

EXTREME LIGHT FIELDS AND THEIR INTERACTION WITH MATTER

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Effect of synchrotron radiation on the dynamics of electron spin precession in the process of laser-plasma acceleration

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Abstract. **A model was developed and tested for studying the dynamics of spin precession and trajectories of charged particles in the wake fields generated in a laser-plasma accelerator. The model takes into account the radiation reaction force in the Landau–Lifshits form. An investigation is made of the effect of synchrotron radiation on the dynamics of energy gaining and spin precession of electrons accelerated in model constant fields as well as on the particle beam polarisation in the acceleration at a separate acceleration stage in the case of self-consistent description of the nonlinear dynamics of a laser pulse and its generated accelerating and focusing plasma wake fields. It is shown that synchrotron radiation hardly affects the energy gain rate and polarisation of the electrons that are accelerated in the fields characteristic for a moderately nonlinear mode of laser-plasma acceleration up to an energy of 3.8 TeV.**

Keywords: *laser-plasma acceleration, electron polarisation, betatron oscillations, synchrotron radiation.*

1. Introduction

The use of multistage laser-plasma accelerators of charged particles may become an alternative to conventional acceleration techniques in high-energy physics (HEP) experiments [1]. One of the schemes proposed for obtaining particle beams with an energy of ~1 TeV is a multistage electron–positron collider consisting of one hundred of accelerating sections [1, 2]. There are several acceleration techniques which may be implemented in a separate accelerating stage of such a collider. Among these, the greatest promise is shown by a moderately nonlinear acceleration regime, which is underlain by a high rate of particle energy gain in weakly nonlinear wake fields generated by a high-intensity short laser pulse in plasma [laser wakefield acceleration (LWFA)]. This regime provides efficient acceleration both of electrons and positrons as well as permits achieving the necessary synchronisation of the accelerating stages [2]. Despite the technical problems related to, in particular, the way of particle transportation between the sections with retention of bunch quality and to the asymmetrical input of laser pulses to the guiding structure [3], the feasibility of a two-stage laser-plasma electron acceleration has been recently demonstrated with the use of an active plasma lens to transport to the second accelerating section the focused electron beam accelerated in the first one [4].

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In modern HEP experiments, one of the most important characteristics of an accelerated electron bunch is its polarisation, because the particle interaction cross section is spin-orientation-dependent [5, 6]. The use of polarised beams permits performing precision tests of the standard model or directly investigating the weak interaction [7]. The objective of our work is to investigate the acceleration of spin-polarised electrons in the moderately nonlinear LWFA regime. The particle motion is considered in the fields generated by a short high-intensity laser pulse in a preformed plasma channel. The radial profile of the electron density in the channel is assumed to be parabolic:

$$n_0 = N_0\left(1 + \frac{r^2}{R_{\rm ch}^2}\right). \quad (1)$$

Here, N_0 is the unperturbed density of free electrons on the axis of the plasma channel; $R_{\rm ch}$ is the characteristic channel radius; and r is the distance from the channel axis. The use of such a channel, provided the channel radius matches the focal spot of the laser pulse, permits the laser radiation to propagate a distance equal to many Rayleigh lengths without diffraction spreading [8].

During acceleration, relativistic particles describe oscillations and emit synchrotron radiation, which may affect the electron beam characteristics [9–11]. In Section 2, we propose a model for calculating the dynamics of electron spin precession in the acceleration in the wake fields generated by a laser pulse in a separate collider accelerating section. This model takes into account the synchrotron radiation by including force of friction in the Landau–Lifshits form in the equation of electron motion. In Section 3, we estimate the effect of synchrotron radiation on the spin precession and energy gain rate for the electrons accelerated in a multisectional collider. To this end, at first we study the acceleration under the action of given constant forces, whose magnitudes correspond to the average magnitudes of the forces obtained from the self-consistent simulation of the moderately nonlinear LWFA regime [12]. Also we describe the evolution of the characteristics of the radiating electrons travelling under the constant forces typical for the bubble acceleration mode [12, 13]. Next, we investigate the effect of synchrotron radiation on the dynamics of electron beam polarisation at one LWFA stage with the self-consistent inclusion of the nonlinear propagation dynamics of the laser pulse and its generated plasma wake fields. The main results of our work are summarised in Section 4.

2. Basic equations

The propagation of laser radiation in the framework of cylindrical symmetry was described by the Maxwell equation for the laser pulse envelope [14]. The wake field generated in the plasma, which varies slightly over a cycle of a laser radiation wave, was determined using the equation for the wake potential $\Phi = \gamma_p - p_{ez}/(mc)$ derived from hydrodynamics equations [14, 15], where $\gamma_p = \sqrt{1 + p_e^2/(m^2c^2) + a^2/2}$; $\boldsymbol{p}_e$ is the momentum of plasma electrons:

$$\left[(\Delta_{\perp\rho} - \nu_0)\frac{\partial^2}{\partial\xi^2} - \frac{\partial \ln\nu_0}{\partial\rho}\frac{\partial^3}{\partial\rho\partial\xi^2} + \nu_0\Delta_{\perp\rho}\right] \times \Phi - \frac{\nu_0^2}{2}\left[1 - \frac{1 + |a|^2/2}{(1+\Phi)^2}\right] = \nu_0\Delta_{\perp\rho}\frac{|a|^2}{4}. \quad (2)$$

Here, $\nu_0 = n_0/N_0$; $\Delta_{\perp\rho}$ is the transverse part of the Laplace operator; $a = a(\xi, \rho, \zeta)$ is the slowly varying (over the period and wavelength of laser radiation) complex amplitude of the laser pulse normalised to $m_e c\omega_0/e$; e and m_e are the electron charge and mass; c is the velocity of light; ω_0 is the frequency of laser radiation;

$$\xi = k_{p0}(z - ct), \ \zeta = k_{p0}z, \ \rho = k_{p0}r_\perp, \ k_{p0} = \omega_{p0}/c \quad (3)$$

are dimensionless variables; and p0= is the plasma frequency.

The accelerating (F_z) and focusing ($F_\perp$) forces, which are normalised to $m_e c\omega_{p0}$, act on the charged particles under acceleration, and are related to the plasma wake field, may be expressed in terms of the wake potential [8, 14, 15]:

$$F_z = \frac{\partial\Phi}{\partial\xi}, \quad F_\perp = \frac{\partial\Phi}{\partial\rho}.$$

The electron spin precession is described by the Thomas–Bargman–Michel–Telegdi (T–BMT) equation [16]. To describe the effect of synchrotron radiation on the wake acceleration of the nth electron in the beam, use was made of the frictional force in the Landau–Lifshits form [11, 17]:

$$\boldsymbol{F}_{\rm rad} = -\frac{2r_e}{3c}\frac{\tilde{F}_\perp^2}{m_e^2c^2}\gamma^{(n)}\boldsymbol{p}^{(n)}, \quad (4)$$

where $\boldsymbol{p}^{(n)} = \{\boldsymbol{p}_\perp^{(n)}, p_z^{(n)}\}$ is the nth electron momentum; $\gamma^{(n)}$ is the gamma factor of the nth electron; $\tilde{r}_e \approx 3\times10^{-13}$ cm is the classical electron radius; and $\tilde{\boldsymbol{F}}_\perp$ is the dimensional focusing force, which acts on the electrons during acceleration.

Since the Lorentz force is far greater than the Stern–Gerlach force in the LWFA case [18], the effect of spin dynamics on the electron motion may be neglected. In this case, in dimensionless variables (ξ, ζ, $\boldsymbol{\rho}$) the system of equations for the trajectory evolution, the momentum, and the spin precession of the nth radiating beam electron, which is obtained by combining the T–BMT equation written in dimensionless coordinates [15], the relativistic Lorentz equation, and force (4), is of the following form:

$$\frac{d\boldsymbol{q}_\perp^{(n)}}{d\zeta} = \frac{1}{\beta_z^{(n)}}\left(\boldsymbol{F}_\perp - \frac{2}{3}r_e\gamma^{(n)}\boldsymbol{F}_\perp^2\boldsymbol{q}_\perp^{(n)}\right), \quad (5)$$

$$\frac{dq_z^{(n)}}{d\zeta} = \frac{1}{\beta_z^{(n)}}\left(F_z - \frac{2}{3}r_e\gamma^{(n)}\boldsymbol{F}_\perp^2 q_\perp^{(n)}\right), \quad (6)$$

$$\frac{d\boldsymbol{x}_\perp^{(n)}}{d\zeta} = \frac{\boldsymbol{q}_\perp^{(n)}}{q_z^{(n)}}, \quad (7)$$

$$\frac{d\xi^{(n)}}{d\zeta} = 1 - \frac{1}{\beta_z^{(n)}}, \quad (8)$$

$$\frac{d\boldsymbol{s}_\perp^{(n)}}{d\zeta} = \frac{s_z^{(n)}}{\beta_z^{(n)}}\left(a_m + \frac{1}{\gamma^{(n)}}\right)\boldsymbol{F}_\perp, \quad (9)$$

$$\frac{ds_z^{(n)}}{d\zeta} = -\frac{1}{\beta_z^{(n)}}\left(a_m + \frac{1}{\gamma^{(n)}}\right)(\boldsymbol{F}_\perp\boldsymbol{s}_\perp^{(n)}), \quad (10)$$

where $\boldsymbol{s}^{(n)} = \{\boldsymbol{s}_\perp^{(n)}, s_z^{(n)}\}$ is the nth electron spin normalised to the absolute magnitude of the electron spin; $a_m = 0.0011$ is the anomalous electron magnetic moment; $\boldsymbol{q}^{(n)} = \{\boldsymbol{q}_\perp^{(n)}, q_z^{(n)}\} = \boldsymbol{p}^{(n)}/(m_e c)$ is the dimensionless nth electron momentum; $\boldsymbol{\beta}^{(n)} = \boldsymbol{v}^{(n)}/c$ is the dimensionless velocity of the nth electron; $\boldsymbol{x}_\perp^{(n)} = \{x^{(n)}, y^{(n)}\}$ are the dimensionless transverse coordinates of the nth electron; $\boldsymbol{F}_\perp = \{F_\perp\cos\phi^{(n)}, F_\perp\sin\phi^{(n)}\}$; $\phi^{(n)} = \arctan(y^{(n)}/x^{(n)})$; and $r_e = \tilde{r}_e k_{p0}$.

Under the assumptions of a constant focusing force $F_\perp = -\alpha\rho$, a constant accelerating force $F_z = F_{ac}$, and $\boldsymbol{F}_{\rm rad} = 0$, the analytical expression for the nth electron spin component $s_z^{(n)}$ may be obtained from the T–BMT equation in cylindrical coordinates (with the spin vector $\boldsymbol{s}^{(n)} = \{s_r^{(n)}, s_\phi^{(n)}, s_z^{(n)}\}$) when $F_{ac}/\sqrt{\alpha\gamma^{(n)}} \ll 1$, $|\boldsymbol{q}_{\perp0}^{(n)}| \ll \sqrt{\alpha\gamma^{(n)}}\,|\boldsymbol{x}_{\perp0}^{(n)}|$ and $\gamma^{(n)} \gg 1$ [18]:

$$s_z^{(n)}[\gamma^{(n)}] = \sqrt{1 - s_{\phi0}^{(n)2}}\sin\left[-r_0^{(n)}\Phi + \arctan\left(\frac{s_{z0}^{(n)}}{s_{r0}^{(n)}}\right)\right], \quad (11)$$

where

$$\Phi[\gamma^{(n)}] = (1 + a_m\gamma^{(n)})\left(\frac{\alpha^2\gamma_0^{(n)}}{\gamma^{(n)3}}\right)^{1/4}\sin\left[\frac{2\sqrt{\alpha}}{F_{ac}}\left(\sqrt{\gamma^{(n)}} - \sqrt{\gamma_0^{(n)}}\right)\right]; \quad (12)$$

$\boldsymbol{s}_0^{(n)}$, $\boldsymbol{x}_{\perp0}^{(n)}$, $\boldsymbol{q}_{\perp0}^{(n)}$, and $\gamma_0^{(n)}$ are the initial parameters of the nth electron; and $r_0^{(n)} = (x_0^{(n)}|x_0^{(n)}| + y_0^{(n)}|y_0^{(n)}|)/\sqrt{x_0^{(n)2} + y_0^{(n)2}}$.

When determining the beam polarisation as the averaged vector of the electron spins in the beam [6],

$$\boldsymbol{P} = \sum_{n=1}^{N_b}\boldsymbol{s}^{(n)}/N_b, \quad (13)$$

where N_b is the total number of beam electrons, the envelope $|\Delta\boldsymbol{P}|_{\rm env} = |\boldsymbol{P} - \boldsymbol{P}_0|_{\rm env}$ of depolarisation oscillations for zero-emittance beams may be estimated using formula (11) (for $\Phi \ll 1$) [18]:

$$\frac{|\Delta\boldsymbol{P}|_{\rm env}}{|\boldsymbol{P}_0|} = \frac{(1 + P_{z0}^2)\sigma_r^2(\alpha\gamma_0\gamma)^{1/2}a_m^2}{8}. \quad (14)$$

Here, $\boldsymbol{P}_0$ is the initial beam polarisation; γ_0 is the average initial gamma factor of the beam electrons; and σ_r is the initial transverse width of the electron beam with a Gaussian density distribution.

3. Simulation results

To estimate the effect of radiation friction on the polarisation and the laser-plasma acceleration of an electron beam, initially we considered the dynamics of the characteristics of one electron, whose motion took place in given constant fields and was described by Eqns (5)–(10). Figure 1 depicts the energy gain by a radiating electron and a nonradiating one, which travel under the action of a dimensionless accelerating force $F_z = 0.47$ and different focusing forces, $F_\perp = -\alpha\rho = -0.75\rho$ or $F_\perp = -0.5\rho$, normalised to $m_e c\omega_{p0}$ for $\omega_{p0} \approx 17.84$ THz, which corresponds to a background plasma electron density $N_0 = 10^{17}$ cm^{-3}. The value $\alpha = 0.5$ corresponds to a strong focusing force in the acceleration of electrons in a bubble regime [9, 12], while $\alpha = 0.075$ corresponds to the average focusing force, which is defined below, in a moderately nonlinear acceleration regime in a wake wave. The initial electron spin $\boldsymbol{s}_0 = \{0.279, -0.334, 0.9\}$ in Cartesian coordinates, the initial transverse electron location $\boldsymbol{x}_{\perp 0} = \{0, 0.5\}$, $\boldsymbol{q}_{\perp 0} = 0$ and $\gamma_0 = 1000$ [18]. One can see in Fig. 1 that the radiating electron, which travels under the action of a high focusing force with $\alpha = 0.5$, accelerates to a lower energy than the nonradiating one and the energy loss amounts to 5% in the acceleration up to ~300 GeV. To estimate the ratio between the radiation friction force $F_{\mathrm{rad}\,z} = -(2r_e/3)\alpha^2\rho^2\gamma p_z$ to the acceleration force $F_z = 0.47$ (these forces act on the electron accelerated to 300 GeV), we take advantage of an approximate formula describing the trajectory of the nonradiating electron with $\boldsymbol{q}_{\perp 0} = 0$ in constant fields [10, 19]:

$$\boldsymbol{x}_\perp = \boldsymbol{x}_{\perp 0}\left(\frac{\gamma_0}{\gamma}\right)^{1/4}\cos\left[\frac{2\sqrt{|\alpha|}}{F_z}\left(\sqrt{\gamma}-\sqrt{\gamma_0}\right)\right]. \quad (15)$$

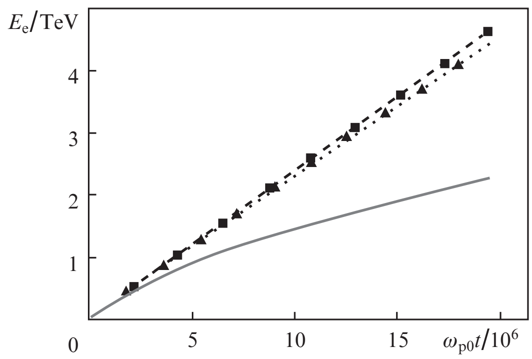

Figure 1. Dynamics of electron acceleration under the action of constant forces, the accelerating force $F_z = 0.47$ and the focusing one $F_{\perp 0} = -\alpha\rho$: time dependences of the electron energy with neglect of electron synchrotron radiation (dashed line and squares) and with its inclusion for $\alpha = 0.075$ (dotted line and triangles) and for $\alpha = 0.5$ (solid curve).

Then $\rho \leqslant |x_{\perp 0}|(\gamma_0/\gamma)^{1/4}$, and

$$\left|\frac{F_{\mathrm{rad}\,z}}{F_z}\right| \approx \frac{2r_e}{3F_z}\alpha^2\rho^2\gamma^2 \leqslant \frac{2r_e}{3F_z}\alpha^2 x_{\perp 0}^2\sqrt{\gamma_0}\,\gamma^{3/2} \approx 0.23 \quad (16)$$

for $\gamma \sim p_z$ and $\alpha = 0.5$. Figure 2a shows the dependence of the depolarisation of the radiating and nonradiating electrons on the acceleration time $\omega_{p0}t$ for $\alpha = 0.5$. One can see that the depolarisation is weaker for the radiating electron, which acquires a lower energy. In this case, the analytical depolarisation curve, which was obtained for the nonradiating electron with the use of formula (11), provides an adequate description of the depolarisation of the radiating particle with an energy of up to 300 GeV. The functional dependence of the depolarisation on the particle oscillation radius and the particle energy follows from formula (14). In accordance with this dependence, the radiating electron exhibits a lower depolarisation than the nonradiating one, since it acquires a lower energy and oscillates with a lower amplitude. The energies gained by the radiating and nonradiating electrons for 0.075 are hardly different up to ~3.8 TeV (see Fig. 1), which corresponds to an estimate $|F_{\mathrm{rad}\,z}/F_z| \approx 0.23$. As follows from Fig. 2b, for relatively weak focusing forces the effect of synchrotron radiation on the depolarisation of particles also manifests itself beginning with an energy of about 3.8 TeV. In the case of forces typical for the bubble mode, it becomes significant for particles with an energy of ~300 GeV.

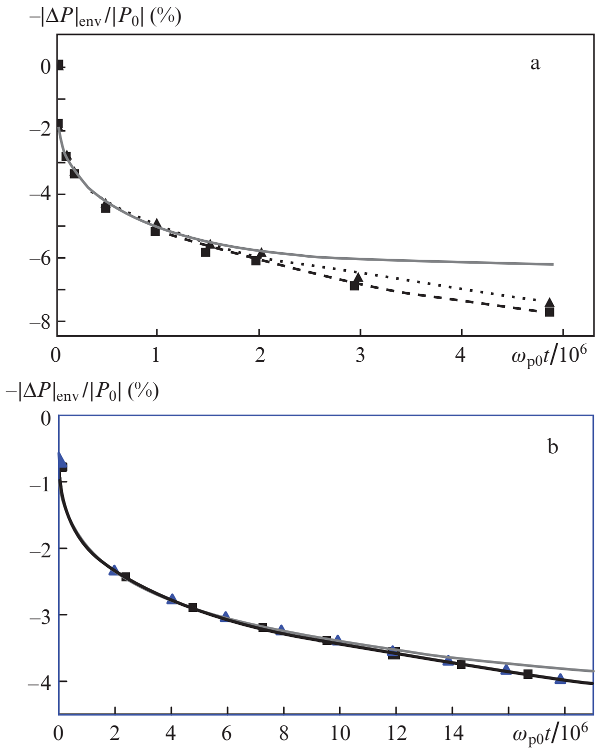

Figure 2. Time evolution of the depolarisation of an electron travelling under the action of constant accelerating, $F_z = 0.47$, and focusing, $F_{\perp 0} = -\alpha\rho$, forces with the inclusion (solid curve) and with neglect (dashed curve, squares) of radiation for α = (a) 0.5 and (b) 0.075 (solid curve, squares); the envelope of the analytical depolarisation value obtained from expression (11) is shown with a dotted line and triangles in Fig. 2a and with a grey curve and triangles in Fig. 2b.

The acceleration of a polarised electron beam with a self-consistent description of the nonlinear dynamics of a laser pulse and its generated accelerating and focusing plasma wake fields was investigated using the LAPLACE code [19] for the following parameters of the laser pulse: radius $r_{\mathrm{las}} = 89.13$ μm of the focal spot of the Gaussian pulse focused on the input of the plasma channel, duration $\tau_{\mathrm{las}} = 56$ fs, wavelength $\lambda_{\mathrm{las}} = 0.8$ μm, intensity $I_{\mathrm{las}} = 4.28\times10^{18}$ W cm^{-2} (dimensionless amplitude: $a_0 = 1.414$), and power $P_{\mathrm{las}} = 534$ TW. The plasma density on the axis of the preformed channel (1) $N_0 = 10^{17}$ cm^{-3}, its radius $R_{\mathrm{ch}} = 305.1$ μm, and $k_{p0} = 0.0595$ μm^{-1} [2, 20]. The electron beam injected from the outside [21] possessed a cylindrically symmetric transverse Gaussian distribution and a zero longitudinal spread. The accelerating force employed for an analytical depolarisation estimate was assumed to be equal to the average field strength acting on the beam electrons in the course of wake acceleration:

$$\overline{F}_z = \frac{1}{N_b}\sum_{n=1}^{N_b}\frac{1}{L_{ac}^{(n)}}\int_0^{L_{ac}^{(n)}} F_z(\boldsymbol{x}_\perp^{(n)},\zeta)\,d\zeta, \quad (17)$$

where $L_{ac}^{(n)}$ is the nth acceleration length normalised to k_{p0}^{-1}. A similar approach is employed to calculate the average coefficient $\overline{\alpha}$ for the focusing force.

The simulation of wake acceleration was performed by integrating the system of equations (2)–(10) for a polarised electron beam with an initial characteristic transverse width σ_r/k_{p0} = 4.2 μm, a polarisation $\boldsymbol{P}_0$ = {0.279, −0.334, 0.9}, a normalised emittance ε = 2.5 mm rad, and the number N_b = 10^5 of electrons in the beam. The electron bunch with an energy $E_{e\,inj}$ = 306.6 GeV was injected into the vicinity of the peak of the accelerating force (ξ_{max} = 3.0, ξ_{inj} = 3.2) [15]. In this case, the forces averaged over the acceleration length $L_{ac}^{(n)} \approx 0.5$ that act on the electron beam are as follows: $\overline{F}_z = 0.47$, $\overline{F}_r = -0.075\rho$. In conformity with the analysis performed above, the depolarisation dynamics obtained by self-consistent calculations is hardly different for the radiating and non-radiating electrons of the beam, and the upper bound of depolarization may be predicted by formula (14) (Fig. 3).

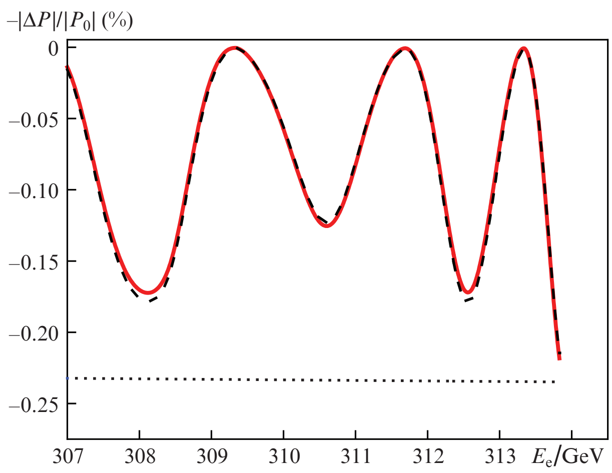

Figure 3. Dynamics of electron beam depolarisation for one LWFA accelerating section (solid curve) with and (dashed curve) without radiating electrons; the dotted line stands for the theoretical prediction of depolarisation (14) for F_z = 0.47 and α = 0.075.

4. Conclusions

We have investigated the effect of synchrotron radiation on the depolarisation of electrons in the acceleration in plasma wake fields generated by a short high-intensity laser pulse. The dynamics of electron characteristics was described by numerical solution of model Eqns (5)–(10). It was shown that radiation friction, which lowers the transverse and longitudinal electron momenta, results in an appreciable lowering of depolarisation in the motion of electrons with energies above 300 GeV in strong focusing fields typical for the bubble acceleration regime and, in this case, significantly lowers the rate of energy acquisition. An analytical estimate of beam depolarisation made by formula (14), which does not take into account the radiation loss, agrees nicely with numerical simulations for particles travelling in relatively low focusing fields as well as for particles travelling under high focusing fields, as long as their final energy does not exceed 300 GeV. It was shown that the synchrotron radiation hardly affects the polarisation and the rate of energy acquisition in the electron acceleration up to 3.8 TeV under the fields typical for the moderately nonlinear laser-plasma acceleration mode.

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