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New modulation method in acousto-optic spectrometers

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Abstract. **A new method is proposed for modulating radiation in collinear acousto-optic (AO) spectrometers, which is based on the sequential diffraction of incident polarised optical radiation using two sound packets with the same frequencies and initial phases. A theory of diffraction of optical radiation on two successive packets of sound waves propagating in a crystal is constructed, and the instrumental function of the AO spectrometer is found. It is established that the instrumental function of such a spectrometer becomes time-dependent, and the modulation frequency of this dependence, in turn, depends on the intensity of the sound wave and on the detuning from the phase-matching conditions. The frequency dependence of the measured photocurrent for some finite number of sound packets is obtained explicitly, and it is shown that its measurement allows the spectral composition of the incident optical radiation to be determined with greater accuracy.**

Keywords: *acousto-optics, diffraction of light on sound, methods of diffracted optical radiation modulation.*

1. Introduction

Modern acousto-optic (AO) methods for measuring the spectrum of optical radiation are based on diffraction of incident light (often polarised) on an acoustic wave propagating in a crystal, with the amplitude and phase of this wave having a nonuniform spatial and/or temporal distribution [1–8]. Taking into account the fact that the characteristic time of a sound wave travelling through a crystal in an AO cell is usually a few microseconds, and also that existing electronic devices are able to quickly change the amplitude and phase of a high-frequency electrical signal exciting a sound wave, it is possible to state that AO spectrometers can be used to develop new methods for measuring the spectrum of optical radiation, inaccessible to conventional diffraction grating spectrometers.

The spectral distribution of the optical radiation characteristics using AO spectrometers, in contrast to diffraction grating devices, is measured in series rather than in parallel, which requires more time to perform measurements. However, this disadvantage is compensated for by a much larger luminosity, since the entrance pupil of a collinear AO spectrometer is determined by the cross section of the sound beam rather than by the width of the entrance slit, as in a grating spectrometer, and, moreover, AO spectrometers also allow for electronic (software) control of the shape and bandwidth, and provide a free choice of the measured wavelength. These properties of AO spectrometers allow one to develop various spectral measurement methods characterised by a large signal-to-noise ratio and by optimal characteristics for a specific measurement task. Many of the above capabilities of AO spectrometers have been implemented as measurement control programmes, in particular, in choosing the radiation wavelength of a tunable dye-based laser with an AO filter inside the resonator [9], in a special 'Kvarts-4T' system for monitoring the chemical composition of plasma in the process plasma chemical etching of silicon wafers during the manufacture of microelectronic products [10, 11].

Presently, AO spectrometers use various techniques for modulating a sound wave: amplitude [12], 'sharp' phase [13], linear frequency [14, 15] and two-frequency [16] methods. These methods solve various problems of spectral measurements: increasing the signal-to-noise ratio; isolating a weak signal against the background of strong illumination; changing the instrumental function (IF) of an AO spectrometer, such as expanding the spectral transmission window of an AO spectrometer or producing an ultra-narrowband optical filter [17]; temporal compressing (or stretching) of a light pulse with a linear frequency modulation [18, 19]; etc. All these modulation methods lead to a change in the photocurrent during the registration of the diffracted optical radiation and have both advantages and disadvantages.

The method of amplitude modulation is the most common technique in which two states are produced in a medium: the first one arises in the presence of a sound wave when the photocurrent from the diffracted radiation and the parasitic signal are measured, and the second one arises in the absence of a sound wave when only the photocurrent of the parasitic light is measured. Then, to eliminate the influence of parasitic illumination, one value of the photocurrent is subtracted from the other. This method can be used only in conditions when the level of illumination by parasitic radiation does not change during the measurement; otherwise, this technique is not applicable.

The use of the modulation method based on a 'sharp' change in the phase of a sound wave during its propagation through a crystal, proposed for the first time in [13], leads not only to a change in the effective interaction length, but also to a change in the amplitude of the diffracted radiation, in particular to its time dependence, which ultimately results in a

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controlled change in the IF of the AO spectrometer itself. Specialised spectrometers fabricated with the help of this modulation method turn out to be very convenient for measuring weak signals at high illumination.

The diffraction of a linear frequency modulated light pulse on a linear frequency modulated sound wave, which was proposed in [18] and allows this pulse to be compressed in time, is widely used today in various applications [19].

In this paper, we consider another, apparently not studied by anyone, modulation method based on the phenomenon of sequential collinear diffraction of the light on two (or a series of) sound packets with the same frequency, separated by a small interval of space in which there is no sound wave. The polarised light successively passes through the first section, where the incident radiation is collinearly diffracted on a sound packet and a new orthogonally polarised light wave appears, and then two light waves with different and mutually orthogonal polarisations pass through the gap in which there is no sound wave and fall into the second region where collinear diffraction is also performed. At the output of the AO cell there is a polariser, which emits the diffracted part of the radiation. It is significant that in the gap the light waves acquire some phase shift, which, due to the optical anisotropy of the medium, is different for the waves with different polarisations, and therefore it is clear that the subsequent diffraction on the second sound packet for both waves will depend on the phase shift in the gap. From a mathematical point of view, this situation corresponds to different initial conditions for the equations describing the diffraction process. Since this intermediate region moves across the crystal at the speed of sound, the amplitudes of the diffracted and nondiffracted light waves will depend on time. The appearance of this time dependence means the emergence of the corresponding time dependence of the IF of the AO filter and in fact represents another type of modulation of the diffracted part of the radiation. The analysis performed in the work shows that the frequency of this modulation depends on the phase of the phase-matching conditions and that for each value of this the modulation frequency is different, which opens up new possibilities for more accurate spectral measurements.

2. Collinear diffraction on sound wave packets

A schematic of a collinear AO filter with the above modulation type is shown in Fig. 1. The polarised light beam is incident on the AO cell on the left and diffracts on the sound wave (the first sound packet), resulting in a new orthogonally polarised light wave. Then both (incident and diffracted) waves fall into the region of the crystal, where the sound wave is absent, and acquire a phase shift. After that, these waves, falling into the crystal region with a sound wave having the same frequency (the second sound packet), again diffract and then leave the crystal. Since the frequencies of the sound waves in the two packets are the same, the phase-matching conditions in both parts of the crystal will also be the same, and therefore when the condition

$$\boldsymbol{k}_{\mathrm{in}} - \boldsymbol{k}_{\mathrm{d}} - \boldsymbol{q}_{\mathrm{s}} = \Delta\boldsymbol{k} \tag{1}$$

is met, collinear diffraction is possible. Here, $\boldsymbol{k}_{\mathrm{in}} = \omega n_{\mathrm{o}}/c$ is the wavenumber of the incident light wave; $\boldsymbol{k}_{\mathrm{d}} = \omega n_{\mathrm{e}}/c$ is the wavenumber of the diffracted light wave; ω is the frequency of the light wave; c is the speed of light in vacuum; n_{o} and n_{e} are the refractive indices for the incident ordinary and diffracted extraordinary waves; and $\boldsymbol{q}_{\mathrm{s}}$ is the wavenumber of the sound wave. The phase-matching condition (1) is written for a uniaxial negative crystal, when $n_{\mathrm{o}} > n_{\mathrm{e}}$ and the wave vector $\boldsymbol{q}_{\mathrm{s}}$ is directed in the same direction as the vectors $\boldsymbol{k}_{\mathrm{in}}$ and $\boldsymbol{k}_{\mathrm{d}}$; if diffraction in a uniaxial positive crystal ($n_{\mathrm{o}} < n_{\mathrm{e}}$) is considered, then the wave vector of the sound wave, $\boldsymbol{q}_{\mathrm{s}}$, is directed along the $\boldsymbol{k}_{\mathrm{in}}$ direction and in relation (1) the sign in front of $\boldsymbol{q}_{\mathrm{s}}$ should be replaced with the opposite one. The amplitude of the diffracted wave in the case under consideration will be maximal if the value of the wave Δk is close to zero.

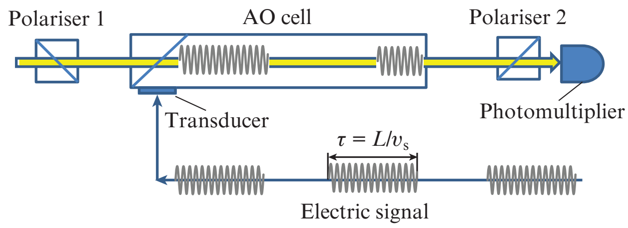

Figure 1. Schematic of a collinear AO filter with the modulation type in question. The duration of the sound packet $\tau = L/v_{\mathrm{s}}$ and the time interval d/v_{s} between the packets satisfy the relation $\tau > d/v_{\mathrm{s}}$.

The presence of a sound wave in two regions of the crystal leads to the exchange of energy between the light waves, the polarisation directions of which are mutually orthogonal. The truncated equations describing this process in the approximation of the theory of coupled modes propagating along the x axis for each region have the form [20]

$$\begin{aligned}\frac{\mathrm{d}E_{\mathrm{e}}(x)}{\mathrm{d}x} &= -\,\mathrm{i}\Gamma\exp(\mathrm{i}\Delta kx)E_{\mathrm{o}}(x),\\ \frac{\mathrm{d}E_{\mathrm{o}}(x)}{\mathrm{d}x} &= -\,\mathrm{i}\Gamma^{*}\exp(-\mathrm{i}\Delta kx)E_{\mathrm{e}}(x),\end{aligned} \tag{2}$$

where $E_{\mathrm{o}}(x)$ and $E_{\mathrm{e}}(x)$ are the amplitudes of ordinary and extraordinary waves;

$$\Gamma = \frac{\omega n_{\mathrm{o}} n_{\mathrm{e}}^{2}}{4c}pS(x), \quad \Gamma^{*} = \frac{\omega n_{\mathrm{o}}^{2} n_{\mathrm{e}}}{4c}pS^{*}(x) \tag{3}$$

are the coupling coefficients, which are expressed in terms of the characteristics of the crystal and the sound wave amplitude $S(x)$; and p is the photoelastic constant corresponding to the interaction geometry in question. The coefficients Γ and Γ^{*} differ somewhat from each other due to the difference in the refractive indices n_{o} and n_{e}, but the final formulae, as we will see below, always include the product $\Gamma\Gamma^{*}$, the root of which will be denoted simply $\Gamma \equiv \sqrt{\Gamma^{*}\Gamma}$, hoping that it will not cause misunderstandings.

Equations (2) strictly describe only the collinear type of diffraction, their use for describing the quasi-collinear case is possible only if the angle between the wave vectors of the incident and diffracted radiation is small and, moreover, with a special choice of the coordinate system such that the dependence on the transverse components in equations (2) has disappeared [20]. Below, we assume that these requirements are satisfied, and then system (2) can also be applied to describe the case of quasi-collinear diffraction. Equations (2) determine the interactions between the light waves in each of the regions of the crystal, where moving packets of sound waves are present, but the boundary conditions for each of the regions will be different:

$$E_o(x=0)=E_o(k_o),\quad E_e(x=0)=0$$

and

$$E_o(tv_s,k_o)=\exp(i\phi_o)E_o^{(2)}(tv_s,k_o), \tag{4}$$

$$E_e(tv_s,k_e)=\exp(i\phi_e)E_e^{(2)}(tv_s,k_e)$$

for the left boundaries of the first and second regions, respectively. It is assumed in (4) that radiation with ordinary polarisation is incident on an AO cell, and the diffracted light has an extraordinary polarisation. Here, t is the time counted from the moment the sound wave packet enters the AO cell; v_s is the speed of sound; k_o and k_e are the wavenumbers of ordinary and extraordinary light waves; $E_o^{(2)}(x)$ and $E_e^{(2)}(x)$ are the amplitudes of the ordinary and extraordinary light waves in the second region; $\phi_o=k_od$ and $\phi_e=k_ed$ are the phase shifts; and d is the distance between the sound wave packets. It is also assumed that the initial phase of the sound wave in the second region coincides with the initial phase of the sound wave in the first region. If this condition is not met, then the initial phase of the sound wave in the second region should be added to the above phase shifts (for more details, see [13]).

The general solution of equations (2) is well known; therefore, the procedure for obtaining a solution for the entire wave interaction region is as follows. For each of these regions, this solution must be ‘sewed’ on the boundary moving with the speed of sound, satisfying boundary conditions (4). As a result, the solution for the diffracted light wave, describing sequential diffraction at the exit of the crystal, will have the form

$$\begin{aligned}E_e(L)&=\frac{E_o}{4\Gamma^2+\Delta k^2}\exp\left[\frac{i(L\Delta k-\Delta\phi)}{2}\right]\Big\{2\sqrt{4\Gamma^2+\Delta k^2}\\&\times\sin(\Delta\phi)\sin\left[\frac{1}{2}(L-2tv_s)\sqrt{4\Gamma^2+\Delta k^2}\right]-i\Delta k(4\Gamma^2+\Delta k^2)\\&\times\sin(\Gamma L\xi)-2i\cos\left(\frac{\Delta\phi}{2}\right)\sin\left(\frac{1}{2}L\sqrt{4\Gamma^2+\Delta k^2}\right)-2i\Delta k\sin\left(\frac{\Delta\phi}{2}\right)\\&\times\left[\sin\left(tv_s\sqrt{4\Gamma^2+\Delta k^2}\right)-\cos\left(\frac{1}{2}L\sqrt{4\Gamma^2+\Delta k^2}\right)\right]\Big\},\end{aligned} \tag{5}$$

where $\Delta\phi=\phi_o-\phi_e$ is the phase shift difference of optical radiation incident between the packets of the sound waves; L is the interaction length; and $\xi\equiv\sqrt{1+\Delta k^2/(4\Gamma^2)}$. It is important to note that the obtained solution of the system of equations (2) with boundary conditions (4) belongs to the time interval $0\leqslant t\leqslant L/v_s$, and it is assumed that the condition $L\gg d$ is satisfied with a large margin. The nondiffracted portion of the radiation can be described as

$$\begin{aligned}E_o(L)&=\frac{E_o}{\xi^6}\exp\left(\frac{iL\Delta k}{2}\right)\{\exp(i\phi_o)\{2\Gamma\xi(2\Gamma^2+\Delta k^2)\cos(\Gamma L\xi)\\&+4\Gamma^3\xi\cos[(L-2tv_s)\Gamma\xi]-i\Delta k(4\Gamma^2+\Delta k^2)\sin(\Gamma L\xi)\}\\&+8\exp(i\phi_e)\Gamma^3\xi\sin(tv_s\Gamma\xi)\sin[(tv_s-L)\Gamma\xi]\}.\end{aligned} \tag{6}$$

If the values of the phases coincide, then the well-known expression for the amplitude of the diffracted radiation, given in a number of papers [1–4], follows from formula (5):

$$E_e(L)=\frac{\sin(\Gamma L\xi)}{\xi}\exp\left(i\phi_e-\frac{i\Delta kL}{2}\right). \tag{7}$$

The phase factor $\exp(i\phi_e-i\Delta kL/2)$ in formula (7) describes the phase change during radiation diffraction, but does not affect the IF of the AO filter.

It can be seen from formulae (5) and (6) that the amplitudes of the extraordinary (5) and ordinary (6) waves at the exit from the AO cell depend significantly on time, with the characteristic frequency of this dependence having the from $\Omega(\Delta k,\Gamma)=\Omega\equiv2\Gamma\xi v_s$. It is important that this frequency depends on the wave and the sound wave power. With a constant sound power, this frequency is determined by the value of the detuning. This means that measuring the photocurrent of a receiver at frequency Ω gives the amplitude of the radiation field at a given wavelength, since the values of the frequency $\Omega(\Delta k,\Gamma)$ for different wavelengths of optical radiation turn out to be different.

From formula (5), one can obtain the relation for the IF of the AO filter, expressed, for the sake of convenience, through trigonometric functions:

$$\begin{aligned}T(\Delta k,t)&\equiv\left|\frac{E_e(\Delta k,t,x=L)}{E_o(x=0)}\right|^2=\frac{1}{16\Gamma^3\xi^5}\Big\{2(2\Gamma\xi)^3\cos^2\left(\frac{\Delta\phi}{2}\right)\\&\times\sin^2\Phi+4\Gamma\xi\Delta k^2\sin^2\left(\frac{\Delta\phi}{2}\right)\cos^2\Phi+4\Gamma\xi\Delta k^2\sin^2\left(\frac{\Delta\phi}{2}\right)\\&\times\sin^2(\Phi-\Omega t)+4\Gamma^2\Delta k\xi^2\sin(\Delta\phi)\\&\times[\sin(\Omega t)-\sin(2\Phi)+\sin(2\Phi-\Omega t)]+16\Gamma^3\xi\sin^2(\Phi-\Omega t)\\&+4\Gamma\xi\Delta k^2\sin^2\left(\frac{\Delta\phi}{2}\right)\cos^2(\Phi-\Omega t)-8\Gamma\xi\Delta k^2\sin^2\left(\frac{\Delta\phi}{2}\right)\\&\times\cos\Phi\cos(\Phi-\Omega t)\Big\},\end{aligned} \tag{8}$$

where $\Phi\equiv\Gamma L\xi$ is the phase. In the general case, the phase difference $\Delta\phi$, which needs to be substituted into formula (8), has the form

$$\begin{aligned}\Delta\phi&=\phi_o-\phi_e=(k_o-k_e)d=q_sd+\Delta kd\\&=\Omega_sd/v_s+\Delta n(\omega/c)d,\end{aligned} \tag{9}$$

where Ω_s is the frequency of the sound wave and $\Delta n=n_o-n_e$. The dependences of the IF (8) at different points in time are shown in Fig. 2. When the phase difference is equal to zero, which is equivalent to the absence of a gap between the sound wave packets, i.e., continuous emission of sound waves, formula (8) gives a known result for the IF of the collinear AO filter [20]:

$$T(\Delta k,L)=\frac{\sin^2(\Gamma L\xi)}{\xi^2}. \tag{10}$$

The temporary structure of the IF, described by formula (8), can also be represented as

$$T(\Delta k,t)=T_0+C_1\cos(\Omega t)+C_2\cos(2\Omega t)+$$

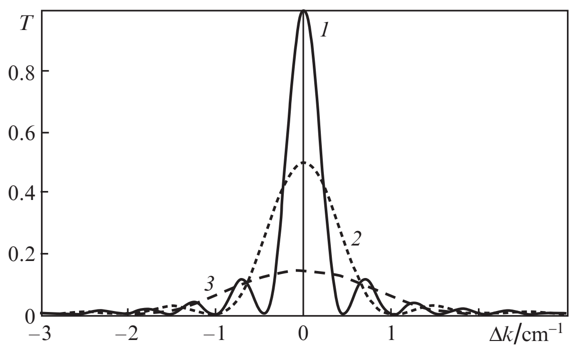

Figure 2. IF (8) at times $t = (1)\ L/v_s$s, (2) $0.5L/v_s$ and (3) $0.25L/v_s$.

$$+ S_1\sin(\Omega t) + S_2\sin(2\Omega t), \qquad (11)$$

where

$$T_0 \equiv T_0(\Delta k, \Delta\phi, \Gamma, \xi) = \frac{1}{\xi^2}\cos^2\left(\frac{\Delta\phi}{2}\right)\sin^2(\Gamma L\xi) + \frac{\sin^2(\Delta\phi/2)}{2\xi^4}$$

$$\times[3\xi^2 - 2 + (\xi^2 - 1)\cos(\Gamma L\xi)] - \frac{\Delta k}{4\Gamma\xi^3}\sin(\Delta\phi)\sin(2\Gamma L\xi); \qquad (12)$$

$$C_1 \equiv C_1(\Delta k, \Delta\phi, \Gamma, \xi) = \frac{\Delta k}{4\Gamma\xi^3}\sin(\Delta\phi)\sin(2\Gamma L\xi)$$

$$-\frac{\Delta k^2}{2\Gamma^2\xi^4}\sin^2\left(\frac{\Delta\phi}{2}\right)\cos^2(\Gamma L\xi); \qquad (13)$$

$$C_2 \equiv C_2(\Delta k, \Delta\phi, \Gamma, \xi) = -\frac{1}{2\xi^4}\sin^2\left(\frac{\Delta\phi}{2}\right)\cos^2(2\Gamma L\xi); \qquad (14)$$

$$S_1 \equiv S_1(\Delta k, \Delta\phi, \Gamma, \xi) = \frac{\Delta k}{2\Gamma\xi^3}\sin(\Delta\phi)\sin^2(\Gamma L\xi)$$

$$-\frac{\Delta k^2}{4\Gamma^2\xi^4}\sin^2\left(\frac{\Delta\phi}{2}\right)\sin(2\Gamma L\xi);\ \text{and} \qquad (15)$$

$$S_2 \equiv S_2(\Delta k, \Delta\phi, \Gamma, \xi) = -\frac{1}{2\xi^4}\sin^2\left(\frac{\Delta\phi}{2}\right)\sin(2\Gamma L\xi). \qquad (16)$$

The dependences of the coefficients (12)–(16) on the detuning Δk are shown in Figs 3–6, with the values of the parameters for which they are obtained indicated in the figure captions. Each figure also shows the IF (10), which allows us to compare the values of the coefficients that determine the temporal behaviour of the IF (11) with its known value. Since the values of the coefficients (13)–(16) decrease rapidly with increasing Δk, all dependences are represented near the region in which the phase-matching conditions are satisfied, as a rule, near $\Delta k \approx 0$.

Formulae (12)–(16) completely determine the time dependence of the IF; in this case, $T_0(\Delta k, \Delta\phi, \Gamma, \xi)$ describes a constant, time-independent, part of the IF, $C_1(\Delta k, \Delta\phi, \Gamma, \xi)$ and $S_1(\Delta k, \Delta\phi, \Gamma, \xi)$ determine the temporal 'amplitude' of IF changes at frequency Ω, and $C_2(\Delta k, \Delta\phi, \Gamma, \xi)$ and $S_2(\Delta k, \Delta\phi, \Gamma, \xi)$ – at doubled frequency, i.e. 2Ω. One can see that when the phase-matching conditions are satisfied, the coefficients C_1 and S_1 vanish and the dependence of the IF on frequency Ω disappears; in this case, only the dependence on the doubled frequency 2Ω persists.

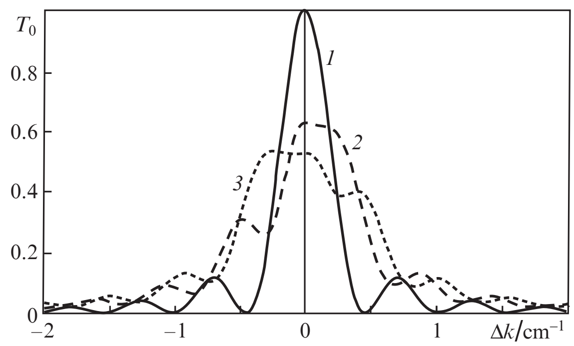

Figure 3. Averaged values of $T_0(\Delta k)$ for the intervals $d = (1)\ m\Lambda_s$ (m is an integer), (2) $(39/7)\Lambda_s$ and (3) $(39/3)\Lambda_s$ with the parameters L = 12 cm, $\Gamma = \pi/(2L)$ and the sound wavelength $\Lambda_s = 2 \times 10^{-3}$ cm.

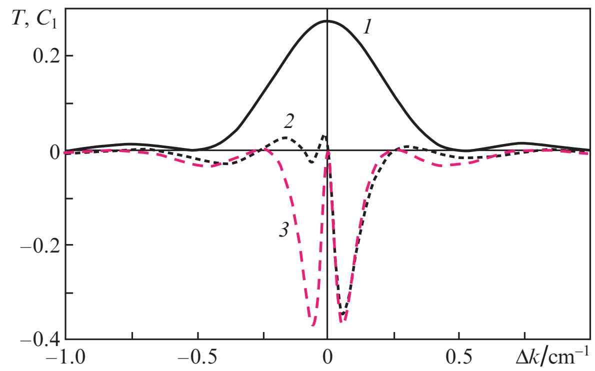

Figure 4. Dependences of (1) the IF (10) and (2, 3) the coefficient C_1 on the detuning Δk for the intervals d = (2) $(11/2)\Lambda_s$ and (3) $(11/4)\Lambda_s$ with the parameters L = 12 cm, $\Gamma = 0.35\pi/(2L)$ (sound wave power equal to 35% of the optimal value), and $\Lambda_s = 1.5 \times 10^{-3}$ cm.

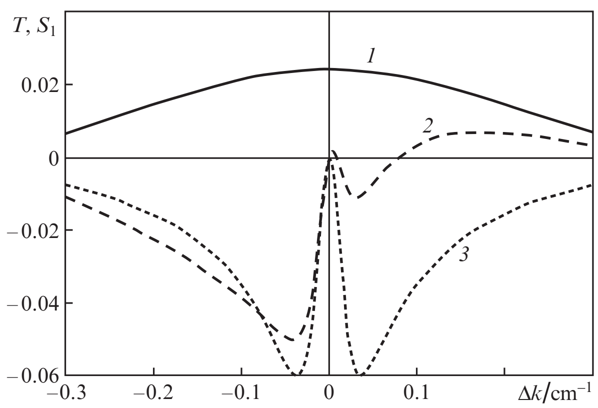

Figure 5. Dependences of (1) the IF (10) and (2, 3) the coefficient S_1 on the detuning Δk for the intervals d = (2) $(17/2)\Lambda_s$ and (3) $(17/4)\Lambda_s$ with the parameters L = 12 cm, $\Gamma = 0.1\pi/(2L)$ (sound wave power equal to 10% of the optimal value), and $\Lambda_s = 1.5 \times 10^{-3}$ cm.

3. Limiting IF values

Consider some features of the behaviour of the IF defined by the general expression (8).

1. It follows from formulae (8) and (9) that when the exact phase-matching condition is met, i.e., when $\Delta k = 0$ and $\xi = 1$, for the IF (8) we obtain the expression

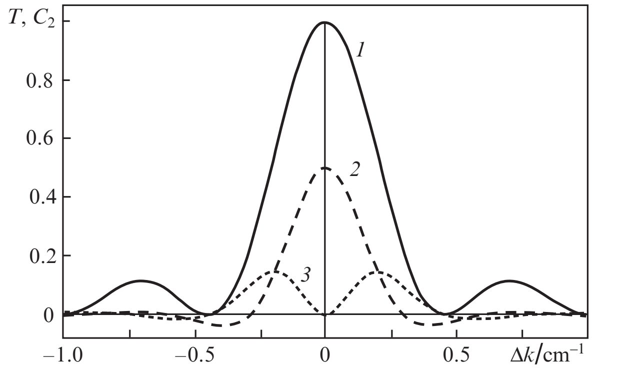

Figure 6. Dependences of (*1*) the IF (10) and the coefficients (*2*) C_2 and (*3*) S_2 on the detuning Δk with the parameters $L = 12$ cm, $\Gamma = \pi/(2L)$, and $\Lambda_s = 1.5\times10^{-3}$ cm.

$$T|_{\Delta k=0,\,\xi=1} = \cos^2\left(\frac{\pi d}{\Lambda_s}\right)\sin^2(\Gamma L) + \sin^2\left(\frac{\pi d}{\Lambda_s}\right)\sin^2(\Gamma L - \Omega t), \qquad (17)$$

where Λ_s is the sound wavelength. It can be seen that if the distance d between the packets is equal to an integer number of the waves of the sound, the time dependence of the IF disappears. At $\Gamma L = \pi/2$, the value of the IF is equal to unity, as in the case of diffraction without phase modulation. If the condition $d = m\Lambda_s$ ($m = 1, 2, 3, \ldots$) is not satisfied, then there appears a time dependence of the IF at frequency Ω at any value ΓL. Examples of this IF behaviour are shown in Fig. 3.

2. It follows from formulae (8) and (11) that the time-averaged IF will have the form

$$\langle T\rangle \equiv \lim_{\theta\to\infty}\left[\frac{1}{\theta}\int_0^{\theta} T(\Delta k, t)\,\mathrm{d}t\right] = T_0(\Delta k, \Delta\phi, \Gamma, \xi). \qquad (18)$$

In the absence of a gap between the packets, that is, when $d = 0$, the averaged IF value $\langle T\rangle$, as it should be, coincides with the well-known expression for IF (10) in the collinear case [20]; if $d \neq 0$, then the stationary value of the IF (18) changes significantly; in particular, the symmetry of the IF with respect to the replacement $\Delta k \to -\Delta k$ disappears. The latter is due to the fact that diffraction of an ordinary optical wave in a crystal on a sound wave and the appearance of an extraordinary wave, when the initial phase is dependent on Δk, is not equivalent to the inverse process: diffraction of an extraordinary wave with the appearance of an ordinary wave. This asymmetry is due to the dependence of the phase shift on the value of the detuning. When the condition for the phase factor

$$\left(\Delta k + \frac{2\pi}{\Lambda_s}\right)d = 2\pi m, \quad m = 1, 2, 3, \ldots \qquad (19)$$

is met, expression (18) coincides with expression (10) for a collinear case in which the value of the detuning Δk is determined by condition (19). If

$$\left(\Delta k + \frac{2\pi}{\Lambda_s}\right)d = \frac{\pi}{2}(2m-1), \quad m = 1, 2, 3, \ldots. \qquad (20)$$

Then from expression (11) we obtain the relation

$$\langle T\rangle = \frac{1}{2\xi^4}\left[1 + 3\left(\frac{\Delta k}{2\Gamma}\right)^2 + \left(\frac{\Delta k}{2\Gamma}\right)^2\cos(2\Gamma L\xi)\right], \qquad (21)$$

and if the phase-matching condition is satisfied, we have $\langle T\rangle = 1/2$. Thus, it follows from (19) and (20) that, by changing the distance d between the packets, it is possible to control the IF.

3. The condition $\Delta\phi = 0$, as is easy to see, is equivalent to the absence of a gap between the sound wave packets, that is, to the continuous emission of these waves, and therefore result (10) follows from (8). If the phase difference is not zero, and the time interval between the packets is greater than the transit time L/v_s, i.e., diffraction occurs only on one packet, then for the IF of the AO filter we obtain from (8) the expression

$$T(\Delta k, t) = \frac{\sin^2(t v_s \Gamma\xi)}{\xi^2} = \frac{1 - \cos[\Omega(\Delta k)t/2]}{2\xi^2}. \qquad (22)$$

Thus, the IF also becomes a periodic function of time. From a physical point of view, result (22) is quite understandable: If the duration of a sound wave packet coincides with the transit time through the crystal, then the characteristic interaction length will increase as the sound packet 'enters' into the AO cell and at the time $t = L/v_s$ the packet will fill the entire region of interaction of the light with the sound wave and the IF will reach a maximum. Then, as the packet 'leaves' the interaction region, the IF, as can be seen from formula (22), will decrease.

4. If the distance between the sound wave packets is zero ($d = 0$), and the difference between the initial phases of sound oscillations of the packets δ is not equal to zero (which can be done using electronic methods for exciting sound waves), then, as shown in [13], the expression for the IF will be expressed as

$$T_\delta(\Delta k, t) = \left|\frac{\sin(\Gamma L\xi)}{\xi} - [1 - \exp(\mathrm{i}\delta)]\frac{\sin[\Gamma L(1 - v_s t/L)\xi]}{\xi} \times\left[\cos(\Gamma v_s t\xi) - \mathrm{i}\frac{\Delta k\sin(\Gamma v_s t\xi)}{2\Gamma\xi}\right]\right|^2. \qquad (23)$$

It is seen that the presence of the phase difference of the initial oscillations of the sound wave packets also leads to the time dependence of the IF, but this dependence differs from (11) and, more significantly, the initial phase difference δ does not depend on the phase detuning Δk. It follows from expression (23) that the time dependence is determined by the value of the frequency $\Omega_\delta \equiv \Gamma\xi v_s = \Omega/2$ [cf. (22)]. Naturally, for $\delta = 2\pi m$ ($m = 0, 1, 2, 3, \ldots$) expressions (10) and (23) coincide.

5. One can see from formulae (8) and (11) that at the time $t = L/v_s$ the IF of the AO filter is independent of the phase difference and coincides with the IF (10), but at other times the IF will depend on time, phase difference, sound wave power and interaction length. The latter property is a consequence of the condition $L \gg d$, which must be satisfied with a large margin. This condition means that at a distance d the change in the amplitude of the diffracted radiation is very small and can be ignored.

Obviously, to complete transition processes under excitation of the sound wave packets, it is necessary that the time interval between the sound packets be greater than the period of the sound wave, and therefore the interval d must satisfy the requirement $d \gg \Lambda_s$. For example, when $d = 50$ μm, $\Omega_s = 2\pi \times 85$ MHz, $\upsilon_s = 6 \times 10^4$ cm s^{-1} and $\Delta k = 0$, we have the phase difference $\Delta\phi = 2\pi \times 7.08$, which is equivalent to $\Delta\phi = 2\pi \times 0.8$. (The chosen value of the sound wave frequency Ω_s for paratellurite crystals corresponds to an approximate fulfilment of the phase-matching conditions at the optical radiation wavelength close to 1 μm [15].) It is important that this phase difference depends on the sound wave frequency and, when the AO filter is tuned to a different wavelength, one needs to take this dependence into account. Similarly, the behaviour of the limiting values of the coefficients C_1, S_1, C_2 and S_2, determining a temporal change of the IF, can be considered. The latter does not present any difficulties, and therefore it is advisable to carry out the examination for a specific experimental situation.

4. Measuring the optical radiation spectra

Unlike conventional collinear AO filters based on diffraction on a sound wave continuous in time, the case of diffraction on sound packets considered in this paper is characterised by an additional parameter, i.e. the distance between the packets, which can vary during the measurements. As was shown above, the presence of even a small, of the order of several wavelengths of sound, gap between sound wave packets leads to a significant change in the initial phases of the light waves [see formula (4)] and, as a result, to a change in the IF of the AO filter. Moreover, the dependence of the initial phase on the detuning causes the appearance of the time dependence of the IF, which can be used to implement new methods of spectral measurements.

Consider in more detail the process of spectral measurements using AO filters. Mathematically, it is reduced to solving the inverse problem: finding the spectral distribution of the intensity $S(k)$ of the incident optical radiation with a wavenumber k for the case of the known IF of the AO filter (or spectrometer), $T(\Delta k, t)$, and the measured photocurrent of the receiver, $J(t)$. This problem is reduced to solving an integral equation for the function $S(k)$ with known functions $T(\Delta k, t)$ and $J(t)$:

$$J(t) = \int_{k_{\min}}^{k_{\max}} S(k)\, T(\Delta k, t)\, \mathrm{d}k. \tag{24}$$

Here it is assumed that the spectral sensitivity of the photodetector does not depend on the radiation wavelength and is included in the value $S(k)$; and $k_{\min} = 2\pi/\lambda_{\min}$ and $k_{\max} = 2\pi/\lambda_{\max}$ are the wavenumbers of optical radiation in vacuum, which determine the measurement range. A photodetector is considered to be sufficiently fast, so that the characteristic time τ_p of its response to a time-varying radiation flux satisfies the relation $\tau_p \Omega(\Delta k, \Gamma) \ll 1$; therefore, there is no time integration in formula (24). Since the IF of the AO filter with exact fulfilment of the phase-matching conditions, i.e., when $\Delta k = 0$, contains a sharp maximum [see formulae (10), (22)], the appropriately normalised delta function is usually used as this function, considering such that $T(k, t) \propto T(k', t)\, \delta(k - k')$, where $\delta(k - k')$ is the Dirac delta function, and k' is the value of the radiation wavenumber corresponding to the exact fulfilment of the phase-matching conditions and, therefore, to the IF maximum. Then the integral equation (24) is greatly simplified, and a simple relation follows from it, which allows one to determine the intensity $S(k')$ of the optical radiation with the wavenumber k':

$$J(k', t) = A S(k') T(k', t), \tag{25}$$

where A is the normalisation constant. The linear dependence of the photocurrent on the intensity of the incident radiation, in which the coefficient is determined by the maximum IF value, i.e. $T(k', t)$, makes it possible to implement a simple method of measuring the spectral distribution of the radiation intensity $S(k')$ and in some cases also to take into account the effect of the parasitic illumination and some noise. As mentioned above, the usual practice is to modulate the intensity of the sound wave, thereby producing two states in the measurements: useful signal and parasitic illumination or only parasitic illumination. Subtracting the result of the second measurement from the result of the first one, we can exclude the contribution of the parasitic illumination, considering that its level has not changed during the measurement. At the same time, the duration of the sound packet is chosen, as a rule, to be much longer than the transit time of sound through the crystal of the AO cell. This method is widely used in many AO spectrometers [12].

Another approach to determining the spectral distribution of the radiation intensity $S(k)$ is also possible if we take into account that the IF of a conventional AO filter (11), when the phase-matching conditions are fulfilled (i.e. at $\Delta k = 0$), has a sufficiently sharp maximum. Then, using the saddle-point method, it is easy to show that integral (24) can be found explicitly:

$$J(k') \simeq S(k') \frac{2\sqrt{\pi}\,\Gamma \sin^2(\Gamma L)}{\Delta n\, |\Gamma L \cot(\Gamma L) - 1|^{1/2}}, \tag{26}$$

where $k' \equiv q_s/\Delta n$. Expression (26) is given only for one maximum, when $\Delta k = 0$, i.e. for a relatively smooth spectral distribution of the radiation intensity; if it is necessary to take into account the influence of side maxima, for which $\Delta k \neq 0$, it is necessary to proceed as follows. The entire region of integration is divided into sections near each maximum, and the integral is written as the sum of the integrals for each section, calculated by the saddle-point method. Assuming, as before, the spectrum of the incident radiation to be sufficiently smooth compared with changes in the IF, we determine the positions of the maxima k_m from the equation

$$\frac{\mathrm{d}f(k)}{\mathrm{d}k} = 0,$$

where

$$f(k) \equiv \ln \frac{\sin^2\{\Gamma L \sqrt{1 + [\Delta k/(2\Gamma)]^2}\}}{1 + [\Delta k/(2\Gamma)]^2}.$$

The solution of the last equation leads to the following sequence of maxima:

$$k_m = (\Delta n)^{-1}[q_s \pm 2\Gamma \sqrt{[r_m/(\Gamma L)]^2 - 1}],$$

where r_m are the roots of the transcendental equation $r \cot r = 1$. Since the 'width' of the function $f(k)$ for each maximum k_m is determined by the expression

$$\sqrt{2\pi\, |\partial^2 f(k)/\mathrm{d}k^2|}\, \Big|_{k_m},$$

for integral (26) we obtain the final expression

$$J(q_s) \approx S(k')\frac{2\sqrt{\pi}\,\Gamma\sin^2(\Gamma L)}{\Delta n[\Gamma L\cot(\Gamma L)-1]^{1/2}}+\sum_{\pm m}\frac{4\sqrt{2\pi}\,\Gamma^2\xi_m^2 S(k_m)\sin^2(\Gamma L\xi_m)}{\left|\Gamma^2[-8+\Gamma L\cos^{-2}(\Gamma L\xi_m)\sin(2\Gamma L\xi_m^2)]+\Delta k_m^2[2-\Gamma L\cos^{-2}(\Gamma L\xi_m)\xi_m^2]\right|^{1/2}}, \tag{27}$$

where $\xi_m = \sqrt{1+[\Delta k_m/(2\Gamma)]^2}$ and the values of the first three roots r_m near the main maximum will be $r_1 = \pm 4.49$, $r_2 = \pm 7.25$ and $r_3 = \pm 10.90$.

Thus, the calculation of integral (24) by the saddle-point method reduces it to an infinite system of linear algebraic equations, and since the contribution to the photocurrent of the secondary maximum decreases with increasing its number, the indicated chain of equations (27) can be terminated at a certain value of m. Then the problem of determining the values of $S(k_m)$ becomes closed and it is possible to construct its solution. The position of each side maximum Δk_m is determined by the specific value of the sound wave frequency, and therefore it is clear that the measurement strategy, more precisely, the choice of the set of frequencies of the acoustic waves at which the photocurrent measurements are made, must correspond to Δk_m values.

Formula (27), in contrast to formulae (25) and (26), makes it possible to take into account the contributions of side maxima to the measured spectral distribution of the radiation intensity and, moreover, the nonuniformity of the distribution of the intensity of the sound wave at different values of its frequency. Note also that the position of the side maxima also depends on the sound wave intensity, which should be taken into account when interpreting the measurement results. This measurement method can also be used for the IF of the AO filter in the form of (22), when diffraction occurs on only one sound packet and the interaction length is time dependent. Since the position of the main maximum of the IF in this case does not change, instead of (26) we obtain the expression

$$J(q_s, t) \approx S(k')\frac{2\sqrt{\pi}\,\Gamma\sin^2(\Gamma v_s t)}{\Delta n|\Gamma v_s t\cot(\Gamma v_s t)-1|^{1/2}},\quad 0 < v_s t < L, \tag{28}$$

in which there is an explicit dependence of the photocurrent on time, which can be used, in particular, to exclude parasitic illumination.

Let us consider the possible methods of spectral measurements using the IF in the form of (8) or (11), when there is its explicit time dependence. Note that the IF in the form of (8) or (11) is implemented for a pair of sound packets with a small gap between them, and the spatial gap between the pairs of the packets themselves exceeds the interaction length L, so that each pair of sound packets 'lives' independently, without affecting each other. In this case, by choosing different combinations of the phase $\Delta\phi$ and the sound packet power (coefficient Γ), one can obtain different values of the IF. Substituting (11) into equation (24), we obtain a time-dependent photocurrent, and, in accordance with the choice of the problem of measurements to be solved and the method of frequency selection, we can implement various measurement algorithms. The dependence of the coefficients C_1, S_1, C_2 and S_2 on the value of detuning is shown in Figs 4–6 and suggests the choice of the most convenient measurement algorithm.

We will now consider the procedure for measuring the spectral distribution of the radiation intensity under conditions of signal accumulation, which makes it possible to obtain a larger signal-to-noise ratio. The IF in the form of (11) describes the intensity of the light diffraction on a sound wave in a time interval that is close in magnitude, but somewhat less than the transit time of a sound wave in the crystal of the AO cell. However, in practice, of more interest is often the case of diffraction of light on a large number of packets of sound waves during a sufficiently large time interval, when it is possible to provide signal accumulation and thereby increase the accuracy of spectral measurements.

Consider this case in more detail. Let N be the number of sound packets with the same frequency, d be the spatial interval between the packets, and τ be the duration of each sound packet. Then the IF $T_N(\Delta k, t)$, which describes the diffraction on N such packets of sound waves, will obviously have the form

$$T_N(\Delta k, t) = \sum_{n=0}^{N} T(\Delta k, t-n\tau)\,\Theta(t-n\tau)\,\Theta((n+1)\tau - t),$$
$$0 < t < N\tau, \tag{29}$$

where $\Theta(x)$ is the Heaviside step function. Writing the IF in the form of (29) means that diffraction on N sound packets occurs independently, with the initial phases of the sound wave at the input of each packet being the same, which ensures that all N sound packets are identical. Therefore, the photocurrent in terms of signal accumulation has the form

$$J(t) = \int_{k_{\min}}^{k_{\max}} \mathrm{d}k S(k)$$
$$\times\sum_{n=0}^{N} T(\Delta k, \Delta\phi, \Gamma, \xi, t-n\tau)\Theta(t-n\tau)\Theta((n+1)\tau - t),$$
$$0 < t < N\tau. \tag{30}$$

Substituting the IF (11) in the formula for the photocurrent (30), we obtain

$$J(t) = J_0 + J_{C_1S_1}(t) + J_{C_2S_2(t)}, \tag{31}$$

where

$$J_0 \equiv \int_{k_{\min}}^{k_{\max}} \mathrm{d}k S(k)\, T_0(\Delta k, \Delta\phi, \Gamma, \xi)\sum_{n=0}^{N}\Theta(t-n\tau)\Theta((n+1)\tau - t)$$
$$= (N+1)\int_{k_{\min}}^{k_{\max}} \mathrm{d}k S(k)\, T_0(\Delta k, \Delta\phi, \Gamma, \xi);$$

$$J_{C_1S_1}(t) \equiv \int_{k_{\min}}^{k_{\max}} \mathrm{d}k S(k)$$
$$\times\left[\frac{\Delta k\sin(\Delta\phi)\sin(L\Gamma\xi)}{2\Gamma\xi^3} - \frac{\Delta k^2\cos(L\Gamma\xi)\sin^2(\Delta\phi/2)}{2\Gamma^2\xi^4}\right]\Psi(t,k);$$

$$J_{C_2S_2}(t) \equiv -\int_{k_{\min}}^{k_{\max}} \mathrm{d}k S(k)\frac{1}{2\xi^4}\sin^2\left(\frac{\Delta\phi}{2}\right) \tag{32}$$

$$\times\sum_{n=0}^{N}\Theta(t-n\tau)\Theta((n+1)\tau - t)\cos[2\Gamma L\xi - 2\Omega(t-n\tau)];\ \text{and}$$

$$\Psi(t,k) \equiv \sum_{n=0}^{N}\Theta(t-n\tau)\Theta((n+1)\tau - t)\cos[\Gamma L\xi - \Omega(t-n\tau)].$$

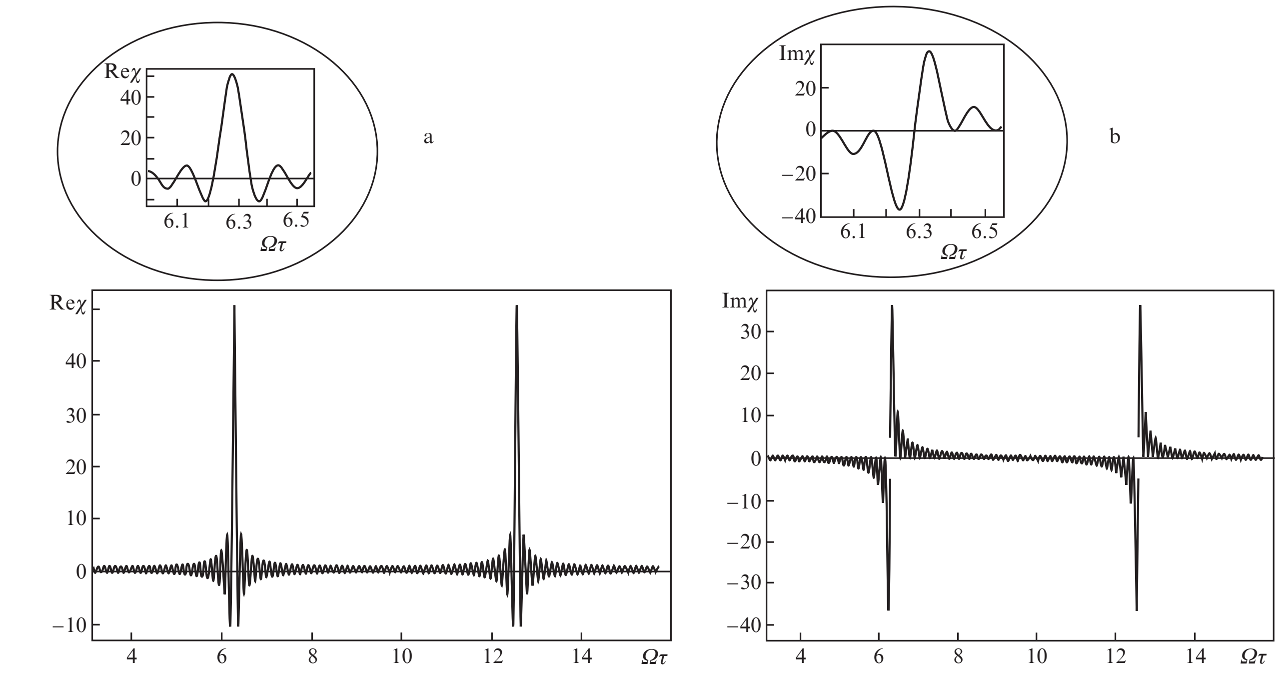

Figure 7. Dependences of (a) real and (b) imaginary parts of the function $\chi(\Omega\tau)$ on its argument.

Here, J_0 is a time constant value of the photocurrent in the interval $0 < t < N\tau$; and $J_{C_1S_1}(t)$ and $J_{C_2S_2}(t)$ are the photocurrent values at frequencies Ω and 2Ω, respectively. Summation in formulae (32) can be done using the relation

$$\sum_{n=0}^{N}\exp(\mathrm{i}n\Omega\tau) \equiv \chi(\Omega\tau) = \cos^{-1}\left(\frac{\Omega\tau}{2}\right)\sin\left[\frac{1}{2}(N+1)\,\Omega\tau\right]$$
$$\times\cos\left(\frac{N\Omega\tau}{2}\right) + \mathrm{i}\left\{\cos^{-1}\left(\frac{\Omega\tau}{2}\right)\sin\left(\frac{N\Omega\tau}{2}\right)\sin\left[\frac{1}{2}(N+1)\,\Omega\tau\right]\right\}. \tag{33}$$

Then for $J_{C_1S_1}(t)$ and $J_{C_2S_2}(t)$ we obtain the expressions

$$J_{C_1S_1}(t) \equiv \int_{k_{\min}}^{k_{\max}}\mathrm{d}kS(k)$$
$$\times\left[\frac{\Delta k\sin(\Delta\phi)\sin(L\Gamma\xi)}{4\Gamma\xi^3} - \frac{\Delta k^2\cos(L\Gamma\xi)\sin^2(\Delta\phi/2)}{4\Gamma^2\xi^4}\right]$$
$$\times\exp(\mathrm{i}\Gamma L\xi - \mathrm{i}\Omega t)\chi(\Omega t) + \text{c.c.}, \tag{34}$$

$$J_{C_2S_2}(t) \equiv -\int_{k_{\min}}^{k_{\max}}\mathrm{d}kS(k)\frac{1}{2\xi^4}\sin^2\left(\frac{\Delta\phi}{2}\right)$$
$$\times\exp(2\mathrm{i}\Gamma L\xi - 2\mathrm{i}\Omega t)\chi(2\Omega t) + \text{c.c.} \tag{35}$$

The dependence of the function $\chi(\Omega\tau)$ on the argument $\Omega\tau$ is presented in Fig. 7. It can be seen that the real part of this function is a series of consecutive maxima at points $\Omega\tau = 2\pi l$, where the integer l is the number of the maxima measured from the value $l = 0$. Since $\tau = L/v_s$ is the transit time of the sound wave along the packet length, it is clear that the maximum number at reasonable values of sound power can only take the value $l = 1$. In other words, the function $\chi(\Omega\tau)$, in addition to the maximum with an argument value equal to zero, which is of no interest to the problem in question, can only have one maximum at $\Omega\tau = 2\pi$. The values of the wavenumber of optical radiation, corresponding to the position of this maximum, have the form

$$k_{1,2} = \frac{q_s}{\Delta n} \pm \frac{2\Gamma}{\Delta n}\sqrt{\left(\frac{\pi}{\Gamma L}\right)^2 - 1}\,, \tag{36}$$

with the 'plus' sign corresponding to the negative crystal that we are considering, and the 'minus' sign – to the positive one. It is easy to show that the real part of the function $\chi(\Omega\tau)$ at the maximum has the form

$$\mathrm{Re}\chi(\Omega\tau)\,|_{\Omega\tau=2\pi} = N + 1. \tag{37}$$

Such a simple value is a direct consequence of the chosen measurement method – signal accumulation. To determine the width of the maximum near the value of k_m, we expand the function $\mathrm{Re}\chi(\Omega\tau)$ into a Taylor series near this maximum:

$$\mathrm{Re}\chi(\Omega\tau) \approx N + 1 - {}^1/_{12}(N + 3N^2 + 3N^3)(\Omega\tau - \pi)^2 + \ldots, \tag{38}$$

and for the width of the maximum along the direction of the wave vector $\boldsymbol{k}$ of the optical radiation we obtain the expression

$$\varDelta_1 = \frac{4\pi}{L}\sqrt{\frac{3}{N + 2N^2}}\left|\left(\frac{\pi}{\Gamma L}\right)^2 - 1\right|^{-1/2}. \tag{39}$$

Thus, when the signal is accumulated, that is, when $N \gg 1$, the function $\chi(\Omega\tau)$ has a sharp maximum. This makes it possible to estimate integral (34), assuming that the region near the maximum makes the largest contribution to it:

$$J_{C_1S_1}(t) \approx -(N+1)\,S(k_1)\,\varDelta_1\left(\frac{\Gamma L}{\pi}\right)^4\left[\left(\frac{\pi}{\Gamma L}\right)^2 - 1\right]\times$$

$$\times \sin^2\left(\frac{\Delta\phi}{2}\right)\cos\left(\frac{2\pi t}{\tau}\right). \quad (40)$$

We take into account that in the maximum

$$\frac{\Delta k_1^2}{4\Gamma^2\xi^4(k_1)} = \left(\frac{\Gamma L}{\pi}\right)^4\left[\left(\frac{\pi}{\Gamma L}\right)^2 - 1\right]. \quad (41)$$

Integral (34) was calculated using the mean value theorem, provided that the function $\chi(\Omega\tau)$ has sharp maxima with a width of Δ_1, and the measured spectrum of optical radiation is sufficiently smooth. Expression (40) means that integral (34) is approximately equal to the sum of the values of the integrand near each maximum multiplied by the width of each maximum (in the case under study, only for the maximum at $\Omega\tau = 2\pi$). Thus, the time dependence of the photocurrent turns out to be periodic with a period equal to the transit time of the sound wave packet in the interaction region. As can be seen from (39) and (40), with increasing N, the amplitude of oscillations of the photocurrent tends to saturation. Reasoning, as above, we find the estimate for integral (35):

$$J_{C_2S_2}(t) \approx -\frac{8(\Gamma L)^4}{\pi^4}(N+1)S(k_1')\Delta_1' \sin^2\left(\frac{\Delta\phi}{2}\right)\cos\left(\frac{2\pi t}{\tau}\right), \quad (42)$$

where

$$k_{1,2}' = \frac{1}{\Delta n}\left[q_s \pm 2\Gamma\sqrt{\left(\frac{\pi}{2\Gamma L}\right)^2 - 1}\right],$$
$$\Delta_1' = \frac{4\pi}{L}\sqrt{\frac{3}{8(N+2N^2)}}\left|\left(\frac{\pi}{2\Gamma L}\right)^2 - 1\right|^{-1/2} \quad (43)$$

are the positions and full width of the maximum, respectively.

Thus, the final expression for the measured total photocurrent will be expressed as

$$J(t) \approx (N+1)\int_{k_{\min}}^{k_{\max}} \mathrm{d}k S(k)\, T_0(\Delta k, \Delta\phi, \Gamma, \xi) - (N+1)S(k_1)$$
$$\times \Delta_1\left[\left(\frac{\Gamma L}{\pi}\right)^2 - \left(\frac{\Gamma L}{\pi}\right)^4\right]\sin^2\left(\frac{\Delta\phi_1}{2}\right)\cos\left(\frac{2\pi t}{\tau}\right) - 8(N+1)\left(\frac{\Gamma L}{\pi}\right)^4$$
$$\times S(k_1')\Delta_1' \sin^2\left(\frac{\Delta\phi_2}{2}\right)\cos\left(\frac{4\pi t}{\tau}\right), \quad (44)$$

where $\Delta\phi_1 \equiv \Delta n k_1 d$ and $\Delta\phi_2 \equiv \Delta n k_1' d$.

Formula (44) describes the temporal behaviour of the photocurrent during diffraction on N sound packets, separated by a small gap, under conditions of signal accumulation. In the absence of a phase difference, i.e., at $d = 0$, formula (42) reduces to the well-known expression for a photocurrent during diffraction on a continuous sound wave for a time $(N+1)\tau$. The accumulation of the signal increases its constant component and has relatively negligible effect on the variable components of the photocurrent. However, signal accumulation, as can be seen from formulae (36) and (43), leads to a narrowing of the region around the maxima, which allows the resolving power of the AO spectral devices to be enhanced.

5. Conclusions

The problem of diffraction of a light flux on two (or more) packets of sound waves propagating in an anisotropic crystal with a small spatial gap between them has been solved, and it has been shown that, due to the phase shift, a time dependence of the IF appears in this gap. In essence, this is another new modulation method, the control parameter in which is the spatial distance between the sound wave packets. It is significant that the phase shift due to the anisotropic properties of the crystalline medium arising in the gap between the packets depends on the wavenumber of the incident optical radiation, and therefore the amplitude of the measured alternating photocurrent signal also becomes dependent on the wavenumber of the incident radiation. A distinctive feature of this approach is that the experiment measures not only the constant component of the photocurrent signal, as it happens in conventional AO spectrometers, but also the amplitudes of the photocurrent signal at frequencies Ω and 2Ω, which, as shown, depend on the wavenumber of the light wave. The fact that the characteristic values of the modulation frequency of the photocurrent depend on the sound wave power makes it possible to take into account its possible changes in the measurement process itself. It is obvious that this modulation method is less sensitive to the noise level, because measurements are performed in a frequency band whose width is obviously smaller than that of noise.

In the case of collinear diffraction on two packets, an exact expression for the IF of the AO filter has been found and some of its possible changes have been analysed, which can be used to modify the methods and algorithms of spectral measurements. Expressions for the measured photocurrent are obtained, which, unlike the known expressions, provide more accurate spectral measurements of the characteristics of the incident radiation. In particular, for the classical case of diffraction on a time constant sound wave [20], it has been shown that the use of the saddle-point method in calculating integral (24) makes it possible to reduce the inverse measurement problem to a system of linear algebraic equations, which allows (for smooth spectra) spectral distributions of the radiation intensity to be determined with greater accuracy. The case of signal accumulation often used in practice under conditions of a new modulation method is considered and expressions for variable components of the photocurrent at different frequencies are found explicitly, the measurement of which makes it possible to obtain more detailed information about the spectral distribution of the radiation characteristics. It is shown that an increase in the total signal accumulation time, i.e. an increase in the number of identical packets of sound waves, leads to a narrowing of the spectral region in which the signal accumulation is effective [see formulae (39), (43)], and this allows the spectral resolution of AO spectrometers to be enhanced without increasing interaction length.

Note that the proposed modulation method is particularly interesting for AO systems of object vision and recognition, since it makes possible to construct new algorithms for detecting 'colour' objects with different polarisation.

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