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TERAHERTZ RADIATION

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Generation of terahertz radiation in the interaction of a femtosecond pulse with a metal film

E.A. Danilov, S.A. Uryupin

Abstract. **The action of an s-polarised femtosecond laser pulse is found to result in nonlinear currents in a metal film, which arise due to the nonuniform heating of the conduction electrons and under the influence of the drag force. The Fourier transforms of a low-frequency magnetic field generated by nonlinear currents are calculated. It is shown that when the film thickness is less than the scale of the low-frequency field nonuniformity, the amplitude of the terahertz signal increases in inverse proportion to the film thickness. If the thickness of the film is less than the depth of the skin layer at the laser frequency, the low-frequency signal is amplified inversely proportional to the square of the film thickness.**

Keywords: *femtosecond pulse, film, metal, terahertz radiation, skin layer.*

1. Introduction

The generation of terahertz radiation by metals has been widely studied experimentally and theoretically (see, for example, [1 – 17]). A number of general properties of terahertz generation in interaction of femtosecond laser pulses with metals can be explained by three generation mechanisms. The first of them dominates under the conditions when the laser pulse duration $\tau_{\rm p}$ is shorter than τ, i.e. the electron free path time, and arises due to the effect of the ponderomotive force on the electrons (see [8, 9, 11 – 13,17]). The other two dominate if the pulse duration is $\tau_{\rm p} \gg \tau$, and arise either as a result of effective heating of electrons [14, 16], leading to a pressure gradient, or under the action of a drag force generating directional motion of electrons along the metal surface [9, 16].

Despite the different physical nature of these mechanisms, for all of them the amplitude of the generated signal markedly depends on the field structure in the metal. Experiments in which the geometry of the metal surface was changed also indicate a strong dependence of the generated signal on the properties of the field distribution in the sample. This is clearly demonstrated in [4, 5], where the generation of terahertz radiation from a corrugated metal surface was studied, and in [7], where radiation was generated by nanoparticles. In a simpler experiment, when the generation in flat iron [1], silver [2] or gold [2, 3, 6, 10] films of various thickness was studied, a strong dependence of the generation efficiency on the film thickness was found. The fields generated by a femtosecond laser pulse in a metal film have been previously studied in connection with the problem of laser heating of a film [18]. It is natural to use the results of paper [18] to calculate nonlinear currents in the film and to subsequently consider the generation of terahertz radiation.

Following [18], we present in this work the expressions for the high-frequency field in a film produced by an s-polarised femtosecond laser pulse. The case is considered when the pulse duration $\tau_{\rm p}$ is much longer than the electron free path time, and there is no need to keep corrections to the field due to a change in the amplitude of the laser pulse envelope. Under conditions of weak nonuniform heating of electrons, an equation was written for a small pressure perturbation and the source of the drag current was found. Equations were also formulated for the Fourier transforms of low-frequency electric and magnetic fields. Using the general solution of these equations and the continuity conditions for the tangential field components at the film boundaries, we found the Fourier transforms of the low-frequency magnetic field generated by a pressure gradient and drag force. The expressions for the Fourier transforms of the magnetic field were analysed. If the duration of the laser pulse is much longer than the free-path time of the electrons, the scale of the change in the low-frequency field is much greater than the depth of the skin layer at the fundamental frequency of the laser radiation. If the film thickness is greater than the scale of the change in the low-frequency field, then the results obtained previously for a massive sample follow from the derived expressions [9, 16]. If the low-frequency field changes weakly in the film thickness and the laser radiation field is localised at the film surface, the Fourier transform of the magnetic field increases proportionally to $1/L$ with decreasing film thickness L. Finally, for very thin films whose thickness is less than the skin depth at the laser radiation frequency, the Fourier transform of the generated field increases in proportion to $1/L^2$. It was suggested that a weak increase in the amplitude of the low-frequency signal and an increase in the transmission of the gold film at the fundamental frequency, which were observed in [6] far from the percolation threshold, are due to the general properties of the field behaviour in the film described by us.

2. High-frequency field in the film

Let us consider the interaction of an s-polarised electromagnetic laser pulse with a metal film occupying a space region $0 < z < L$. We represent the electric field strength of the pulse as

$$\boldsymbol{E}_{\rm en}(t - \boldsymbol{kr}/\omega)\sin(\omega t - \boldsymbol{kr}). \quad (1)$$

E.A. Danilov, S.A. Uryupin P.N. Lebedev Physical Institute, Russian Academy of Sciences, Leninsky prosp. 53, 119991 Moscow, Russia; National Research Nuclear University 'MEPhI', Kashirskoe sh. 31, 115409 Moscow, Russia; e-mail: uryupinsa@lebedev.ru

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Here, ω is the carrier frequency of radiation; $\boldsymbol{k} = (\omega/c)(\sin\theta, 0, \cos\theta)$; θ is the angle between the wave vector $\boldsymbol{k}$ and the z axis orthogonal to the film surface; c is the speed of light; and $\boldsymbol{E}_{\mathrm{en}}(t) = (0, E_{\mathrm{en}}(t), 0)$ is the pulse envelope changing during the time $t_{\mathrm{p}} \gg 1/\omega$. The electromagnetic pulse penetrates into the film, is reflected from it, and passes into the region $z > L$. Under conditions when the characteristic spatial scale of the change in the field is much larger than $v_{\mathrm{F}}/|\omega + \mathrm{i}\nu|$, where v_{F} is the Fermi velocity and ν is the effective collision frequency in the field of the incident pulse, one can use the results of [18] in considering the field in the film. By neglecting small corrections $\sim 1/\omega t_{\mathrm{p}} \ll 1$, in accordance with relation (15) from [18], for the electric field strength in the film we have $\boldsymbol{E}(z, \tau) = (0, E(z, \tau), 0)$,

$$E(z,\tau) = E_{\mathrm{en}}(\tau)[\mathrm{Re}\,F(z,\omega)\sin\omega\tau - \mathrm{Im}\,F(z,\omega)\cos\omega\tau], \quad (2)$$

where $\tau = t - (x/c)\sin\theta$, and the function $F(z, \omega)$ has the form

$$F(z,\omega) = \frac{2\omega\cos\theta\{\omega\cos\theta\sinh[\kappa(z-L)] - \mathrm{i}\kappa c\cosh[\kappa(z-L)]\}}{(\kappa^2c^2 - \omega^2\cos^2\theta)\sinh(\kappa L) - 2\mathrm{i}\omega\kappa c\cos\theta\cosh(\kappa L)}. \quad (3)$$

The spatial structure of the field in the film depends on the value of the parameter κ, which is related by the expression

$$\kappa^2 = (\omega^2/c^2)[\sin^2\theta - \varepsilon(\omega)] \quad (4)$$

with the permittivity of the metal

$$\varepsilon(\omega) = \varepsilon_0(\omega) - \omega_{\mathrm{p}}^2/\omega(\omega + \mathrm{i}\nu), \quad (5)$$

where ω_{p} is the plasma frequency of the electrons, and $\varepsilon_0(\omega)$ is the contribution to the permittivity from the lattice. Below, we confine ourselves to considering such frequencies ω, when the imaginary part of $\varepsilon_0(\omega)$ can be neglected.

For further consideration, we need the current density of the conduction electrons, $\boldsymbol{j}(z, \tau) = (0, j(z, \tau), 0)$, and the magnetic field component along the z axis, $B_z(z, \tau)$. In accordance with relation (2) and the equation $c\partial E/\partial x = -\partial B_z/\partial t$, we have

$$B_z(z, \tau) = E(z, \tau)\sin\theta. \quad (6)$$

In turn, the expression for $j(z, \tau)$ follows from the equation for the velocity of the directed motion of electrons in a high-frequency field (see, for example, equation (3) from [18]):

$$j(z,\tau) = -\frac{\omega_{\mathrm{p}}^2}{4\pi}\frac{E_{\mathrm{en}}(\tau)}{\omega^2+\nu^2}\{[\omega\,\mathrm{Re}F(z,\omega) + \nu\,\mathrm{Im}\,F(z,\omega)]\cos\omega\tau + [\omega\,\mathrm{Im}\,F(z,\omega) - \nu\,\mathrm{Re}\,F(z,\omega)]\sin\omega\tau\}. \quad (7)$$

The current density $j(z, \tau)$ and the electric field strength $E(z, \tau)$ allow us to find the average power obtained by the conduction electrons during the period $2\pi/\omega$. Taking into account the inequality $\omega t_{\mathrm{p}} \gg 1$, for the absorbed power we have

$$\frac{\omega}{2\pi}\int_0^{2\pi/\omega}\mathrm{d}t\, j(z,\tau)E(z,\tau) \cong \frac{\omega_{\mathrm{p}}^2}{8\pi} \times \frac{\nu}{\omega^2+\nu^2}E_{\mathrm{en}}^2(\tau)\,|F(z,\omega)|^2 \equiv Q(z,\tau). \quad (8)$$

Absorption of laser radiation leads to nonuniform heating of electrons and the associated pressure perturbation. Under conditions of relatively weak heating, for a small pressure perturbation $\Delta p(z, \tau)$, one can use the equation

$$\frac{\partial}{\partial\tau}\Delta p(z,\tau) - \frac{\partial}{\partial z}\left[\frac{v_{\mathrm{F}}^2}{3\nu_{\mathrm{s}}}\frac{\partial}{\partial z}\Delta p(z,\tau)\right] = \frac{2}{3}Q(z,\tau), \quad (9)$$

where ν_{s} is the effective frequency of electron collisions, whose velocity of directed motion varies over times longer than the duration of the laser pulse. The pressure perturbation is related to the heat capacity of electrons by the expressions $\Delta p(z, \tau) = C(z, \tau)T(z, \tau)/3$, $C(z, \tau) = \pi^2 n k_{\mathrm{B}}^2 T(z, \tau)/2\varepsilon_{\mathrm{F}}$, where ε_{F} is the Fermi energy; k_{B} is the Boltzmann constant; and n and $T(z, \tau)$ are the concentration and temperature of electrons, respectively.

The high-frequency motion of electrons in the fields $E(z, \tau)$ and $B_z(z, \tau)$ leads to the appearance of a low-frequency drag current along the film surface. In accordance with relations (6) and (7), after averaging over a period $2\pi/\omega$, the source of the low-frequency drag current density along the x axis has the form

$$\frac{e}{mc}\frac{\omega}{2\pi}\int_0^{2\pi/\omega}\mathrm{d}t\, j(z,\tau)B_z(z,\tau) = \frac{e}{mc}Q(z,\tau)\sin\theta. \quad (10)$$

Relations (8), (10) and the equation for $\Delta p(z, \tau)$ form the basis for further consideration of the generation of low-frequency radiation in a thin film.

3. Low-frequency field in the film

The drag force density and the pressure gradient along the film surface lead to the appearance of a directed motion of electrons along the x axis. In the case, when ν_{s} satisfies the inequality

$$\nu_{\mathrm{s}}t_{\mathrm{p}} \gg 1, \quad (11)$$

for nonlinear current density along the film surface we have

$$J_x(z,\tau) = \frac{e\sin\theta}{mc\nu_{\mathrm{s}}}\left[Q(z,\tau) + \frac{\partial}{\partial\tau}\Delta p(z,\tau)\right] \equiv J_{\mathrm{cd}}(z,\tau) + J_{\mathrm{grad}}(z,\tau). \quad (12)$$

Relation (12) does not contain a contribution to $J_x(z, \tau)$ due to the ponderomotive effect of the laser pulse. Under the conditions of inequality (11), this contribution is $\nu_{\mathrm{s}}t_{\mathrm{p}} \gg 1$ times less than the contribution due to the influence of the drag force. If in equation (9) we can neglect the effect of heat transfer across the film on the pressure gradient along the x axis, the contribution to $J_x(z, \tau)$ due to the drag force, as can be seen from relation (12) and equation (9), is 3/2 times greater than that due to the pressure gradient along the film surface. Considerable heat transfer across the film leads to an even greater relative decrease in the contribution from $\partial\Delta p(z, \tau)/\partial\tau$ into the current density in $J_x(z, \tau)$. For example, under the conditions of the experiment performed in [6], there is an additional decrease of about two times.

Relation (12) allows us to consider the generation of low-frequency radiation under conditions of inequality (11). From Maxwell's equations we find the low-frequency field in the film by using a system of equations for the Fourier transforms of the electric field component $E_z(z, \Omega)$ and the magnetic field component $\boldsymbol{B}(z, \Omega) = (0, B(z, \Omega), 0)$:

$$\varepsilon(\Omega)E_x(z,\Omega) = -\frac{\mathrm{i}c}{\Omega}\frac{\partial}{\partial z}B(z,\Omega) - \frac{4\pi i}{\Omega}J_x(z,\Omega), \tag{13}$$

$$\frac{\partial^2}{\partial z^2}B(z,\Omega) - \kappa_s^2 B(z,\Omega) = -\frac{4\pi}{c}\frac{\partial}{\partial z}J_{\mathrm{cd}}(z,\Omega). \tag{14}$$

Here, Ω is the frequency resulting from the Fourier transform;

$$\varepsilon(\Omega) = \varepsilon_0(\Omega) - \omega_p^2/[\Omega(\Omega + \mathrm{i}\nu_s)] \tag{15}$$

is the low-frequency permittivity; and

$$\kappa_s^2 = (\Omega^2/c^2)[\sin^2\theta - \varepsilon(\Omega)]. \tag{16}$$

The change in ν_s over time is neglected, which is possible under conditions of weak heating. The general solution of equation (14) has the form

$$B(z,\Omega) = C_1\exp(-\kappa_s z) + C_2\exp(\kappa_s z) + \frac{2\pi}{c\kappa_s}\left\{\int_z^L \mathrm{d}z'\exp[\kappa_s(z-z')]\frac{\partial}{\partial z'}J_{\mathrm{cd}}(z',\Omega) + \int_0^z \mathrm{d}z'\exp[-\kappa_s(z-z')]\frac{\partial}{\partial z'}J_{\mathrm{cd}}(z',\Omega)\right\}. \tag{17}$$

The low-frequency radiation is emitted from the film in the region of space $z < 0$ and $z > L$. The Fourier components of the fields of the radiation emitted from the film are found from equations (13) and(14), in which the term with $J_x(z, \Omega)$, is omitted, k_s^2 is replaced by $(-\Omega^2/c^2)\cos^2\theta$, and $\varepsilon(\Omega)$ is substituted by unity. In this case, in the region $z < 0$

$$B_r(z,\Omega) = B_r(\Omega)\exp\left[-\mathrm{i}z\frac{\Omega}{c}\cos\theta\right], \tag{18}$$

$$E_r(z,\Omega) = -B_r(z,\Omega)\cos\theta, \tag{19}$$

and in the region $z > L$

$$B_t(z,\Omega) = B_t(\Omega)\exp\left[\mathrm{i}(z-L)\frac{\Omega}{c}\cos\theta\right], \tag{20}$$

$$E_t(z,\Omega) = B_t(z,\Omega)\cos\theta. \tag{21}$$

Unknown quantities C_1, C_2, $B_r(\Omega)$, $B_t(\Omega)$ are found from the conditions of continuity of $E_x(z, \Omega)$ and $B(z, \Omega)$ at the film boundaries $z = 0$ and L. Because the radiation patterns in the region of space $z < 0$ and $z > L$ are very similar, we confine ourselves to the analysis of radiation in the region $z < 0$, which is either greater than the radiation in the region $z > L$, or comparable to it if the film is sufficiently thin.

The nonlinear currents $J_{\mathrm{cd}}(z, \Omega)$ and $J_{\mathrm{grad}}(z, \Omega)$ make an additive contribution to the magnetic field strength emitted from the surface $z = 0$. The contribution from the drag current has the form

$$B_{\mathrm{cd}}(\Omega) = \frac{4\pi\mathrm{i}\kappa_s}{\Omega\varepsilon(\Omega)}\int_0^L \mathrm{d}z\,J_{\mathrm{cd}}(z,\Omega)\left\{\cos\theta\cosh[\kappa_s(z-L)] - \frac{\mathrm{i}\kappa_s c}{\Omega\varepsilon(\Omega)}\sinh[\kappa_s(z-L)]\right\}\left\{\left[\cos^2\theta + \left(\frac{\mathrm{i}\kappa_s c}{\Omega\varepsilon(\Omega)}\right)^2\right]\times\sinh(\kappa_s L) + 2\cos\theta\frac{\mathrm{i}\kappa_s c}{\Omega\varepsilon(\Omega)}\cosh(\kappa_s L)\right\}^{-1}. \tag{22}$$

The result of (22) follows from the continuity conditions of the tangential components of the electromagnetic field and from expressions (12), (13), and (17)–(21), if $J_{\mathrm{grad}}(z, \tau)$ is omitted from (12). In turn, by omitting $J_{\mathrm{cd}}(z, \tau)$, from the same relations we find the contribution to $B(0, \Omega)$ from the pressure gradient

$$B_{\mathrm{grad}}(\Omega) = \frac{4\pi\mathrm{i}}{\Omega\varepsilon(\Omega)}\left\{J_{\mathrm{grad}}(0,\Omega)\left[\cos\theta\sinh(\kappa_s L) + \frac{\mathrm{i}\kappa_s c}{\Omega\varepsilon(\Omega)}\cosh(\kappa_s L)\right] - 2J_{\mathrm{grad}}(L,\Omega)\frac{\mathrm{i}\kappa_s c}{\Omega\varepsilon(\Omega)}\right\} \times\left\{\left[\cos^2\theta + \left(\frac{\mathrm{i}\kappa_s c}{\Omega\varepsilon(\Omega)}\right)^2\right]\sinh(\kappa_s L) + 2\cos\theta\frac{\mathrm{i}\kappa_s c}{\Omega\varepsilon(\Omega)}\cosh(\kappa_s L)\right\}^{-1}. \tag{23}$$

Relations (22) and (23) allow us to consider the effect of film thickness on the generation of low-frequency radiation due to the drag current and the pressure gradient along the film surface.

4. Discussion

Let us discuss the properties of low-frequency radiation by a metal film. For a typical metal in the low frequency region:

$$\frac{\omega_p^2}{\Omega\nu_s} \gg |\sin^2\theta - \varepsilon_0(\Omega)|,\ \nu_s \gg \Omega. \tag{24}$$

At frequencie $\Omega \sim 1/t_p \ll \nu_s$, from (16) we approximately have $\kappa_s = (1 - \mathrm{i})(\omega_p/c)\sqrt{\Omega/2\nu_s}$. In addition, in the visible frequency range, conditions

$$\omega_p^2/\omega^2 \gg |\sin^2\theta - \varepsilon_0(\omega)|,\ \omega \gg \nu, \tag{25}$$

are relatively simply realised, in which $\kappa \approx \omega_p/c$ and exceeds $|\kappa_s| = (\omega_p/c)\sqrt{\Omega/\nu_s}$ by $\sqrt{\nu_s/\Omega} \gg 1$ times.

First, we consider relation (22) describing the generation of low-frequency radiation by the drag current. If the film is thick, then $\kappa L \gg \kappa_s L \gg 1$. In this case,

$$F(z,\omega) = \frac{2\omega\cos\theta}{\omega\cos\theta + \mathrm{i}\omega_p}\exp\left(-\frac{\omega_p}{c}z\right). \tag{26}$$

Taking into account relations (8), (12) and (26), for a thick film we find from (22)

$$B_{\mathrm{cd}}(\Omega) = 16\pi\mathrm{i}\frac{e\nu}{mc^2\nu_s}\frac{\kappa_s}{2\kappa + \kappa_s}\frac{\omega_p^2}{\omega_p^2 + \omega^2\cos^2\theta}\times\frac{\sin\theta\cos^2\theta}{\Omega\varepsilon(\Omega)\cos\theta + \mathrm{i}\kappa_s c}I(\Omega), \tag{27}$$

where

$$I(\Omega) = \frac{c}{8\pi}\int_{-\infty}^{+\infty}\mathrm{d}\tau\exp(\mathrm{i}\Omega\tau)E_{\mathrm{en}}^2(\tau)$$

is the Fourier transform of the energy flux density. Taking into account the inequalities $v_s \gg \Omega, \omega \gg v, v \gg \sqrt{v_s \Omega}$ and the explicit form of κ and κ_s, expression (27) coincides with expression (37) from [9], obtained for a massive sample. For a thinner film, conditions are possible when the high-frequency field is localised at the surface ($z = 0$), and the low-frequency field is weakly nonuniform across the film thickness, i.e., $\kappa L \gg 1 \gg \kappa_s L$. In this case, at

$$\cos\theta \gg |\kappa_s c/(\Omega\varepsilon(\Omega))| \simeq \sqrt{\Omega v_s}/\omega_p,$$

when $B_{cd}(\Omega)$ is not abnormally small, instead of (27) we have

$$B_{cd}(\Omega) = 16\pi i \frac{ev}{mc^2 v_s} \frac{\kappa_s}{2\kappa} \frac{\omega_p^2}{\omega_p^2 + \omega^2 \cos^2\theta} \times \frac{\sin\theta\cos^2\theta}{\kappa_s L \Omega\varepsilon(\Omega)\cos\theta + i\kappa_s c} I(\Omega). \quad (28)$$

Comparing relations (27) and (28), we see that at

$$1 \gg \kappa_s L \gg \sqrt{\Omega v_s}/(\omega_p \cos\theta) \quad (29)$$

radiation from a thin film is more efficient. According to (28), with film thicknesses satisfying inequalities (29), the Fourier transform of $B_{cd}(\Omega)$ increases proportionally to $1/L$ with decreasing film thickness.

An even stronger increase in $B_{cd}(\Omega)$ with a decrease in the film thickness is realised under the conditions when $1 \gg \kappa L$, $\kappa_s L$. In such a thin film

$$F(z,\omega) = \frac{2\omega\cos\theta}{2\omega\cos\theta + i\omega_p \kappa L}, \quad (30)$$

and the Fourier transform of $B_{cd}(\Omega)$ has the form:

$$B_{cd}(\Omega) = 16\pi i \frac{ev}{mc^2 v_s} \frac{\omega_p^2 \kappa_s L}{\omega_p^2 (\kappa L)^2 + 4\omega^2 \cos^2\theta} \times \frac{\sin\theta\cos^2\theta}{\kappa_s L \Omega\varepsilon(\Omega)\cos\theta + i\kappa_s c} I(\Omega). \quad (31)$$

When changing the film thickness in the interval (29) and the interval

$$1 \gg \kappa L \gg 2(\omega/\omega_p)\cos\theta \quad (32)$$

the Fourier transform of $B_{cd}(\Omega)$ increases proportionally to $1/L^2$ with decreasing L.

The generation of low-frequency radiation due to the pressure gradient is described by relation (23). The explicit form of $B_{grad}(\Omega)$, as can be seen from formula (12), depends on the type of solution of equation (9). We assume that the change in pressure is determined by a small change in the temperature of the electrons. Then, neglecting the heat loss at the film boundaries, which meets the boundary conditions $\partial\Delta p(z,\tau)/\partial z|_{z=0} = \partial\Delta p(z,\tau)/\partial z|_{z=L} = 0$, from equation (9) we have

$$\Delta p(z,\Omega) = \frac{\kappa_T}{3i\Omega}\Big\{-\exp(-\kappa_T z + \kappa_T L)\int_0^L dz' Q(z',\Omega)\frac{\cosh(\kappa_T z')}{\sinh(\kappa_T L)} - \exp(\kappa_T z)\int_0^L dz' Q(z',\Omega)\frac{\cosh[\kappa_T(z'-L)]}{\sinh(\kappa_T L)} + \int_0^z dz' Q(z',\Omega)\exp(\kappa_T z - \kappa_T z') + \int_z^L dz' Q(z',\Omega)\exp(-\kappa_T z + \kappa_T z'), \quad (33)$$

where $Q(z, \Omega)$ is the Fourier transform of the absorbed power $Q(z,\tau)$ (8) and use is made of the notation

$$\kappa_T^2 = -3i\Omega v_s/v_F^2. \quad (34)$$

According to relation (12), the pressure perturbation $\Delta p(z, \tau)$ determines the nonlinear current density $J_{grad}(z, \tau)$. Taking into account the explicit form of $\Delta p(z, \Omega)$ (33), from (12) we find the Fourier transform of $J_{grad}(0, \Omega)$ on the illuminated surface of the film:

$$J_{grad}(0,\Omega) = -i\Omega \frac{2e\sin\theta}{m\kappa_T v_F^2}\int_0^L dz \frac{\cosh[\kappa_T(z-L)]}{\sinh(\kappa_T L)} Q(z,\Omega). \quad (35)$$

The expression for $J_{grad}(L, \Omega)$ is given by relation (35), if we replace $\cosh[\kappa_T(z-L)]$ by $\cosh(\kappa_T z)$.

For thick films, $\kappa_T L \gg 1, \kappa_s L \gg 1$ and $\kappa L \gg 1$. Under these conditions, using relations (8), (26) and (35), from (23) we have

$$B_{grad}(\Omega) = 16\pi i \frac{ev}{mc^2 v_s} \frac{2\kappa_T/3}{2\kappa + \kappa_T} \frac{\omega_p^2}{\omega_p^2 + \omega^2\cos^2\theta} \times \frac{\sin\theta\cos^2\theta}{\Omega\varepsilon(\Omega)\cos\theta + i\kappa_s c} I(\Omega). \quad (36)$$

Expression (36) differs from (27) in that instead of $\kappa_s/(2\kappa + \kappa_s)$, expression (36) contains the factor $(2\kappa_T/3)(2\kappa + \kappa_T)^{-1}$, which is less than 2/3 in absolute value. We also note that under the conditions of $\omega_p \gg \omega\cos\theta$ and $\omega_p \cos\theta \gg \sqrt{\Omega v_s}$ adopted in [16], expression (36) coincides with the Fourier transform of the field E_T (see formula (25) in [16]) arising due to the temperature gradient, if there is no significant difference between v_{eff} and v, as was considered above. At $\omega_p \gg \omega\cos\theta$, $\omega_p\cos\theta \gg \sqrt{\Omega v_s}$ relation (27) also coincides with the Fourier transform of the field E_d (see formula (24) in [16]) generated by the drag current.

For typical metals $|\kappa_s| = (\omega_p/c)\sqrt{\Omega/v_s} \sim |\kappa_T| = \sqrt{3\Omega v_s}/v_F$, since $v_F/(\sqrt{3}v_s) \sim c/\omega_p$. For example, for gold at room temperature, $v_s = 4\times10^{13}$ s^{-1}, $v_F = 1.4\times 10^8$ cm s^{-1} and $\omega_p = 1.37\times10^{16}$ s^{-1}. In this case, $v_F/(\sqrt{3}v_s) \simeq 0.8\times10^{-6}$ cm, and $c/\omega_p \simeq 2\times10^{-6}$ cm. Therefore, as the film thickness decreases, there arise conditions under which $\kappa_s L \ll 1$ and $\kappa_T L \ll 1$ simultaneously, but κL can be large, since $v_s \gg \Omega$. Under these conditions and in fulfilling the right-hand inequality (29) we find

$$B_{grad}(\Omega) \simeq \frac{2}{3}B_{cd}(\Omega),\ \kappa L \gg 1 \gg \kappa_s L \gg \frac{\sqrt{\Omega v_s}}{\omega_p\cos\theta}, \kappa_T L \ll 1. \quad (37)$$

Here, $B_{cd}(\Omega)$ is given by (28), in the denominator of which one can omit the small term $i\kappa_s c$. That is, like $B_{cd}(\Omega)$, the Fourier transform of $B_{grad}(\Omega)$ increases with decreasing L in proportion to $1/L$. If, under the conditions of applicability of formula (37), the inequality $\kappa L \ll 1$ is satisfied instead of $\kappa L \gg 1$, then relation (37) still holds. Only in this case $B_{cd}(\Omega)$ is given by relation (31), in the denominator of which the small term $i\kappa_s c$ is omitted. For such thin films, the Fourier transforms of

$B_{cd}(\Omega)$ and $B_{grad}(\Omega)$ increase proportionally to $1/L^2$ with decreasing L.

Ramakrishnan and Planken [6] studied the generation of terahertz radiation by illuminating gold films of various thickness on a glass substrate with weak femtosecond laser pulse at a wavelength of 800 nm. For such films, the percolation threshold is ~7 nm [6]. In [6], the pulse duration was $\tau_p =$ 50 fs. For a Gaussian pulse, this value of τ_p corresponds to $t_p = \tau_p/(2\sqrt{\ln 2}) \simeq 30$ fs. Under these conditions, for the characteristic scales of the change in the field we have the following estimates: $1/\kappa \simeq c/\omega_p \simeq 22$ nm, $1/\mathrm{Re}\kappa_T \simeq v_F\sqrt{t_p/(3v_s)}$ $\simeq 22$ nm and $1/\mathrm{Re}\kappa_s = \sqrt{v_s t_p}\, c/\omega_p \simeq 24$ nm, where it is assumed that $\Omega \simeq 2/t_p$. According to [6], at film thicknesses of less than 20 nm, but larger than 16 nm, i.e. far from the percolation threshold, one can observe an increase in the film transmission at the fundamental frequency and a relatively weak increase in the amplitude of the terahertz signal. The reason for such an increase, as shown above, may be a relative increase in the field strength of the laser pulse in a thin film, which leads to an increase in the transmittance at the fundamental frequency (see [18]). A more detailed quantitative comparison with the data from [6] is difficult.

Firstly, for gold, the mean free path of conduction electrons is $\sim v_F/v_s \simeq 35$ nm, i.e., somewhat larger than the field inhomogeneity scales, and the theory should be supplemented with allowance for spatial dispersion. Secondly, the theory does not take into account the graininess of the film structure, which can also lead to errors in the calculations of the terahertz signal. And, thirdly, most data were obtained in [6] in the vicinity of the percolation threshold, where the theory is not applicable. At the same time, the above-established peculiarities of the generation of terahertz radiation by the drag current and the pressure gradient of electrons can be realised in films of metals of higher quality, when the granular structure is absent. In this case, it is desirable to have films with hot electrons, when, due to an increase in the effective collision frequency, the mean free path of electrons becomes small compared with the characteristic scales of field changes.

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