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Spontaneous phase symmetry breaking in a gas coupled-cavity ring laser

I.I. Zolotoverkh, E.G. Lariontsev

Abstract. **A theoretical model is proposed that describes the lasing dynamics in a gas ring laser (GRL) with coupled cavities. The conditions are found under which spontaneous phase symmetry of counterpropagating waves is broken in the GRL with an antiphase optical coupling of the cavities. It is shown that in the case of spontaneous phase symmetry breaking, two branches appear on the GRL frequency characteristic. In some region of frequency nonreciprocity of a ring cavity, both branches can exist under the same conditions. In this case, radiation bistability appears in the GRL, and hysteresis phenomena can be observed.**

Keywords: *gas ring laser, coupled cavities, beat mode, phase nonreciprocity, spontaneous symmetry breaking.*

1. Introduction

The study of nonlinear dynamics, amplitude and frequency characteristics of radiation of ring coupled-cavity lasers is an important task. In such lasers, part of the intracavity field of the main cavity containing the active medium is introduced into an additional ring cavity and then returns from it to the main cavity. As a result, external optical feedback arises in the main cavity, which, as theoretical and experimental studies of semiconductor coupled-cavity ring lasers have shown, has a great influence on the lasing dynamics and output characteristics [1–3]. In these studies, it was found that the optical feedback phase plays an important role in the lasing dynamics and under conditions of stable generation regimes [3]. In the case of selective optical feedback, when radiation is spectrally filtered in an additional cavity, single-mode generation can be obtained, the radiation frequency can be tuned, and the generation modes can be controlled in semiconductor lasers [4, 5].

In theoretical and experimental studies of a solid-state coupled-cavity ring Nd:YAG laser, the effect of the optical feedback on the self-modulation oscillations of the intensities of the counterpropagating waves was examined [6, 7].

Coupled-cavity ring lasers are of interest for use in gyroscopy. It was theoretically shown in [8–10] that in gas ring lasers (GRLs) with coupled cavities it is possible to control the intracavity dispersion and create conditions for the occurrence of anomalous dispersion, leading to an increase in the scale factor and sensitivity of the laser gyroscope. The disadvantage of these works is that they do not consider the lasing dynamics and the stability of the generation regimes. To do this, it is necessary to improve the theoretical model of the coupled-cavity GRL, as was done in studies of semiconductor [1–5] and solid-state [6, 7] lasers.

One of the nonlinear effects observed in ring lasers and nonlinear ring cavities is spontaneous symmetry breaking of the fields of counterpropagating waves (see, for example, [11–13]). The purpose of this work is a theoretical study of spontaneous phase symmetry breaking, which, as shown below, can occur in coupled-cavity GRLs.

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2. System of equations

Figure 1 shows the scheme of a coupled-cavity GRL. Inside the main ring cavity containing the active medium (AM), two counterpropagating waves $\tilde{E}_{1,2}$ propagate. The radiation emitted from the main cavity through a partially transmitting coupling mirror M excites the optical fields $\tilde{E}_{c1,c2}$ in the external ring cavity and returns again to the main cavity through the same mirror.

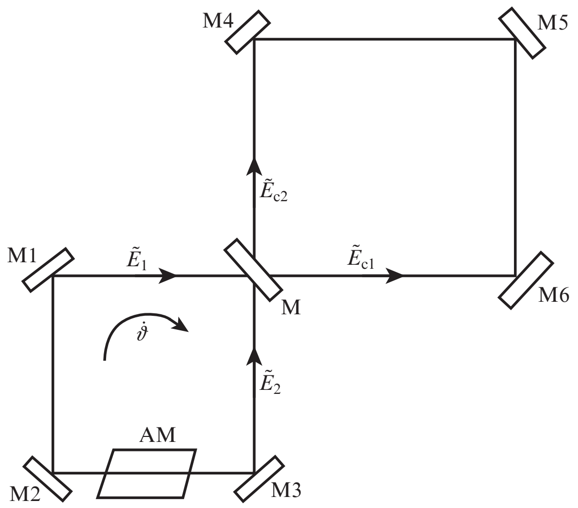

Figure 1. Scheme of a coupled-cavity GRL.

Intracavity fields in the main and additional cavities are written in the form

$$\tilde{E}_{1,2}(t) = E_{1,2}(t)\exp(\mathrm{i}\omega_n t),$$
$$\tilde{E}_{c1,c2}(t) = E_{c1,c2}(t)\exp(\mathrm{i}\omega_n t), \qquad (1)$$

where ω_n is the optical frequency of the generated mode. The complex amplitudes $E_{1,2}(t)$ are slow functions of time and change little over the period of optical oscillations.

For coupled-cavity GRLs, based on similar studies of semiconductor [4] and solid-state [6, 7] ring lasers, we write the system of ordinary differential equations:

$$\dot{E}_{1,2} = \frac{\Delta\omega_{\rm r}}{2}\Big[\frac{\kappa_{1,2}(1+\eta)}{\kappa_0} - 1 - \alpha_{1,2}|E_{1,2}|^2 - \beta_{1,2}|E_{2,1}|^2\Big]E_{1,2}$$

$$\pm {\rm i}\frac{\Omega}{2}E_{1,2} + \frac{\rm i}{2}\tilde{m}_{1,2}E_{2,1} + \frac{k_{\rm c}\exp({\rm i}\varphi)}{T}E_{\rm c1,c2}, \quad (2)$$

$$\dot{E}_{\rm c1,c2} = -\frac{\Delta\omega_{\rm c}}{2}E_{\rm c1,c2} \pm {\rm i}\frac{\Omega_{\rm c}}{2}E_{\rm c1,c2} + \frac{\rm i}{2}\tilde{m}_{\rm c1,c2}E_{\rm c2,c1}$$

$$+\frac{k_{\rm c}\exp({\rm i}\varphi + \omega_n T_{\rm c})}{T_{\rm c}}E_{1,2}. \quad (3)$$

Equations (2) describe the generation of counterpropagating waves $E_{1,2}$ inside the main cavity taking into account the effect of the fields $E_{\rm c1,c2}$, and equation (3) describes the excitation of counterpropagating waves in the external cavity by waves $E_{1,2}$. Here $\Delta\omega_{\rm r}$, $\Delta\omega_{\rm c}$ are the bandwidths of the main and additional cavities (intracavity losses for counterpropagating waves are assumed to be equal); $\tilde{m}_{1,2}$ and $\tilde{m}_{\rm c1,c2}$ are the complex coupling coefficients, which determine the linear coupling of the counterpropagating waves in the main and additional cavities; the coefficients $\kappa_{1,2}$ describe the linear polarisability of the GRL active medium, and $\alpha_{1,2}$, $\beta_{1,2}$ describe its saturation with fields of counterpropagating waves; $(1+\eta)/\kappa_0$ is the ratio of the gain to the intracavity losses at the maximum of the gain curve; $\kappa_0 = \max\{{\rm Re}\kappa_{1,2}\}$; η is the excess of the gain over the threshold; and $k_{\rm c}\exp({\rm i}\varphi)/T$ and $k_{\rm c}\exp({\rm i}\varphi + \omega_n T_{\rm c})/T_{\rm c}$ are the optical coupling coefficients between the fields in the main and additional cavities, depending on the amplitude transmittance of the coupling mirror $k_{\rm c}$, on the round-trip transit times T and $T_{\rm c}$ for the light in the main and additional cavities, on the phase shift φ between the reflected and transmitted waves at the coupling mirror, and also on the phase shift per round-trip transit of the additional cavity, $\Phi = \omega_n T_{\rm c}$.

To justify the considered model, which describes the dynamics of the coupled-cavity GRL emission, one can also use models of coupled lasers (see, for example, [14, 15]). If in the model of coupled lasers we proceed to a particular case when there is no gain medium in one of the cavities, then we come to the model under consideration. It is applicable provided that in each of the counterpropagating directions in the GRL there is a single-mode generation.

The sensitivity to rotation is associated with the Sagnac effect, when the main and additional cavities due to rotation with an angular velocity $\dot{\vartheta}$ exhibit the difference between the eigenfrequencies of the counterpropagating waves:

$$\Omega = \frac{8\pi S\dot{\vartheta}}{\lambda L}, \quad \Omega_{\rm c} = \frac{8\pi S_{\rm c}\dot{\vartheta}}{\lambda L_{\rm c}}, \quad (4)$$

where S, $S_{\rm c}$ are the projections of the areas of the main and additional cavities on the axis of rotation; and L, $L_{\rm c}$ are the perimeters of the ring cavities.

To describe the interaction of counterpropagating waves in an active medium, we will use the GRL vector theory [16, 17], which is valid in the weak field approximation with an arbitrary ratio of the widths of the homogeneous and Doppler lines.

3. Beat frequency

Consider the regime of the beating of counterpropagating waves. We assume that the beat frequency $\omega_{\rm b}$ is significantly greater than the coupling of counterpropagating waves through backscattering ($|\omega_{\rm b}| \gg |\tilde{m}_{1,2}|, |\tilde{m}_{\rm c1,c2}|$). In this case, when solving equations (2) and (3), the coupling coefficients $|\tilde{m}_{1,2}|$ and $|\tilde{m}_{\rm c1,c2}|$ can be neglected. For simplicity, we restrict ourselves to the case when the additional cavity is insensitive to rotation (the projection of the area vector on the axis of rotation $S_{\rm c}$ is zero or small).

In the regime of beatings, the time dependence of the complex amplitudes of the counterpropagating waves $|E_{1,2}|$ will be expressed as

$$E_{1,2}(t) = |E_{1,2}|\exp(\pm{\rm i}\omega t), \quad (5)$$

where the wave amplitudes $|E_{1,2}|$ are constant and $\omega = \omega_{\rm b}/2$. From equations (3) we find

$$E_{\rm 1c,2c} = \frac{k_{\rm c}\exp({\rm i}\varphi + \omega_n T_{\rm c})/T_{\rm c}}{\pm{\rm i}\omega + \Delta\omega_{\rm c}/2}E_{1,2}. \quad (6)$$

Substituting (6) into equations (2) and assuming for simplicity that the saturation coefficients $\alpha_{1,2}$ and $\beta_{1,2}$ are real values, we obtain the equation for ω:

$$\omega + \frac{A\omega}{\omega^2 + (\Delta\omega_{\rm c})^2/4} = \frac{\Omega}{2}, \quad (7)$$

where $A = {\rm Im}\{k_{\rm c}^2\exp[{\rm i}(2\varphi + \omega_n T_{\rm c})]\}$.

In (7) there are two parameters characterising the optical feedback phase: $\Phi = \omega_n T_{\rm c}$ and the phase shift φ between the reflected and transmitted waves at the coupling mirror. The optical feedback phase can be changed in the range of $0-2\pi$ when adjusting the perimeter of the additional cavity by a value of the order of the wavelength. We will further assume that $2\varphi + \omega_n T_{\rm c} = 2\pi p \pm \pi$, where p is an integer. In this case, the optical coupling between the cavities increases the losses in the main cavity and reduces the amplitudes $|E_{1,2}|$ of the intracavity fields. This optical coupling of the cavities will be called antiphase.

In the case of antiphase optical coupling, we rewrite equation (7) in the form

$$\omega - \frac{\omega_0^2\omega}{\omega^2 + (\Delta\omega_{\rm c})^2/4} = \frac{\Omega}{2}, \quad (8)$$

where $\omega_0 = k_{\rm c}/\sqrt{TT_{\rm c}}$.

From (8) it follows that in the absence of rotation ($\Omega = 0$) in the case of antiphase optical coupling, the beat frequency $\omega_{\rm b} = 2\omega$ is determined by the expression

$$\omega_{\rm b} = \pm 2\sqrt{\omega_0^2 - (\Delta\omega_{\rm c})^2/4}. \quad (9)$$

Thus, the antiphase optical coupling in the absence of nonreciprocity in the main and additional cavities (due to rotation or under the action of magnetic fields) leads to the emergence of the inequality of the frequencies of the counterpropagating waves. In other words, it can be said that optical coupling leads to the emergence of a frequency bias, which, according to (9), can take two opposite signs. This effect is called the spontaneous phase symmetry breaking of the counterpropagating waves.

From (9) it follows that spontaneous phase symmetry breaking occurs when the condition

$$k_c/\sqrt{TT_c} > \Delta\omega_c/2 \qquad (10)$$

is satisfied.

Figure 2a shows the dependence of the beat frequency $f_b = \omega_b/2\pi$ on the frequency nonreciprocity of the main ring cavity $\Omega/2\pi$. Here, the solid curve corresponds to the beat frequency $\omega_b = 2\omega$, calculated by formula (8). As can be seen from Fig. 2a, there are two regimes with beat frequencies of opposite sign. Figures 2b and 2c show the same results in narrower frequency intervals.

In the calculations, the following parameters were chosen: the perimeters of the main and additional cavities, $L = 10$ cm and $L_c = 40$ cm; losses per round-trip transit of the main and additional cavities, 0.005 and 0.002, respectively; and the amplitude transmittance of the coupling mirror, $k_c = 0.001$. With the indicated values of the parameters, in the case of antiphase optical coupling due to the spontaneous phase symmetry breaking of the counterpropagating waves, there appears a frequency bias; in accordance with formula (9), $f_b = \omega_b/2\pi = \pm 413.5$ kHz. As a result, two branches appear on the GRL frequency response (see Fig. 2a). There is a region of frequency nonreciprocities Ω in which both branches can coexist. In this case, radiation bistability occurs in the GRL, and hysteresis phenomena can be observed.

The system of equations (2) and (3) was also solved numerically in the case of antiphase optical coupling. In this case, all the parameters of the cavity were chosen the same as in the calculations using formula (8). The coefficients $\kappa_{1,2}$, describing the linear polarisability of the GRL active medium, and the saturation coefficients $\alpha_{1,2}$ and $\beta_{1,2}$ were calculated using the formulas given in [16, 17] for a ring He–Ne laser on the $3s_2-2p_4$ neon transition with a wavelength of 0.63 μm. A single-isotope laser was considered at a pressure of $p = 700$ Pa, with a Doppler linewidth of $ku/2\pi = 800$ MHz, a homogeneous transition linewidth $\gamma_{ab}/2\pi = 357$ MHz, an upper level width $\gamma_a/2\pi = 32$ MHz and a lower level width $\gamma_b/2\pi = 85$ MHz. It was assumed that the magnetic field H, which produces the splitting of the magnetic sublevels of neon, is absent ($H = 0$).

The points in Fig. 2a show the results obtained by numerically solving equations (2) and (3) with the coupling coefficients of the counterpropagating waves, $\tilde{m}_{1,2} = m\exp(-i\pi/2)$ and $m/2\pi = 100$ Hz. The excess of the gain over the threshold η was assumed to be equal to 0.5.

As noted above, the antiphase optical coupling between the cavities reduces the intensity of the intracavity field in the main cavity. With the above parameters, as shown by the numerical solution of equations (2) and (3), the intensity reduction was 20%. The results obtained allow us to conclude that the analytical solution of (8), which does not take into account the effect of the coupling of counterpropagating waves through backscattering, agrees well with the results of calculations with allowance for coupling.

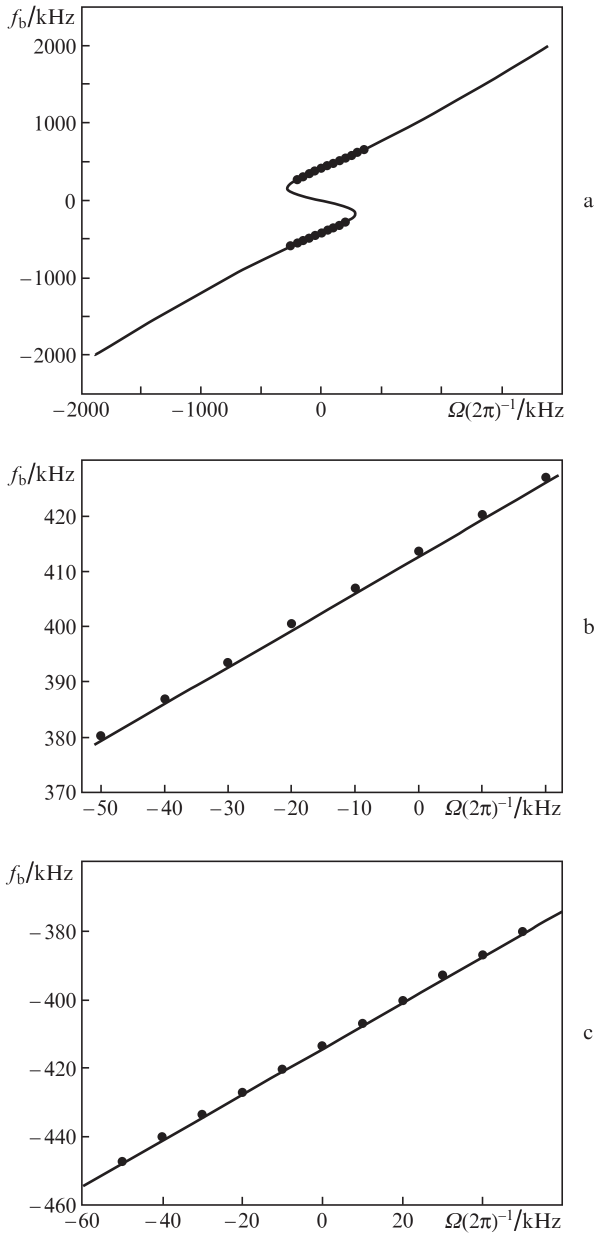

Figure 2. (a) Dependence of the beat frequency of counterpropagating waves $f_b = \omega_b/2\pi$ on the frequency nonreciprocity of the main ring cavity $\Omega/2\pi$ in the presence of bistability; (b, c) bistable regimes with opposite values of beat frequencies (shown in narrower frequency intervals): solid curves show beat frequency $\omega_b = 2\omega$ calculated using formula (8), and points are the results obtained by numerically solving equations (2) and (3); the parameter values are given in the text of the paper.

4. Conclusions

A theoretical model describing the lasing dynamics of a coupled-cavity GRL is proposed. It is shown that in the framework of this model, in the case of antiphase optical coupling of the cavities, spontaneous phase symmetry breaking of the counterpropagating waves can be observed, leading to the emergence of a frequency bias and the formation of two branches on the frequency response of the GRL. These branches correspond to two bistable regimes of beatings of counterpropagating waves with beat frequencies of opposite sign. An approximate analytical solution (8) is obtained, which is in fairly good agreement with the exact results calculated on the basis of the considered coupled-cavity GRL model. Conditions (10) are found, under which spontaneous phase symmetry of the counterpropagating waves is broken in the GRL with antiphase optical coupling of the cavities.

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