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Generalised hyper-Ramsey resonance with spinors*

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Abstract. **The generalised hyper-Ramsey resonance formula originally published in *Phys. Rev. A*, 92, 023416 (2015) is derived using a Cayley–Klein spinor parametrisation. The shape of the interferometric resonance and the associated composite phase shift are reformulated including all individual laser pulse parameters. Potential robustness of signal contrast and phase-shift of the wavefunction fringe pattern can now be arbitrarily explored tracking any shape distortion due to systematic effects from the probe laser. An exact and simple analytical expression describing Ramsey's method of separated composite oscillating laser fields with quantum state control allows us to accurately simulate all recent clock interrogation protocols under various pulse defects.**

Keywords: *Ramsey spectroscopy, optical clock, composite pulses, spinor, phase shift, quantum mechanics, atomic interferometry.*

1. Introduction

In 1949, N.F. Ramsey developed a method of separated oscillating fields in magnetic resonance in order to improve the Rabi pulse technique measuring atomic transition frequencies with higher precision [1, 2]. Ramsey's method was thus intensively applied in time and frequency metrology leading to modern atomic clocks with the highest precision [3]. The method was also extended to matter-wave interferometry using the atomic recoil effect induced by lasers to realise beam splitters and mirrors [4]. Atomic gravito-inertial sensors based on Ramsey–Bordé interferometers have thus rapidly improved the measurement sensitivity to external fields or rotations [5].

The present paper focuses on quantum engineering of a Doppler-recoil-free laser pulsed Ramsey resonance associated with the atomic phase shift driven by a sequence of three composite pulses applied around a single free evolution time [6, 7]. The original scheme called hyper-Ramsey (HR) spectroscopy demonstrates a very strong efficiency to eliminate the sensitivity to residual probe induced light shift in a single $^{171}Yb^+$ ion clock [8]. A new frequency standard based on the very narrow electric octupole (E3) transition leads to an unprecedented relative accuracy at $\sim 3 \times 10^{-18}$ [9].

Such a remarkable result has thus stimulated intensive research in the field of Ramsey spectroscopy leading to other sequences of composite pulses even more robust to residual light shift [10]. An exact analytical transition probability denoted as the generalised hyper-Ramsey resonance was initially presented in [7]. Such a resonance was an extension of the original hyper-Ramsey scheme taking into account a few modifications of laser parameters during laser probe interrogation. But the exact influence of the laser phase step on each pulse was not clearly provided and the effect of a residual coherence was not taken into account.

We report in this paper, for the first time, a new derivation of the same resonance by the spinor formalism. An exact and simple closed-form solution for both the resonance shape and the central fringe phase shift are established including all laser parameters from each individual tailored pulse. The formalism of spinor matrices would provide an accurate quantum simulation platform testing other sequences of composite pulses within Ramsey spectroscopy dedicated to a next generation of robust atomic clocks and spectrometers against laser probe perturbations [10, 11].

We start the formal derivation of the generalised hyper-Ramsey (GHR) transition probability by introducing Cayley–Klein parametrisation of rotation spinors as following [10, 12, 13]:

$$M(\tilde{\vartheta}_l) = \begin{pmatrix} \cos\tilde{\vartheta}_l e^{\mathrm{i}\phi_l} & -\mathrm{i}e^{-\mathrm{i}\varphi_l}\sin\tilde{\vartheta}_l \\ -\mathrm{i}e^{\mathrm{i}\varphi_l}\sin\tilde{\vartheta}_l & \cos\tilde{\vartheta}_l e^{-\mathrm{i}\phi_l} \end{pmatrix}, \tag{1}$$

where $l = 1', 1, 2$; and

$$\tilde{\vartheta}_l = \arcsin\left(\frac{\Omega_l}{\omega_l}\sin\tilde{\theta}_l\right), \tag{2a}$$

and

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$$\phi_l = \arctan\left(\frac{\delta_l}{\omega_l}\tan\tilde{\theta}_l\right) \tag{2b}$$

are the phase angles. Here, $\tilde{\theta}_l = \theta_l/2$; $\theta_l = \tau_l\omega_l$ is the effective pulse area; and $\omega_l = (\delta_l^2 + \Omega_l^2)^{1/2}$ is the generalised Rabi frequency. Such a parametrisation is very convenient to simulate any composite three-pulse configuration shown in Fig. 1. Applying spinors to the Schrödinger equation with $c_g(0)$, $c_e(0)$ as initial conditions, a closed-form solution is derived for the complex GHR amplitude:

$$\begin{pmatrix} c_g \\ c_e \end{pmatrix} = M(\tilde{\vartheta}_2)M(\tilde{\vartheta}_1)\begin{pmatrix} \exp(i\delta T/2) & 0 \\ 0 & \exp(-i\delta T/2) \end{pmatrix} \times M(\tilde{\vartheta}_{1'})\begin{pmatrix} c_g(0) \\ c_e(0) \end{pmatrix}. \tag{3}$$

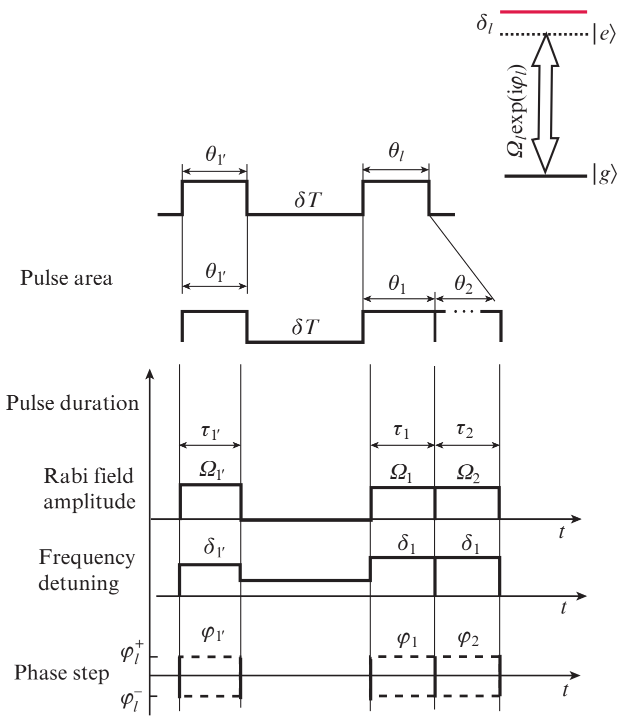

Figure 1. Composite three-pulse spectroscopy probing an ultra-narrow clock transition. Optical pulses are defined by a generalised area θ_l ($l = 1', 1, 2$), a frequency detuning δ_l, a Rabi field frequency $\Omega_l\exp(i\varphi_l)$ including a laser phase step, a pulse duration τ_l and a single free evolution time T. The general clock frequency detuning is defined by $\delta_l = \delta - \Delta_l$, where Δ_l is the residual frequency shift after pre-compensation of the laser probe and δ is the frequency detuning from the frequency of the unpertubed transition.

The clock transition can be detected by measuring the atomic population fraction in the ground state, $P_g = |c_g|^2$, or in the excited state, $P_e = |c_e|^2 = 1 - P_g$.

2. Exact generalised hyper-Ramsey resonance formula

The corresponding transition probability of the ground state for initial population initialisation given by $c_g(0) = 1$, $c_e(0) = 0$ can be recast in the canonical form:

$$P_g = |\alpha|^2|\{1 - |\beta|\exp[-i(\delta T + \Phi)]\}|^2, \tag{4a}$$

including a composite phase shift accumulated over the entire sequence of laser pulses given by:

$$\Phi = \varphi_1 - \varphi_{1'} + \phi \tag{4b}$$

with an additional complex phase-shift contribution:

$$\phi = \phi_1 + \phi_{1'} - \arg\beta. \tag{4c}$$

Parameters α and β driving the overall envelop and composite phase shift can be separated in two contributions from the left pulse driven by a pulse area $\tilde{\vartheta}_{1'}$ and two right pulses driven by $\tilde{\vartheta}_1$, $\tilde{\vartheta}_2$ around the free evolution time matrix (Fig. 1). We introduced then the reduced notations:

$$\alpha \equiv \alpha(\tilde{\vartheta}_{1'})\alpha(\tilde{\vartheta}_{12}), \tag{5a}$$

$$\beta \equiv \beta(\tilde{\vartheta}_{1'})\beta(\tilde{\vartheta}_{12}), \tag{5b}$$

where

$$\alpha(\tilde{\vartheta}_{1'}) = \cos\tilde{\vartheta}_{1'}; \tag{6a}$$

$$\alpha(\tilde{\vartheta}_{12}) = \cos\tilde{\vartheta}_1\cos\tilde{\vartheta}_2[1 - \exp(-i\Xi_{12})\tan\tilde{\vartheta}_1\tan\tilde{\vartheta}_2]; \tag{6b}$$

$$\beta(\tilde{\vartheta}_{1'}) = \tan\tilde{\vartheta}_{1'}; \tag{6c}$$

$$\beta(\tilde{\vartheta}_{12}) = \frac{\tan\tilde{\vartheta}_1 + \exp(-i\Xi_{12})\tan\tilde{\vartheta}_2}{1 - \exp(-i\Xi_{12})\tan\tilde{\vartheta}_1\tan\tilde{\vartheta}_2} \tag{6d}$$

with $\Xi_{12} = \varphi_2 - \varphi_1 + \phi_1 + \phi_2$.

From Eqn (4c), few composite phase-shift formulae can be straightforwardly recovered: identical two pulse Ramsey phase shift,

$$\phi = 2\phi_1; \tag{7a}$$

different two pulse Ramsey phase shift,

$$\phi = \phi_1 + \phi_{1'}; \tag{7b}$$

and generalised hyper-Ramsey phase shift,

$$\phi = \phi_1 + \phi_{1'} - \arg[\beta(\tilde{\vartheta}_{12})]. \tag{7c}$$

These analytical expressions extend previous perturbative or exact solutions related to Ramsey and hyper-Ramsey lineshapes from Refs [2,7,11]. The dependence of $P_e(\delta)$ related to the original HR scheme [Eqns (4)–(6)] on the clock detuning is shown in Fig. 2a.

Note that in order to lock the laser frequency to the atomic transition, a phase-step modulation with opposite sign $\varphi_l^{\pm}$ can also be applied to the ground or excited-state transition probabilities [14] generating a dispersive error signal given by:

$$\Delta E = P_g(\varphi_l^+) - P_g(\varphi_l^-). \tag{8}$$

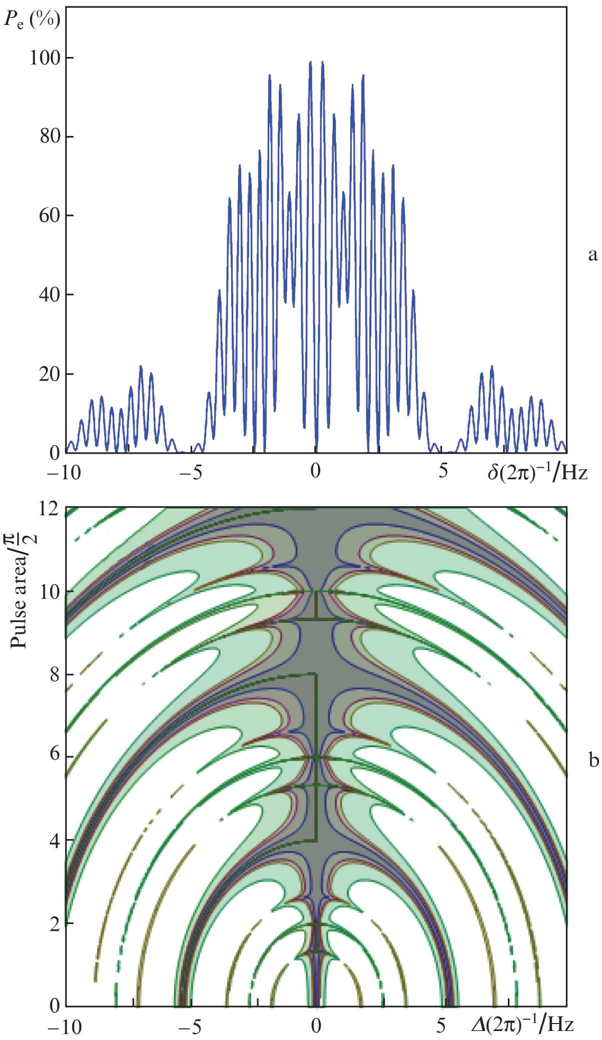

Figure 2. (Colour online) Population fraction P_e and associated clock frequency shift vs. clock detuning: (a) Hyper-Ramsey (HR) resonance and (b) 2D map representation of the HR clock frequency-shift $\delta\nu_2$ as a function of residual uncompensated probe light shift $\Delta/2\pi$ and pulse area in units of $\pi/2$ for initial conditions $c_g(0) = 1$, $c_e(0) = 0$. Colored contour plots are drawn by superposition of layers for different frequency shifts starting from 10 mHz (light zone) to below 0.05 mHz (dark zone, which means a very low sensitivity to residual probe light shift). Spectroscopic parameters are the Rabi field $\Omega = \pi/(2\tau)$ with a pulse duration $\tau = 3/16$ s, a free evolution time $T = 2$ s and a laser probe-induced residual frequency shift $\Delta_{1'} \equiv \Delta_1 \equiv \Delta_2 = \Delta$.

The subscript $l = 1', 1, 2$ is related to any kind of a phase step applied during left or right interaction pulses. Several sequences of phase steps have been proposed with different error signal sensitivity to the composite phase shift. The original hyper-Ramsey scheme (HR) is based on $\varphi_{1'}^{\pm} = \pm\pi/2$ during the first left pulse and on $\varphi_1 = \pi$ during the middle pulse as reported in [6, 7]. Another possibility called the modified hyper-Ramsey (MHR) technique is to apply $\varphi_{1'}^{\pm} = +\pi/2$ during the first left pulse, keeping $\varphi_1 = \pi$ during the middle pulse while using an opposite phase step $\varphi_2^{-} = -\pi/2$ during the last pulse [15]. Another protocol is the generalised hyper-Ramsey (GHR) scheme where phase steps are applied only during the middle pulse with $\varphi_1 = \pm\pi/4$ or $\varphi_1 = \pm 3\pi/4$ [16]. All error signals are easily retrieved within the spinor formalism following phase-step protocols previously listed and are fully consistent with results already presented in review [10].

3. GHR transition probability including arbitrary initial amplitudes

We establish the full lineshape expression of the transition probability for arbitrary initial amplitudes $c_g(0)$, $c_e(0)$ satisfying the condition $|c_g(0)|^2 + |c_e(0)|^2 = 1$. New expressions for coefficients $\alpha(\tilde{\vartheta}_{1'})$ and $\beta(\tilde{\vartheta}_{1'})$ driving the first laser pulse excitation are now given by:

$$\alpha(\tilde{\vartheta}_{1'}) = \cos\tilde{\vartheta}_{1'}\{c_g(0) + \mathrm{i}c_e(0)\exp[\mathrm{i}(\varphi_{1'} - \phi_{1'})]\tan\tilde{\vartheta}_{1'}\}, \quad (9a)$$

$$\beta(\tilde{\vartheta}_{1'}) = \frac{c_g(0)\tan\tilde{\vartheta}_{1'} - \mathrm{i}c_e(0)\exp[\mathrm{i}(\varphi_{1'} - \phi_{1'})]}{c_g(0) + \mathrm{i}c_e(0)\exp[\mathrm{i}(\varphi_{1'} - \phi_{1'})]\tan\tilde{\vartheta}_{1'}}. \quad (9b)$$

The envelop and phase shift driving the entire GHR transition probability are replaced by complex trigonometric functions including imperfect initialisation of quantum states. The generalised composite phase-shift expression becomes:

$$\phi = \phi_1 + \phi_{1'} - \{\arg[\beta(\tilde{\vartheta}_{12})] + \arg[\beta(\tilde{\vartheta}_{1'})]\}, \quad (10)$$

wheree $\beta(\tilde{\vartheta}_{12})$ and $\beta(\tilde{\vartheta}_{1'})$ are respectively given by Eqns (6d) and (9b). The clock frequency shift associated to this phase shift is given by the relation:

$$\delta\nu = -\frac{\phi \pm k\pi}{2\pi T}, \quad (11)$$

where k is an integer selected to ensure the expression continuity by correcting quadrant jumps.

Figure 2b shows the HR clock frequency shift [Eqn (11)] associated to the resonance minimum versus residual uncompensated probe-induced shifts and pulse area. This is the original signature feature of the ideal HR scheme already reported in [17] with a density matrix formalism. The distortion of the HR clock frequency shift displayed in Figs 3a and 3b are produced when state initialisation is incorrect. Laser phase steps are fixed to be $\varphi_{1'} = 0, \varphi_1 = \pi, \varphi_2 = 0$ for a simplified simulation. A modification of the relative phase shift between composite pulses is due to a residual coherence between atomic states $c_g(0), c_e(0) \neq 0$ in a quantum mechanical approach. This new result might emphasise that a weak distortion of the lineshape may coexist with an additional phase-shift contribution related to a lack of appropriate quantum state engineering between multiple sequences of two-level system probe interrogation [18]. Finally, Fig. 3c demonstrates the effect of a systematic small error in the phase-step inversion $\varphi_1 = 0.99\pi$ realised during the middle pulse of the HR scheme. The clock frequency shift experiences a small offset against residual probe-induced shifts if phase steps or state initialisation are not well controlled as shown in Fig. 4. The clock frequency shift is assumed to be a $\Delta\theta/\theta = \pm 10\%$ pulse variation during the probe interrogation.

4. Conclusions

We have derived the generalised hyper-Ramsey resonance with spinors. The exact lineshape and the central fringe phase

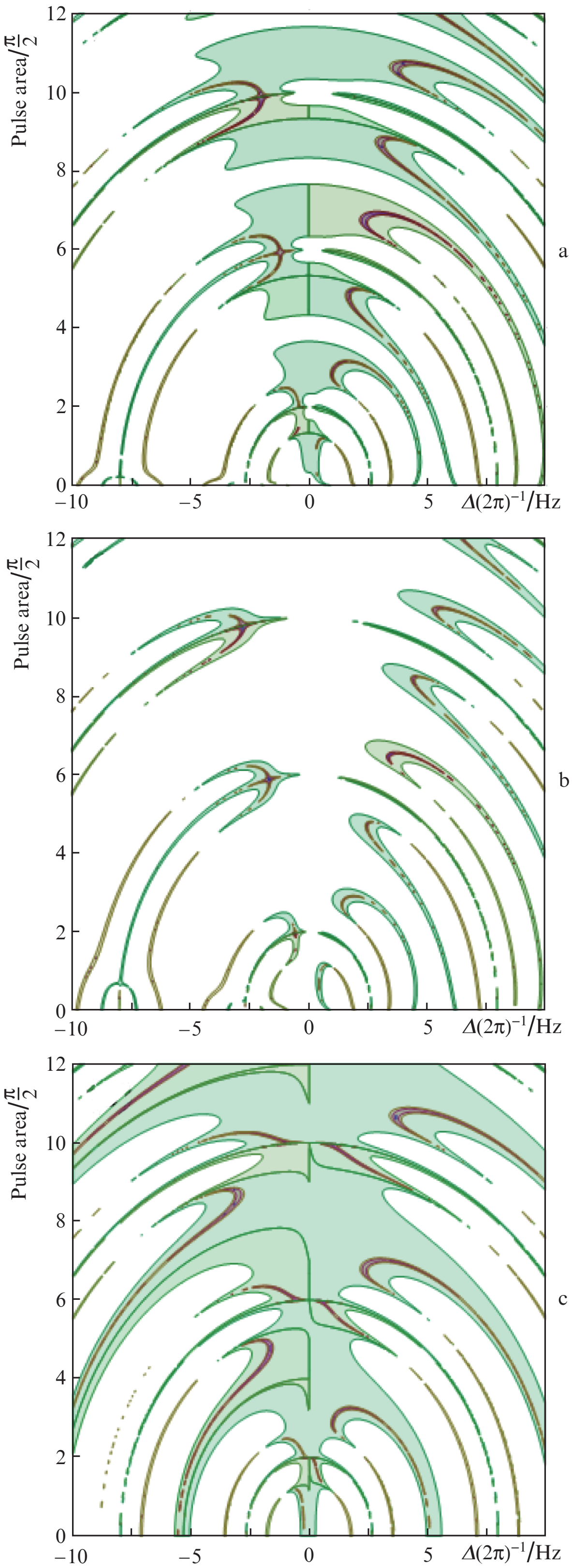

Figure 3. Distortion of the 2D map representation of the HR clock frequency shift $\delta\nu$ due to imperfect (a, b) state initialisation or (c) phase-step error: the frequency shift related to Eqn (11) is plotted vs. residual uncompensated probe light shift $\Delta/2\pi$ and pulse area in units of $\pi/2$ at (a) $c_g(0) = \sqrt{0.999}$, $c_e(0) = \sqrt{0.001}$, (b) $c_g(0) = \sqrt{0.99}$, $c_e(0) = \sqrt{0.01}$ and (c) phase-step error $\varphi_1 = 0.99\pi$ during the middle pulse. Others parameters and coloured contour plots are the same as in Fig. 2.

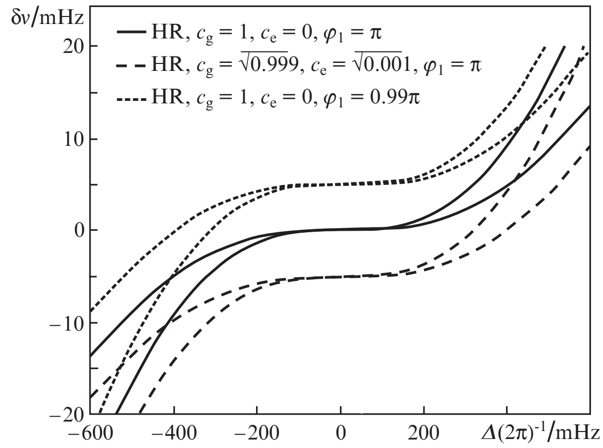

Figure 4. HR clock frequency shift $\delta\nu$ of the population fraction P_e vs. residual uncompensated probe-induced light shift $\Delta/2\pi$ with a small offset due to imperfect state initialisation and phase-step error. Different curves of the same type are produced by a pulse area variation of $\Delta\theta/\theta = \pm 10\%$. Others parameters are the same as in Fig. 2.

shift have been rewritten in a compact form allowing any distortion from pulse defects to be tracked. For the first time, a composite phase shift includes an arbitrary initial amplitude of quantum states. We have employed spinors to establish an exact and tractable model of the GHR resonance which might be very helpful when intensity fluctuations are taken into account [19]. Merging the original method of Ramsey spectroscopy [2] with more complex sequences of composite laser pulses [20, 21] would allow optimal control of laser parameters in the field of clock interferometry and precision spectroscopy for tests of fundamental physics with ultracold quantum particles.

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