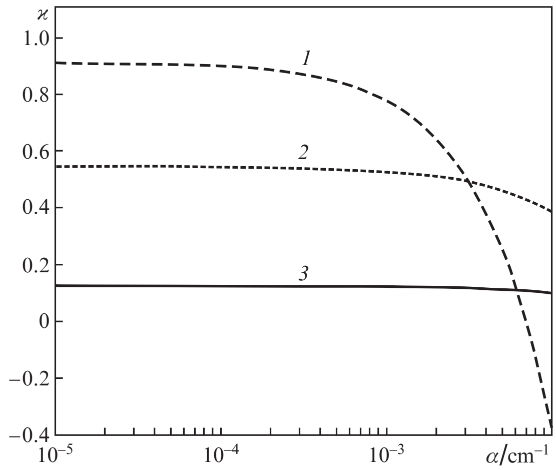


**Figure 3.** The same as in Fig. 1 but at $Q = 1.23 \times 10^{21}$ photon cm$^{-3}$ s$^{-1}$ (300 W cm$^{-3}$).

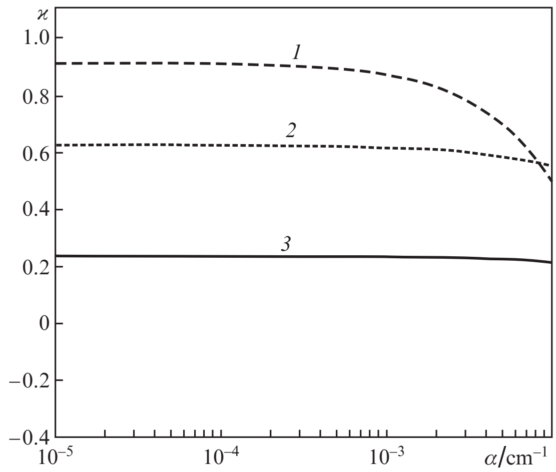


**Figure 4.** The same as in Fig. 1 but at $Q = 4.1 \times 10^{21}$ photon cm$^{-3}$ s$^{-1}$ (1000 W cm$^{-3}$).

As one would expect, the stored energy extraction efficiency increases with increasing radiation intensity at the amplifier input because, in the high-gain regime, $c\Phi(0) \gg (\tau\sigma)^{-1}$, the population inversion decreases to a minimum value corresponding to the optical transparency threshold, i.e., to the absence of absorption from the ground state [see also (4)]. Negative values of $\varkappa$ mean that the power losses due to absorption and scattering are not compensated for by amplification in the optical material, i.e., the output beam power becomes lower that the input power. This effect is especially pronounced for high-intensity beams and low specific pump powers (Figs 1, 2). In general, the obtained relations can be useful for optimisation of configurations of optical amplifiers tacking into account specific losses in optical materials. In addition, it should be noted that photon number density $\Phi$ depends on $\alpha$ exponentially in the unsaturated-gain regime and linearly in the high-gain regime [see (5)]. Thus, systematic measurements of the gain of an optical material as a function of the incident radiation intensity provide the potential possibility to determine specific optical losses without applying other methods. The method accuracy and the possible range of measurable specific losses will be considered in future publications.

## 3. Conclusions

The analytical solution obtained using the rate equations describes the single-pass gain as a function of the optical material spectroscopic parameters, specific losses, and amplified radiation intensity. These results are of scientific and practical interest, in particular, for optimisation of optical amplifiers and estimation of specific losses based on measurements of the gain of the optical material.

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## References

1. Osipov V.V., Khasanov O.L., Shitov V.A., et al. *Ross. Nanotekh.*, **3**, 98 (2008).
2. Bagayev S.N., Osipov V.V., Solomonov V.I., et al. *Opt. Mater.*, **34**, 1482 (2012).
3. Bagaev S.N., Osipov V.V., Shitov V.A., et al. *Atmos. Oceanic Opt.*, **25**, 292 (2012).
4. Bagayev S.N., Osipov V.V., Solomonov V.I., et al. *Persp. Mater.*, **4**, 18 (2012).
5. Bagayev S.N., Osipov V.V., Vatnik S.M., et al. *Quantum Electron.*, **45**, 492 (2015) [*Kvantovaya Elektron.*, **45**, 492 (2015)].
6. Bagayev S.N., Osipov V.V., Vatnik S.M., et al. *Quantum Electron.*, **45**, 23 (2015) [*Kvantovaya Elektron.*, **45**, 23 (2015)].
7. Li M., Hu H., Gao Q., et al. *IEEE Photonics J.*, **9**, 1504010 (2017).
8. Chen Y., Fan Z., Guo G., et al. *Opt. Mater.*, **71**, 125 (2017).
9. Peroni M., Tamburrini M. *Opt. Lett.*, **15**, 842 (1990).
10. Svelto O. Principles of Lasers (New York: Springer, 2010).
11. Rustad G., Stenersen K. *IEEE J. Quantum Electron.*, **32**, 1645 (1996).
12. Payne S.A., Chase L.L., Smith L.K., et al. *IEEE J. Quantum Electron.*, **28**, 2619 (1992).
13. Fan T.Y., Huber G., Byer R.L., Mitzscherlich P. *IEEE J. Quantum Electron.*, **24**, 924 (1988).
14. Gruber J.B., Sardar D.K., Yow R.M. *J. Appl. Phys.*, **96**, 3050 (2004).
15. Kumar G.A., Lu J., Kaminskii A.A., et al. *IEEE J. Quantum Electron.*, **40**, 747 (2004).
16. Pokhrel M., Ray N., Kumar G.A., Sardar D.K. *Opt. Mater. Express*, **2**, 235 (2012).