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Physical mechanism of electron bunch generation by an ultrarelativistic-intensity laser pulse passing through a sharp plasma boundary

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Abstract. **The physical mechanism of the interaction process of a laser pulse of ultrarelativistic intensity with semi-bounded plasma having a sharp boundary is studied in one-dimensional geometry. It is shown that electron bunches are generated by the laser pulse due to the multiflow motion of plasma electrons with crossing trajectories. An analytical relation that allows an electron bunch charge to be estimated as a function of plasma characteristics and electron trajectory parameters is derived and confirmed by the results of numerical simulations.**

Keywords: *laser pulse, wake wave, semi-bounded plasma, self-injection of electrons, laser-plasma acceleration of electrons.*

1. Introduction

The characteristics of electron bunches arising under laser-plasma acceleration are largely determined by the method of electron injection into the accelerating system. Among the previously known methods of electron injection into an accelerating wake wave, one can mention optical [1, 2] and ionisation [3, 4] techniques, as well as self-injection of electrons into the wake wave of the laser pulse, caused by its passing through the plasma density jump [5, 6] or its nonlinear dynamics in the so-called bubble regime of propagation through plasma [7–9].

The present papers is devoted to the study of injection of electrons into the wake wave, generated by a laser pulse in plasma, based on the generation of electron bunches that arise under the interaction of the laser pulse with the plasma boundary. For the first time this method was considered in Ref. [10], where using numerical simulation it was found that a laser pulse of ultrarelativistic intensity passing through the boundary of a rarefied plasma target generates ultrashort electron bunches with high energy, the process being quasi-one-dimensional.

On this basis, the authors of Refs [11–13] proposed a one-dimensional physical model to describe the process of electron bunch generation under the interaction of a laser pulse with a sharp plasma boundary. It was shown that in the regime considered in Ref. [10] the electron bunch generation is possible only when after the interaction with the laser pulse the energy of longitudinal oscillations of electrons $E_{\rm os}$ attains a certain value. The threshold energy of oscillations for the generation of electron bunches is $E_{\rm os\,th} = E_{\rm e}/\sqrt{1 - V_{\rm gr}^2/c^2}$, where $E_{\rm e}$ is the rest energy of electrons and $V_{\rm gr}$ is the group velocity of the laser pulse.

However, to simplify the theoretical analysis of the electron bunch generation performed in Refs [11–13], the energy was assumed to be only slightly above the threshold ($E_{\rm os} - E_{\rm os\,th} \ll E_{\rm os\,th}$), which allowed the study of the formation of electron bunches as a result of the laser pulse action on the plasma, neglecting the electrons that leave the ion background region. Using the obtained results it was established that in the determination of the bunch charge the difference between theoretical predictions and numerical simulation results increases with increasing electron oscillation energy, and under the condition $E_{\rm os} - E_{\rm os\,th} \sim E_{\rm os\,th}$ becomes considerable. One can expect that when the energy strongly exceeds the threshold, the mechanism of plasma electron self-injection into the wake wave of the laser pulse is more complex than that assumed in Refs [11–13].

The goal of the present paper is to present a thorough study of the process of electron bunch generation by a laser pulse interacting with semi-bounded plasma and to reveal the specific features of accumulation of different groups of electrons in a laser-induced bunch, when their total oscillation energy is significantly above the threshold value.

2. Physical and mathematical model of laser pulse interaction with semi-bounded plasma

Consider a semi-bounded plasma, in which only its electron component is mobile, while ions comprise an immobile uniform positively charged background. The sharp plasma boundary coincides with the origin of the coordinate axis z. A short one-dimensional laser pulse of circularly polarised electromagnetic radiation with frequency ω_0 much greater than the plasma frequency $\omega_{\rm p}$ propagates along the positive direction of the z axis across the plasma boundary. Let us assume that the shape of the laser pulse envelope does not change as it propagates into the plasma at the spatial scale of interest for the present problem. This corresponds to the quasi-static approximation, in which the driver that causes the motion of electrons evolves much slower than the response of plasma electrons. From this fact it follows that for the constant velocity $V_{\rm gr}$ of the laser pulse propagation in the homogeneous plasma its impact on each next electron is completely similar to the impact on the preceding ones.

In one-dimensional geometry for circular polarisation of the electromagnetic waves of the laser radiation the longitudinal motion of electrons along the z axis has no high-frequency component and is described by the equations

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$$\frac{\mathrm{d}P}{\mathrm{d}t} = |e|\frac{\partial\varphi}{\partial z} - mc^2\frac{\dfrac{\partial}{\partial z}\left(\dfrac{eA}{mc^2}\right)^2}{2\sqrt{1+\dfrac{P^2}{m^2c^2}+\left(\dfrac{eA}{mc^2}\right)^2}}, \quad (1)$$

$$\frac{\mathrm{d}z}{\mathrm{d}t} = u = \frac{P/m}{\sqrt{1+\dfrac{P^2}{m^2c^2}+\left(\dfrac{eA}{mc^2}\right)^2}}, \quad (2)$$

where $A(z,t)$ is the envelope amplitude of the laser pulse vector potential; $\varphi(z,t)$ is the scalar potential of the charge separation field; P and u are the momentum and the velocity of the electron; and $-|e|$ and m are its charge and mass, respectively.

The charge separation field arises due to the laser pulse action on the electrons, which shifts each electron from its initial position z_0. After the interaction with the laser pulse, the electron transits into the regime of free oscillatory motion with the oscillation centre z_0. It is important to note that at the initial stage of electronic motion, the order of mutual arrangement of electrons is conserved, and if the laser pulse characteristics are unchanged, then after the action of the laser pulse the background plasma electrons move along the trajectories that are similar, but possess a certain time shift. This shift is determined by the distance z_0 from the plasma boundary, at which each electron was located before the laser pulse impact.

For further mathematical analysis of motion of the electrons that form the bunch generated by the laser pulse, let us accept a limitation related to the character of electron motion. This limitation implies that the plasma electrons stop to experience the laser pulse action before they cross the ion background boundary. This implies that the laser pulse must be short enough. For clarity, Fig. 1 illustrates an example calculation of interaction with plasma electrons for a laser pulse of circularly polarised radiation with the envelope, whose time dependence can be presented in the form $a = a_0\cos^2(t/\tau)\times\mathrm{sgn}(\pi\tau/2 - |t|)$, where $a_0 = |e|A_0/(mc^2) = 7.867$ is the dimensionless amplitude of the vector potential; τ is the duration of the laser pulse, corresponding to $\tau_{\mathrm{FWHM}} = 1.143\tau = 12$ fs. It was assumed that the group velocity V_{gr} of laser pulse propagation in plasma corresponds to the gamma-factor $\gamma_{\mathrm{ph}} = 1/\sqrt{1 - V_{\mathrm{gr}}^2/c^2} = 5$. The plasma concentration n_0 is determined from the relation $\omega_0/\omega_{\mathrm{p}} = \gamma_{\mathrm{ph}} = 5$, where $\omega_{\mathrm{p}} = \sqrt{4\pi e^2 n_0/m}$ is the plasma frequency, ω_0 is the high carrier frequency of the laser pulse, corresponding to the wavelength $\lambda_0 = 1$ μm. Consequently, we obtain $\tau = 3.956\omega_{\mathrm{p}}^{-1}$. Figure 1 shows the vector potential $|e|A/(mc^2)$ of the laser pulse and the potential $|e|\varphi/(mc^2)$ of the charge separation field generated by the laser pulse versus the dimensionless coordinate $k_{\mathrm{p}}z$, where $k_{\mathrm{p}} = \omega_{\mathrm{p}}/c$, as well as the distribution of the selected family of electron macroparticles (points) in the phase plane z, P (before the laser pulse action the electrons are assumed to be uniformly distributed in plasma with the step $k_{\mathrm{p}}\Delta z_0 = 0.1$). Because of the high concentration of electron macroparticles in some areas, their distribution merges into a solid line.

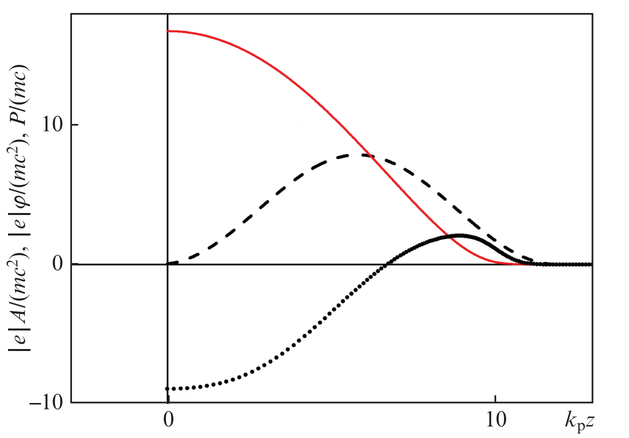

Figure 1. Distributions of the vector potential of the laser pulse $|e|A/(mc^2)$ (dashed line) and the wake wave potential $|e|\varphi/(mc^2)$ (solid line) along the coordinate z, as well as distribution of electron macroparticles (points) in the phase plane z, P at the moment $\omega_{\mathrm{p}}t = 0$. The parameters of the laser pulse are $a_0 = |e|A_0/(mc^2) = 7.867$, $\tau_{\mathrm{FWHM}} = 12$ fs; $\gamma_{\mathrm{ph}} = 1/(1 - V_{\mathrm{gr}}^2/c^2)^{1/2} = 5$, $\lambda_0 = 1$ μm.

This figure corresponds to the time moment, chosen as a zero point of the time scale for the process of laser pulse interaction with plasma ($t = 0$). At this moment of time, the leftmost electron of the plasma in its backward motion after the laser pulse action crosses the boundary of the ion background, and its further motion occurs in the spatial region beyond the ion background. It is seen that for the chosen parameters of the laser pulse its action on the motion of the given electron at $t = 0$ is absent, and, therefore, the limitation accepted above is valid. The estimates obtained in Ref. [12] have shown that at the relativistic velocity of the electron motion in the longitudinal direction after the impact of the laser pulse, for which $V_{\mathrm{gr}} \approx c$, the accepted limitation is valid, if the length of the laser pulse does not exceed the amplitude of the longitudinal oscillation of electrons, induced by the pulse.

The accepted limitation essentially simplifies the mathematical analysis of the electron motion, because when the electron becomes free of the laser pulse action, one can consider it as a free plasma oscillator. Thus, the laser pulse propagating through plasma leaves behind it an ensemble of free plasma oscillators with the similar total energy E_{os} for all electrons, the value of which is determined by the amplitude of the laser pulse vector potential $a_0 = |e|A_0/(mc^2)$, the characteristic pulse duration τ, the gamma-factor $\gamma_{\mathrm{ph}} = 1/\sqrt{1 - V_{\mathrm{gr}}^2/c^2}$, and the plasma concentration n_0.

The equation of motion for this oscillator has the form

$$\frac{\mathrm{d}P}{\mathrm{d}t} = -4\pi e^2 n_0(z - z_0), \quad (3)$$

where the force acting on the electron is determined exclusively by the charge separation field

$$E_z = 4\pi|e|n_0(z - z_0), \quad (4)$$

arising due to the displacement of the electron from its equilibrium position z_0. Equation (3) has the integral that corresponds to the energy conservation law

$$\sqrt{m^2c^4 + c^2P^2} + 2\pi e^2 n_0(z - z_0)^2 = E_{\mathrm{os}}. \quad (5)$$

The principle of similarity of electron trajectories is conserved until the electrons cross the boundary of the ion background $z = 0$. If an electron leaves the ion background region, then the electric field that returns it to the initial position, is determined by the expression

$$E_z = -4\pi|e|n_0 z_0, \quad (6)$$

and the appropriate integral of the equation of motion has the form

$$\sqrt{m^2c^4 + c^2P^2} + 2\pi e^2 n_0(z_0^2 - 2zz_0) = E_{\rm os}. \qquad (7)$$

Expressions (5) and (7) allow exact integral formulation of the trajectory for any electron of the plasma at any point of space and at any moment of time after the end of its interaction with the laser pulse and before the beginning of the electron mixing process. However, below it is convenient to use the notation of electron trajectories that simultaneously presents the entire collection of all trajectories of plasma background electrons, differing from each other by the value of the coordinate z_0 of the oscillation centre:

$$t = \frac{z_0}{V_{\rm gr}} + \frac{1}{c}I(z, z_0) + T_{\rm lft}(z_0), \qquad (8)$$

where the following notations are introduced:

$$cT_{\rm lft}(z_0) = -2I(z_0, z_0 - A_{\rm m}) \quad \text{for } z_0 \geqslant A_{\rm m}, \qquad (9)$$

$$cT_{\rm lft}(z_0) = -2I(z_0, 0) - 2I_{\rm vac}(z_0) \quad \text{for } z_0 < A_{\rm m}, \qquad (10)$$

$$I(z, z_0) = \int_{z_0}^{z} \frac{{\rm d}z'}{\sqrt{1 - m^2c^4/[E_{\rm os} - 2\pi e^2 n_0(z' - z_0)^2]^2}}, \qquad (11)$$

$$I_{\rm vac}(z_0) = \int_0^{z_{\rm bn}(z_0)} \frac{{\rm d}z'}{\sqrt{1 - m^2c^4/[E_{\rm os} + 2\pi e^2 n_0(2z'z_0 - z_0^2)]^2}}. \qquad (12)$$

Here the expression $I(z, z_0)$ corresponds to the electron motion in the region of ion background; $I_{\rm vac}(z_0)$ corresponds to the motion beyond the region of ion background after passing the boundary $z = 0$ by the electron; $z_{\rm bn}(z_0) = z_0/2 - (E_{\rm os} - mc^2)/(z_0 4\pi e^2 n_0)$ is the coordinate of maximal penetration into the region $z < 0$ for the electron that before the laser pulse impact was located at point $z_0 < A_{\rm m}$; $T_{\rm lft}(z_0)$ is the total length of the time interval necessary for the electron to execute the oscillatory motion to the left from its oscillation centre z_0 and back to z_0 again with the ion background boundary crossing taken into account; and $A_{\rm m} = \sqrt{(E_{\rm os} - mc^2)/(2\pi e^2 n_0)}$ is the oscillation amplitude for the electrons that do not leave the region, occupied by the ion background.

Equation (8) describes the trajectories in the region $z > 0$ for all plasma electrons during the time interval, when the electrons again move in the direction of the laser pulse propagation, but are beyond the spatial region occupied by the pulse and do not interact with it. The integration constant in Eqn (8) according to Fig. 1 is determined from the condition that the electron with the centre of oscillation coincident with the origin of coordinates ($z_0 = 0$) at $t = 0$ is located at its centre of oscillation and moves to the left, i. e., outwards the ion background. All other electrons of the plasma begin their motion along similar trajectories (until the plasma boundary is crossed) with the delay of $z_0/V_{\rm gr}$, determined by the positions of their oscillation centres.

Equations (8)–(12) reflect the fact that according to the character of trajectories, the electrons of plasma can be divided into two groups: those that cross the boundary of the ion background in the process of their motion and those that do not. When an electron crosses the plasma boundary, then even when the order of electrons is conserved, its trajectory is no more similar to that of the electrons that did not cross the ion background boundary. After crossing the boundary by the considered electron in the opposite direction, the similarity of its trajectory to the trajectories of electrons that did not cross the boundary becomes restored. However, in this case the value of the time shift with respect to similar trajectories is changed.

3. Wake wave breaking and self-injection of electrons

The manifold of motions of individual plasma electrons as plasma oscillators constitutes a longitudinal plasma wave. In infinite plasma, where the thermal motion of charged particles is neglected, this wave is an eigenmode of longitudinal one-dimensional electromagnetic oscillations of cold plasma, if the amplitude of the plasma wave or the oscillation amplitude of electrons does not exceed a certain limiting value [14]. The plasma wave has a regular character, i. e., propagates in the plasma without changing its shape at a constant phase velocity $V_{\rm ph}$.

In the present paper, the oscillations of plasma electrons and, therefore, the plasma wave are induced by the laser pulse. For this reason, the plasma wave is referred to as the wake wave. As shown above, the plasma electrons that do not move beyond the background boundary possess similar trajectories at least at the initial stage of their motion under the condition that the shape and velocity of the laser pulse exciting the motion of the plasma electrons remain unchanged. If the similarity of the electron trajectories is conserved during the entire cycle of their oscillations, then in the plasma the laser pulse will be followed by a regular plasma wave, propagating with the phase velocity equal to the group velocity of the pulse, $V_{\rm ph} = V_{\rm gr}$. From the results of the study carried out in Refs [12, 13] it follows that the condition for the regular wake wave formation can be presented in the form

$$E_{\rm os} < E_{\rm os\,th} = mc^2\gamma_{\rm ph}. \qquad (13)$$

In the case when condition (13) is not satisfied, i. e., when $E_{\rm os} > E_{\rm os\,th} = mc^2\gamma_{\rm ph}$, the similarity of trajectories will be violated starting from a certain moment of time even for electrons that do not move beyond the ion background. This occurs due to the crossing of trajectories of plasma electrons in the process of their motion. The crossing of trajectories with the violation of condition (13) occurs behind the laser pulse and begins near the first minimum of the wake wave potential. Therefore, already during the first period of electron oscillations after the laser pulse, the profile of the plasma wave becomes nonstationary, or, as they commonly say, the wake wave breaking takes place.

The crossing of electron trajectories disturbs the initial order of the relative positions of electrons, i. e., the mixing of electrons occurs. The mixing of electrons causes the appearance of plasma regions, where the concentration of electrons is singular. Because of redistribution of charges in the plasma, the plasma fields change so that some groups of electrons from the regions of their concentration singularity transit to the regime of acceleration by the wake wave of the laser pulse, i. e., become self-injected into the accelerating wake field, thus forming an electron bunch generated by the laser pulse. Thus, the process of electron bunch generation has threshold character.

The phenomenon of breaking of the laser pulse wake wave, propagating through rarified plasma, for which $\gamma_{ph} \gg 1$, and the accompanying phenomenon of electron self-injection into the wake wave and capture of electrons in it, occur under the condition that the total energy of the plasma electron oscillation exceeds the rest energy of the electrons $E_e = mc^2$ at least by an order of magnitude. Therefore, the electron bunch generation requires ultrarelativistic intensity of the laser pulse $a_0 = |e|A_0/(mc^2) \sim 10$. The calculations performed in Refs [11–13] confirm this estimate.

Let us consider the process of electron self-injection into the wakefield of the laser pulse in more detail. Expression (8) obtained for the trajectories of electrons after the laser pulse impact completely and exactly describes the motion of all plasma electrons after the passage of the laser pulse at any oscillation energy E_{os}, but only before the beginning of the electron trajectory crossing process. The theoretical analysis, carried out in Refs [12, 13] has shown that for a small excess over the threshold of electron bunch generation ($E_{os} - E_{os\,th} \ll E_{os\,th}$) the process of crossing of electron trajectories, mixing of electrons and their capture by the accelerating wakefield begins from the electron located before the laser pulse impact inside the plasma at the distance $z_{0\,ld} = A_m$ from the boundary, which is equal to the electron oscillation amplitude at the given value E_{os} of the total oscillation energy, provided that the plasma boundary is sharp. This electron is called a leading electron, since it is the first one in the leading part of the electron bunch generated by the laser pulse. According to its initial position, the leading electron only touches the boundary of the ion background, but does not cross it. The electrons whose initial position is $z_0 < A_m$ comprise a negligibly small part of the captured bunch under the condition that $E_{os} - E_{os\,th} \ll E_{os\,th}$, and were not taken into account earlier in Refs [12, 13].

In the case considered by us, when the energy is much higher than the threshold, i.e., $E_{os} - E_{os\,th} \approx E_{os\,th}$, it is not obvious which electron will be a leader in the electron bunch, generated by the laser pulse. The condition for trajectory crossing of any electron with the trajectory of the adjacent one reads as $dz/dz_0 = 0$. Differentiating expression (8), we arrive at the relation

$$u(z, z_0)/c = \sqrt{1 - m^2c^4/[E_{os} - 2\pi e^2 n_0 (z - z_0)^2]^2}$$
$$= \left[\frac{c}{V_{gr}} + c\frac{dT_{lft}(z_0)}{dz_0}\right]^{-1}. \tag{14}$$

However, condition (14) is only necessary, but not sufficient to determine the electron, from which the process of electron self-injection into the wake wave will begin. The leadership in the electron bunch will be won by the electron with such position z_0 of the oscillation centre, that moving along its trajectory (8) in time it will first achieve the coordinate z, satisfying relation (14) together with z_0. In other words, under condition (14) it is necessary to find the minimum of the function

$$t(z, z_0) = \frac{z_0}{V_{gr}} + \frac{1}{c}I(z, z_0) + T_{lft}(z_0). \tag{15}$$

The mathematical analysis shows that in the case of high excess over the threshold the electron with $z_0 = A_m$ again is the first to enter into regime of trajectory crossing, although the close electrons with $z_0 < A_m$ are ahead in phase of oscillations and to the considered moment of time possess greater velocity and energy of motion $E = \sqrt{m^2c^4 + c^2P^2}$. As in the case of small excess over the threshold, this electron appears the first in the head of the bunch generated by the laser pulse. The energy of the leading electron injection into the wakefield, $E_{inj} = \gamma_{ph}mc^2$, is resonance, i.e., the electron velocity at the moment of self-injection is equal to the phase velocity of the wake wave.

Figure 2 shows the distribution of electron macroparticles of plasma (points) in the phase space z, P near the point of self-injection of the leading electron $z_{inj\,ld}$ (marked by a large circle) at the time moment $\omega_p t_{inj\,ld} = 9.736$ when the process of self-injection of electrons into the wake wave of the laser pulse begins. The accelerating force $F = -|e|E_z/(mc\omega_p)$ acting on the electrons at the given moment of time and determined by the magnitude of the electric field E_z of the wake wave is also shown (solid curve). The laser pulse and plasma parameters here and below are the same as indicated above.

We showed [12, 13] that the condition $dz/dz_0 = 0$ specifies the coordinate of the leading electron self-injection into the wake wave by the expression

$$z_{inj\,ld} = z_{0\,ld} - \sqrt{(E_{os} - \gamma_{ph}mc^2)/(2\pi e^2 n_0)}, \tag{16}$$

where $z_{0\,ld} = A_m$ is the initial position of the leading electron before the action of the laser pulse; correspondingly, the time moment of the beginning of the wake wave breaking process is determined as

$$t_{inj\,ld} = \frac{z_{0\,ld}}{V_{gr}} + \frac{1}{c}I(z_{inj\,ld}, z_{0\,ld}) + T_{lft}(z_{0\,ld}). \tag{17}$$

The momentum value possessed by the leading electron at the time moment of self-injection, $P_{res} = mc\sqrt{\gamma_{ph}^2 - 1}$, is marked in Fig. 2 by the horizontal dash-dot line. At this moment, the accelerating force acting on the leading electron is positive, since the electric field at the point of self-injection of the leading electron is determined by the expression

$$E_z = -\sqrt{8\pi n_0 (E_{os} - \gamma_{ph}mc^2)}. \tag{18}$$

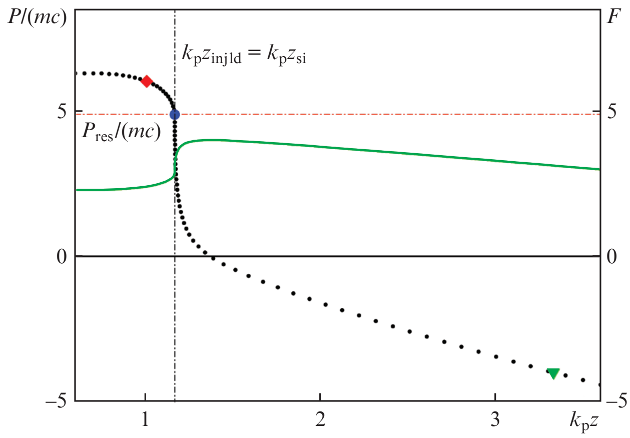

Figure 2. Distribution of electrons in the phase plane z, P (point) at the beginning of the electron self-injection process $\omega_p t_{inj\,ld} = 9.736$. The solid curve represents the force $F = -|e|E_z/(mc\omega_p)$ acting on electrons in the wake wave (see text).

The direction of the electric field at the point of the leading electron self-injection indicates the character of the charge distribution in plasma at this moment of time. Namely, the total charge of the ions to the right of the leading electron at this moment is greater than the total charge of electrons. That is why the force acting on the leading electron is accelerating, and immediately after the self-injection into the wake wave, the leading electron transits to the acceleration regime in the wakefield of the laser pulse.

Moreover, from Fig. 2 it is seen that immediately before the beginning of the self-injection process the oscillating plasma electrons near the leading electron form an accumulation point, in which their trajectories draw together in such a way that the concentration of plasma electrons acquires a singularity at this point. The point of the leading electron self-injection $z_{\rm inj\,ld}$ is a point of accumulation for both groups of electrons, which have crossed the ion background boundary and which have not.

Further motion of electrons leads to the crossing of their trajectories; the electrons become mixed and injected into the wakefield of the laser pulse, which is confirmed by the numerical simulation. Both the electrons initially located to the right of the leader with $z_0 > A_{\rm m}$ and the electrons initially located to the left of the leader with $z_0 < A_{\rm m}$ are involved in the self-injection process. This process is not infinite, and the conditions for its termination will be considered below. Figure 2 shows the electrons, leftmost and rightmost with respect to the leading electron (large circle), thus indicating the region, from which the electrons will compose the bunch, generated by the laser pulse. The triangle and the diamond mark the electrons that have the oscillation centres at the utmost points $z_{0\,\rm r}$ and $z_{0\rm l}$, initially (before the interaction with the laser pulse) located to the right and to the left of the leading electron, respectively.

4. Mixing of electrons and electron bunch generation

The process of electron mixing leading to the formation of a bunch of electrons captured by the wake wave is a very complex phenomenon that can hardly be described exactly. The above expressions (8), (9) remain exact only for the electrons located outside the mixing region.

In further study let us assume that while the size of the electron mixing region is much smaller than other characteristic scales in the longitudinal direction, the electron order violation in this region does not affect their motion in it, and one can use the formulae (8)–(12) with sufficient accuracy. The substantiation of this assumption consists in the following: as seen by the example of the leading electron, the entrance of electrons into the mixing region occurs at relativistic energies of their motion $E = \sqrt{m^2c^4 + c^2P^2}$. This implies the electron velocity close to the velocity of light. Therefore, at small dimensions of the mixing region the real trajectory of electrons slightly differs from the relativistic electron motion, described by Eqns (8)–(12). The required condition related to the size of the mixing region is satisfied with great margin, since from the results of Refs [12, 13] it follows that the size of the generated electron bunch that coincides with the size of the mixing region by the order of magnitude is extremely small as compared to the length of the nonlinear plasma wave and the amplitude of electron oscillations.

The mechanism of electrons grouping into the bunch generated by the laser pulse depends on their initial position relative to the leading electron. The electrons from the segment $(z_{0\rm ld}, z_{0\,\rm r})$ initially located to the right of the leading electron are injected into the wake wave at the self-injection point, located deeper in plasma with respect to the point of self-injection of the leading electron, and with a time delay relative to it. From the condition of trajectory crossing for any adjacent electrons ($dz/dz_0 = 0$), applied to the ensemble of electrons described by Eqns (8)–(12), it follows that the point of self-injection of electrons into the wake wave $z_{\rm si}$ moves following the laser pulse with the velocity, equal to its group velocity, according to the relation

$$z_{\rm si} = z_{\rm inj\,ld} + V_{\rm gr}(t - t_{\rm inj\,ld}). \tag{19}$$

In Fig. 2, the vertical dash-dot line marks the position of the self-injection point for the leading electron, which at the time moment $t_{\rm inj\,ld}$ is located at the point with the coordinate $z_{\rm inj\,ld}$ and has the velocity $V_{\rm gr}$. It is seen that since the electrons from the interval $(z_{0\rm ld}, z_{0\,\rm r})$ have a phase lag relative to the leading electron, they approach the self-injection point having the energy below the resonance one. In the accepted approximation, the theoretical analysis of the trajectories of the electrons passing the mixing region shows that at the moment of self-injection the energy of the electrons coincides with the resonance one. According to the results of exact numerical simulation, this energy is close to the resonance one, and correspondingly, the velocity of electrons is close to the velocity of movement of the self-injection point $z_{\rm si}$. As a result, the electrons are accumulated in the bunch of very small length, since the electron is self-injected into the wake wave at the spatial point and at the moment of time, when the bunch of previously injected electrons appears at the same point.

The electrons from the interval $(z_{0\rm l}, z_{0\rm ld})$ initially located to the left of the leading electron (see Fig. 2), near the accumulation point possess the velocity exceeding the group velocity of the laser pulse. Therefore, as shown by the numerical modelling, although these electrons lag behind the leading electron, they have the possibility to catch up with the electron bunch after some time. For approximate estimation of the process of incorporation of these electrons into the electron bunch generated by the laser pulse, one can assume that if the trajectory of an electron from the interval $(z_{0\rm l}, z_{0\rm ld})$ at some moment of time overtakes the running self-injection point $z_{\rm si}$ where the self-injection of electrons from the interval $(z_{0\rm ld}, z_{0\,\rm r})$ occurs, then the given electron is also integrated into the generated bunch.

For illustration Fig. 3 presents the numerically simulated distribution of electron macroparticles in the phase plane z, P at the moment of time $\omega_{\rm p}t = 11.063$, where the vertical dash-dot line marks the current position of the self-injection point of the electrons from the interval $(z_{0\rm ld}, z_{0\,\rm r})$, determined using Eqn (19). From Fig. 3 it is seen that the real self-injection point is sufficiently close to it. Below we assume that in the accepted approximation the electron bunch comprises the electrons that have the momentum $P > P_{\rm res}$ (shown in Fig. 3 by the horizontal dash-dot line) and are located to the right of the position of the running self-injection point $z_{\rm si}$ at the considered moment of time. We neglect the dimension of the electron bunch and assume that its position coincides with $z_{\rm si}$.

The comparison of Figs 2 and 3 shows that at the considered stage of the electron bunch generation process the velocity of electrons from the interval $(z_{0\rm l}, z_{0\rm ld})$ that pursue the bunch increases. This is because their motion is not affected

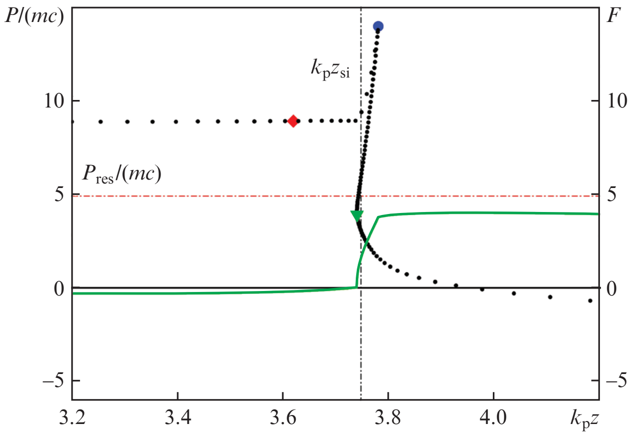

Figure 3. Distribution of electron macroparticles (points) in the phase plane z, P at the moment of time $\omega_p t = 11.063$. The solid curve presents the force $F = -|e|E_z/(mc\omega_p)$ acting on electrons in the wake wave (see text).

by the mixing of electrons in the generated bunch. After returning from the region of space $z < 0$, these electrons execute the same oscillatory motion as they did before crossing the ion background boundary after the laser pulse action. The only difference is that due to some time spent in the region $z < 0$ they acquire a phase shift with respect to the adjacent electrons. The situation shown in Fig. 3 corresponds to the moment of time when these electrons approach the centre of their oscillations. That is why their velocity is increased and, therefore, the probability to catch up with the bunch and to join it is increased, too.

From Fig. 3 it is seen that the accelerating force acting on the electrons of the bunch trailing part decreases with increasing its charge. This is due to the displacement of the bunch in the positive direction of the z axis, which leads to a decrease in the total charge of ions to the right of the coordinate z_{si}, which in the accepted approximation coincides with the current position of the bunch, while the total charge of electrons in the same region increases because of the supply of electrons from the interval (z_{01}, z_{01d}). As a result, at a certain moment of time the charge separation field in the trailing part of the bunch can change the sign, and at the self-injection point of electrons having the oscillation centres in the region $z_0 > z_{01d}$ the force acting on the electrons can become negative. Then the electrons from the region $z_0 > z_{01d}$ will be no more captured in the bunch and will not be able to join its composition, since they approach the self-injection point with the energy smaller than the resonance one. The distribution of electrons at the time moment when the capture of electrons from the region $z_0 > z_{01d}$ into the bunch is terminated ($\omega_p t_q = 12.37$) is shown in Fig. 4. For the last electron, initially located to the right of the leading electron and captured by the bunch, the centre of oscillations is z_{0r}.

Note that the real picture of the phase-space distribution of electron macroparticles captured in the bunch and their characteristics are somewhat different from the approximate ones, used in the theoretical analysis. Thus, from Fig. 4 it follows that in reality the position of point z_{si} of the electrons self-injection into the bunch slightly differs from the one predicted by Eqn (19), but is sufficiently close to it. Under a more careful consideration, the numerical simulation also shows that the capture of electrons in the generated bunch is not so unambiguously determined by the equality of the accelerating force to zero in the trailing part of the bunch, since the distribution of fields in this region of plasma and its further dynamics possess the character that is more complex. Besides that, the last electron from the region $z_0 > z_{01d}$ captured in the bunch and marked by the triangle, has the energy smaller than the resonance one. However, the used approximation facilitates the qualitative understanding of the process of electron capture from the region (z_{01d}, z_{0r}) into the generated bunch and, as will be seen below, allows the analytical derivation of the result, close to the results of numerical simulation.

Let z_q be the coordinate at which at a certain moment of time t_q the accelerating force at the trailing part of the pulse turns into zero. Neglecting the small thickness of the bunch, we assume that z_q also coincides with the position of the self-injection point z_{si} at the time moment t_q, when the capture of electrons with the oscillation centres at $z_0 > z_{01d}$ is finished.

The ensemble of bunch electrons at the time moment t_q incorporates all electrons from the plasma layer (z_{0q}, z_{0r}) according to their position in plasma before the action of the laser pulse, where z_{0q} is the oscillation centre of the electron from the interval (z_{01}, z_{01d}), which at the time moment t_q finds itself at the point z_q. From Fig. 4 it is seen that $z_{0q} > z_{01}$, since at the time moment t_q not all electrons from the region (z_{01}, z_{01d}) have reached the bunch. For the rightmost one of the electrons from the layer (z_{0q}, z_{0r}) that form the bunch captured by the wake wave to the time moment t_q, according to Eqns (8)–(12) the following relation can be written:

$$t_q = \frac{z_{0r}}{V_{gr}} + \frac{1}{c} I(z_q, z_{0r}) + T_{lft}(z_{0r}). \tag{20}$$

For the electron from the region (z_{01}, z_{01d}) with the oscillation centre at point z_{0q} that finds itself in the bunch at the time moment t_q it follows from Eqns (8)–(12) that

$$t_q = \frac{z_{0q}}{V_{gr}} + \frac{1}{c} I(z_q, z_{0q}) + T_{lft}(z_{0q}). \tag{21}$$

Relations (20), (21) are completed by the condition

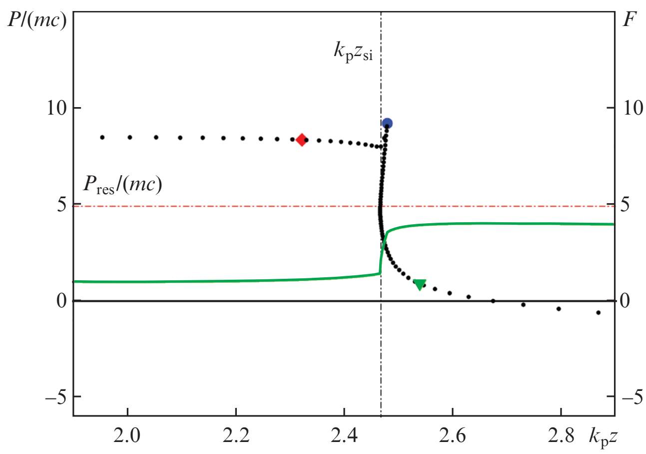

Figure 4. Distribution of electrons (points) in the phase plane z, P at the moment of time $\omega_p t_q = 12.37$, when the self-injection of electrons with the oscillation centres at $z_0 > A_m$ is terminated. The solid curve presents the force $F = -|e|E_z/(mc\omega_p)$ acting on electrons in the wake wave (see text).

$$z_{0\,\mathrm{r}} - z_{0\,\mathrm{q}} = \sqrt{(E_{\mathrm{os}} - \gamma_{\mathrm{ph}} mc^2)/(2\pi e^2 n_0)}\,, \tag{22}$$

which means that the field in the trailing part of the bunch at the time moment t_{q} is equal to zero.

Combining relations (20)–(22), taking into account that

$$z_{\mathrm{q}} = z_{0\,\mathrm{r}} - \sqrt{(E_{\mathrm{os}} - \gamma_{\mathrm{ph}} mc^2)/(2\pi e^2 n_0)}\,, \tag{23}$$

and introducing the notation for the half-period of oscillation of the electron that does not cross the ion background boundary

$$T_{\mathrm{h}} = \frac{1}{c}\int_{-A_{\mathrm{m}}}^{A_{\mathrm{m}}} \frac{\mathrm{d}z'}{\sqrt{1 - m^2c^4/[E_{\mathrm{os}} - 2\pi e^2 (z')^2 n_0]^2}}\,, \tag{24}$$

we arrive at the following equation for the determination of $z_{0\mathrm{q}}$:

$$T_{\mathrm{lft}}(z_{0\mathrm{q}}) = \frac{\sqrt{(E_{\mathrm{os}} - \gamma_{\mathrm{ph}} mc^2)/(2\pi e^2 n_0)}}{V_{\mathrm{gr}}} + \frac{1}{c} I(z_{\mathrm{ld\,inj}}, z_{0\mathrm{ld}}) + T_{\mathrm{h}}\,. \tag{25}$$

Note that from Eqns (22) and (23) it follows that $z_{\mathrm{q}} = z_{0\mathrm{q}}$. It means that to the time moment t_{q} the last electron from the region $(z_{01}, z_{0\mathrm{ld}})$ included in the bunch is the one, through the oscillation centre of which the running point $z_{\mathrm{si}}(t_{\mathrm{q}})$ of electron self-injection passes at this moment of time.

From Fig. 4 it follows that at $t > t_{\mathrm{q}}$ the process of electron accumulation in the generated bunch will continue at the expense of receiving electrons from the region $(z_{01}, z_{0\mathrm{q}})$, since their velocity at the considered moment of time exceeds the phase velocity of the wake wave, and they continue to approach the electron bunch. However, these electrons have already passed through their oscillation centre, and their velocity begins to decrease. The last electron captured by the bunch will be the one for which the decrease in velocity to the phase velocity of the wake wave will occur at the same time moment t_{ex} and the point z_{ex} of the z axis, where at this time moment the self-injection point z_{si} will find itself. In Fig. 5 this situation is illustrated at $\omega_{\mathrm{p}} t_{\mathrm{ex}} = 15.085$. It is seen that the above statement is approximate, but, as will be shown below, the approximation is good enough to get sufficiently precise numerical estimates.

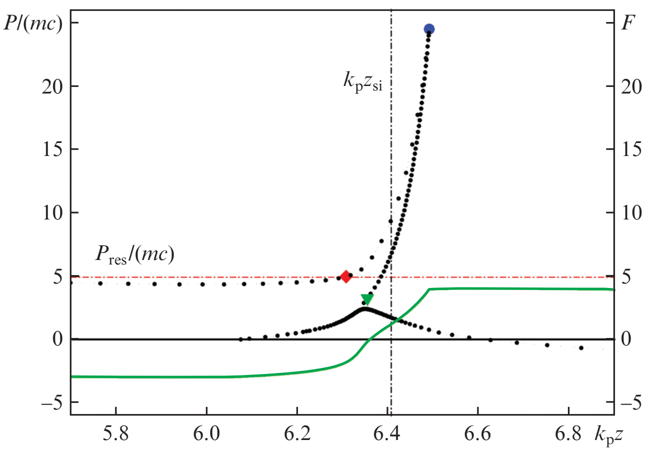

Figure 5. Distribution of electrons (points) in the phase plane z, P at the moment of time $\omega_{\mathrm{p}} t_{\mathrm{ex}} = 15.085$, when the self-injection of electrons with the oscillation centres at $z_0 < A_{\mathrm{m}}$ is terminated. The solid curve presents the force $F = -|e|E_z/(mc\omega_{\mathrm{p}})$ acting on electrons in the wake wave.

From the solution of electron oscillation equations it follows that the leftmost electron with the centre of oscillations at z_{01} at the moment of self-injection will have the velocity equal to that of the wake wave, if it is located at the point separated from the oscillation centre by the interval

$$z_{\mathrm{ex}} - z_{01} = \sqrt{(E_{\mathrm{os}} - \gamma_{\mathrm{ph}} mc^2)/(2\pi e^2 n_0)}\,. \tag{26}$$

On the other hand, from Eqn (8) for the electron trajectory we have

$$t_{\mathrm{ex}} = \frac{z_{01}}{V_{\mathrm{gr}}} + \frac{1}{c} I(z_{\mathrm{ex}}, z_{01}) + T_{\mathrm{lft}}(z_{01})\,. \tag{27}$$

Completing Eqns (26) and (27) with the relation obtained from Eqn (19),

$$z_{\mathrm{ex}} = z_{\mathrm{inj\,ld}} + V_{\mathrm{gr}}(t_{\mathrm{ex}} - t_{\mathrm{inj\,ld}})\,, \tag{28}$$

we arrive at the equation for z_{01}

$$T_{\mathrm{lft}}(z_{01}) = \frac{2\sqrt{(E_{\mathrm{os}} - \gamma_{\mathrm{ph}} mc^2)/(2\pi e^2 n_0)}}{V_{\mathrm{gr}}} + \frac{2}{c} I(z_{\mathrm{injld}}, z_{0\mathrm{ld}}) + T_{\mathrm{h}}\,. \tag{29}$$

Solving Eqns (25) and (29) under the conditions $A_{\mathrm{m}} - z_{01} \ll A_{\mathrm{m}}$ and $A_{\mathrm{m}} - z_{0\mathrm{q}} \ll A_{\mathrm{m}}$ one can find the additional term $z_{0\mathrm{q}} - z_{01}$, which together with Eqn (22) determines the thickness of the layer of unperturbed plasma electrons that enter the composition of the bunch. Finally, we find that to the time moment of separation of the electron bunch from the electron background the bunch accumulates the electrons that occupy in the unperturbed plasma the layer as thick as

$$k_{\mathrm{p}}\Delta z_{\mathrm{tr}} = k_{\mathrm{p}}(z_{0\,\mathrm{r}} - z_{01}) = \sqrt{2[E_{\mathrm{os}}/(mc^2) - \gamma_{\mathrm{ph}}]} + \left(\frac{3}{4\sqrt{2}}\right)^{2/3} (2^{\,2/3} - 1)\sqrt{2[E_{\mathrm{os}}/(mc^2) - 1]} \times [2(E_{\mathrm{os}}/(mc^2) - \gamma_{\mathrm{ph}})]^{1/3} \times \left[\frac{1}{\sqrt{1 - 1/\gamma_{\mathrm{ph}}^2}} - \frac{1}{\sqrt{1 - m^2c^4/E_{\mathrm{os}}^2}}\right]^{2/3}. \tag{30}$$

Consequently, the value of the charge per unit cross section area of the bunch generated by the laser pulse passing through the sharp plasma boundary is determined by the expression $\sigma = -|e|n_0\Delta z_{\mathrm{tr}}$.

Figure 6 presents the calculated dependences of the thickness of the layer of laser-unperturbed plasma electrons, captured in the generated bunch, on the energy of oscillations E_{os} for the plasma with different values of γ_{ph}. For comparison Fig. 6 also presents the thickness variations of the layer of electrons, captured in the bunch, calculated using the formula of Ref. [12], where the contribution of electrons crossing the boundary of the ion background to the bunch charge is not taken into account. From Fig. 6 it follows that the approach

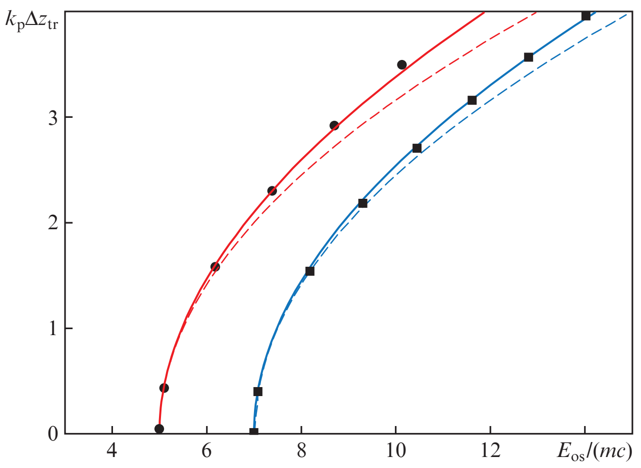

Figure 6. Thickness of the layer of captured background electrons vs. the energy of plasma oscillators for the threshold energy $E_{os\,th} = 5mc^2$ (circles) and $7mc^2$ (squares). The solid curves correspond to the calculations using Eqn (30), the dashed ones present the calculations using the formula from Ref. [12], derived under the condition $E - E_{os\,th} \ll E_{os\,th}$.

to the analysis of the process of electron bunch generation by the laser pulse, presented in this paper, reflects the real physical picture more precisely and reveals the mechanism of electron bunch generation by the ultrarelativistic-intensity laser pulse passing through the sharp boundary of plasma in more detail.

Nevertheless, it is worth noting that Eqn (30) does not cardinally change the general character of the dependence of the layer thickness on the energy of plasma oscillators. As a result, for the laser pulse parameter values chosen above, for which the corresponding electron oscillation energy is $E_{os} = 9mc^2$, the error of the estimate obtained using the simpler formula from Ref. [12] does not exceed 10%. This fact means that in the description of electron self-injection into the wake wave of the laser pulse for the large energy excess over the threshold ($E_{os} - E_{os\,th} \approx E_{os\,th}$) one can use the formulae of Refs [12, 13] to estimate the rest characteristics of the electron bunch, namely, the length and the energy difference between the electrons.

To finalise, we also note that Eqn (30) allows determination of the applicability limits for the assumption of similarity of the electron trajectories under the conditions, when the laser pulse evolves slowly in the process of propagation through plasma. The utmost electrons of the segment Δz_{tr}, as well as all electrons between them, will move along the approximately similar trajectories in the process of interaction with the evolving laser pulse, if during the time $\Delta z_{tr}/V_{gr}$ the characteristics of the laser pulse do not change essentially. The character of motion of other electrons that are not included in the bunch does not matter, since at the stage of interaction with the laser pulse the mixing of electrons is absent. On the other hand, according to Ref. [15], the characteristics of the laser pulse having relativistic amplitude (the characteristic longitudinal size and the steepness of the pulse profile) in the course of propagation through the plasma change at the length estimated by the order of magnitude as $L_{ev} \sim k_p^{-1}\gamma_{ph}^2$. Thus, under the condition $k_p\Delta z_{tr} \ll \gamma_{ph}^2$ the evolution of the laser pulse having the group velocity close to the velocity of light is sufficiently slow, so that the theoretical approach used in the present paper is applicable to the analysis of the process of generation of electron bunches by the laser pulse.

5. Conclusions

The performed study of generation of short electron bunches by the laser pulse of ultrarelativistic intensity ($|e|A_0/mc^2 \sim 10$) passing through the sharp plasma boundary allows detailed determination of the mechanism underlying this physical phenomenon.

It is revealed that the total charge of the electron bunch is formed of two different groups of electrons. One group consists of electrons that do not cross the boundary of ion background in the process of their motion. The electrons of the other group cross the plasma ion background boundary and return. It is found that independent of the degree of the total electron oscillation energy excess over the threshold value, the bunch formation always begins from the electron initially located at the distance from the ion background boundary equal to the amplitude of its oscillations, caused by the subsequent interaction with the laser pulse.

The main contribution to the bunch charge, particularly in the case of small excess over the threshold, comes from the electrons that do not cross the boundary of the ion background in the process of their motion. The mechanism of their accumulation in the bunch generated by the laser pulse implies that the self-injection of every electron from this group into the wake wave occurs at the time moment and space point, where the earlier injected electrons composing the bunch already are.

Because of interaction with the laser pulse, the electrons that move beyond the ion background make the most considerable contribution when the process of electron bunch generation is characterised by large excess over the threshold. The mechanism of their accumulation into the bunch is such that after the return into the region of ion background these electrons move with greater velocities and overtake the bunch.

We derived a formula that determines the bunch charge per unit of its cross section area. It is determined by the thickness of the unperturbed plasma layer, the electrons from which form the generated electron bunch. It is established that the charge accumulated in the bunch is completely determined by two parameters: the energy of the longitudinal oscillations of electrons and the gamma-factor of the laser pulse wake wave.

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