Quantum Electronics **49** (8) 773–776 (2019)

OPTICAL FIBRES

https://doi.org/10.1070/QEL16936

Effect of a magnetic field on polarisation of light in an optical fibre with a random distribution of linear birefringence

V.A. Konyshev, S.N. Lukinykh, O.E. Nanii, A.G. Novikov, V.N. Treshchikov, R.R. Ubaydullaev

Abstract. **A numerical model is proposed to describe the evolution of polarisation of a light wave propagating through a telecommunication fibre with random linear birefringence in a magnetic field. As a result of statistical processing of a set of numerical simulation results, a convenient phenomenological formula is obtained for the first time for the dependence of the average value of the polarisation rotation angle on the magnetic field, the fibre parameters and its length. It is found that the average value of the polarisation rotation angle in a long telecommunication fibre in the representation of the Stokes vectors linearly depends on the applied longitudinal magnetic field (as in the classical Faraday effect for an isotropic medium) but is proportional to the root of the fibre length. It is theoretically shown and experimentally confirmed that the polarisation rotation angle for an extended segment of a telecommunication fibre (50 km) is two orders of magnitude less than that for an isotropic fibre of the same length and material.**

Keywords: *Faraday effect, standard single-mode fibre, SSMF, beat length, correlation length, Jones matrix, polarisation state, Stokes vector, polarimeter.*

1. Introduction

The effect of a magnetic field on polarisation of light in an isotropic medium, first discovered by M. Faraday in 1845, is well studied and widely used in polarimetry and magnetic field sensors [1–3]. In particular, under the action of a magnetic field directed along the light propagation axis the polarisation plane of linearly polarised radiation is rotated in an isotropic medium by an angle Θ, which linearly depends on the magnetic field induction B and fibre length L [4]

$$\Theta = VBL, \tag{1}$$

where the proportionality coefficient V, called the Verdet constant, is determined by the medium properties. The presence of constant linear birefringence in the medium leads to a significant change in the effect of the magnetic field on the polarisation of light [5].

In this paper, we investigate the effect of the magnetic field on the polarisation of light propagating in a telecommunication optical fibre characterised by a random distribution of linear birefringence along the fibre. As far as we know, studies on the effect of the magnetic field on the polarisation of light in such media have not previously been performed.

Our numerical model allowed us to derive a convenient phenomenological formula describing the dependence of the average value of the polarisation rotation angle on the magnetic field and fibre parameters. The results can be used to assess the impact of the magnetic field caused by a lightning stroke on the operation of high-speed fibre-optic communication lines in optical ground wires [6–8].

2. Model of fibre with random distribution of linear birefringence in a magnetic field

Rectilinear fibre of an ideal cylindrical shape, made of an isotropic material, in the absence of an external physical impact has circular symmetry and is isotropic. However, in the production of fibre and optical cable, there arises birefringence with random characteristics along the fibre [9–11]. Birefringence in fibre is usually characterised by the beat length

$$L_{\mathrm{B}} = 2\pi/\Delta\beta, \tag{2}$$

(in our case, $\Delta\beta$ is the difference in the propagation constants for the slow and fast modes, averaged over the fibre length [12]).

Consider the model of a telecommunication fibre in which the birefringence value is constant, and the orientation of the principal axes changes randomly along the fibre. Strictly speaking, both the orientations of the principal axes of birefringence f and s (fast and slow) and the birefringence value change randomly. However, as shown in work [9, 13], statistical polarisation properties of fibre can be adequately described using a model in which the linear value of birefringence is fixed, and only the orientation of the slow and fast axes of the birefringent element changes randomly.

In the model under study, the fibre of length L has M sections of equal size, $\Delta z = L/M$. To account for the impact of the magnetic field on the polarisation of light, we use the split-step Fourier method [14] (Fig. 1). With this aim in view, each section is represented by two subsections, the first of which describes weak linear birefringence caused by the imperfection of the cylindrical profile of fibre, and the second corresponds to weak circular birefringence induced by an external

V.A. Konyshev, S.N. Lukinykh, A.G. Novikov, R.R. Ubaydullaev T8 Ltd., ul. Krasnobogatyrskaya 44, stroenie 1, office 826, 107076 Moscow, Russia, e-mail: rru@t8.ru;
O.E. Nanii T8 Ltd., ul. Krasnobogatyrskaya 44, stroenie 1, office 826, 107076 Moscow, Russia; Faculty of Physics, M.V. Lomonosov Moscow State University, Vorob'evy Gory, 119991 Moscow, Russia;
V.N. Treshchikov T8 Ltd., ul. Krasnobogatyrskaya 44, stroenie 1, office 826, 107076 Moscow, Russia; Kotel'nikov Institute of Radio Engineering and Electronics (Fryazino Branch), Russian Academy of Sciences, pl. Akad. Vvedenskogo 1, 141701 Fryazino, Moscow region, Russia

Received 18 December 2018; revision received 2 April 2019
Kvantovaya Elektronika **49** (8) 773–776 (2019)
Translated by M.A. Monastyrskiy

magnetic field due to the Faraday effect. Then the Jones matrix of the mth section ($m = 1, 2, ..., M$) is defined as $J_m = FG_m$, where the matrix G_m describes weak internal linear birefringence of the fibre, and the matrix F describes circular birefringence. Thus, the resulting Jones matrix of the entire fibre is

$$J = J_M J_{M-1} ... J_2 J_1 = FG_M FG_{M-1} ... FG_2 FG_1. \tag{3}$$

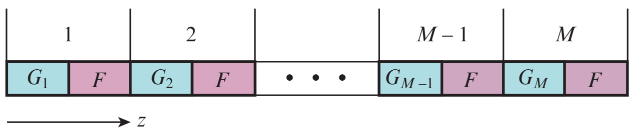

Figure 1. Splitting of the fibre into alternating subsections G and F describing linear and circular birefringence.

In general case, the Jones matrix G_m in the Cartesian coordinate system with x, y axes is described by two parameters: the difference between the phase shifts of the components linearly polarised along the principal axes of the element, $\psi = \psi_s - \psi_f$, and the angle α_m between the slow axis s of the element and the axis x of the Cartesian coordinate system [15]. Then the Jones matrix is expressed as

$$G_m = R_m G R_m^{-1}, \tag{4}$$

where

$$R_m = \begin{pmatrix} \cos\alpha_m & \sin\alpha_m \\ -\sin\alpha_m & \cos\alpha_m \end{pmatrix} \text{ and } G = \begin{pmatrix} \exp(i\psi/2) & 0 \\ 0 & \exp(-i\psi/2) \end{pmatrix}$$

are the matrix of rotation by the angle α_m and the matrix of the birefringent element, respectively; and $\psi = 2\pi z/L_B$ is the difference between the phase shifts.

The random orientation distribution of the principal axes of sections in the model is described by a random process with a white noise spectrum [13]. Each fibre section is rotated relative to the preceding one around the z axis by a random angle $\Delta\alpha_m$ obeying a normal distribution with dispersion σ and zero mean value. The dependence of the probability density on the angle $\Delta\alpha_m$ has the form:

$$f(\Delta\alpha_m) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(\Delta\alpha_m)^2}{2\sigma^2}\right). \tag{5}$$

Thus, an individual random fibre sample $(\Delta\alpha_1, \Delta\alpha_2, ..., \Delta\alpha_M)$ is formed. The orientation angles of the section axes are related to the relative angles of the section rotation by the expression

$$\alpha_m = \sum_{i=1}^{m} \Delta\alpha_i, \quad m = 1, 2, ..., M. \tag{6}$$

It can be shown that if the square of dispersion is proportional to the section size ($\sigma^2 \sim \Delta z$) and the section sizes are smaller than the characteristic sizes of the change in light polarisation, the statistical properties of fibre do not depend on the section size; i.e., in the limiting case, when the section length Δz tends to zero, the dispersion also tends to zero, and the square of dispersion remains proportional to the section length:

$$\sigma^2 = \aleph \Delta z. \tag{7}$$

Wai and Menyuk [13] introduced the correlation length $L_C = 1/(2\aleph)$ through the proportionality coefficient $\aleph$. Our numerical experiments, as well as the calculations performed in work [13], have shown that the correlation length L_C introduced in this way coincides with high accuracy with the widely used depolarisation length employed for describing the properties of optical fibres with random birefringence, measured with respect to the local polarisation eigenstates (see Appendix). The obtained relationship between the correlation and depolarisation lengths is of great importance, since the latter can be measured experimentally, while we know nothing about the existing methods for direct measurement of the correlation length.

In the presence of a magnetic field, the Jones matrix F describing the circular birefringence induced by the magnetic field due to the Faraday effect has the form [15]

$$F = \begin{pmatrix} \cos(\varphi/2) & -\sin(\varphi/2) \\ \sin(\varphi/2) & \cos(\varphi/2) \end{pmatrix}, \tag{8}$$

where $\varphi = VB\Delta z$ is the phase difference between the left and right circular polarisations in a section of the length Δz.

Since the light polarisation at the fibre output can be arbitrary, it is convenient to describe its change under the magnetic field action in terms of the Stokes vectors. If depolarisation is neglected, the Stokes vector can be represented by a point on the Poincaré sphere, and is determined by three components: $\boldsymbol{s} = (S_1, S_2, S_3)$, which are uniquely defined as [15]

$$S_1 = E_x^2 - E_y^2, \quad S_2 = 2E_x E_y \cos\delta, \quad S_3 = 2E_x E_y \sin\delta, \tag{9}$$

where E_x and E_y are the amplitudes of the x and y components of the Jones vector of light in fibre; and δ is the phase difference between the y and x components of the Jones vector. With a fixed polarisation state at the input and a change in the magnetic field from zero to B, the output Stokes vector $\boldsymbol{s}_{out}(B)$ moves along a certain trajectory on the Poincaré sphere.

We define the cumulative angle as an angle numerically equal to the trajectory length of the Stokes vector on the Poincaré sphere, averaged over the family of random fibre samples as the magnetic field changes:

$$\theta = \left\langle \sum_{n=1}^{N} \Delta\theta_n^k \right\rangle_k, \tag{10}$$

where $\Delta\theta_n^k$ is the angle between successive Stokes vectors in the kth sample with a stepwise increase in the field (n is the step number, and N is the total number of steps). The angle $\Delta\theta$ between the Stokes vectors $\boldsymbol{s}_A$ and $\boldsymbol{s}_B$ is calculated by the formula:

$$\Delta\theta = \arccos(\boldsymbol{s}_A, \boldsymbol{s}_B) = \arccos(S_{1A}S_{1B} + S_{2A}S_{2B} + S_{3A}S_{3B}). \tag{11}$$

In an isotropic medium, the cumulative angle θ defined by formula (10) is related to the rotation angle Θ of the polarisation plane of linearly polarised radiation, which is included in the classical formula for the Faraday effect (1):

$$\theta = 2\Theta. \tag{12}$$

Figure 2 shows the dependences of the cumulative angle on the optical fibre length, obtained by numerical simulation with averaging over the number of samples equal to 20 and 400. The following parameters were used in the calculations: fibre length, L = 50 km; section length, Δz = 0.1 m; correlation length, L_C = 1 m [11]; beat length, L_B = 23.5 m, magnetic field induction, B = 4.6 mT; and Verdet constant for telecommunication fibre, V = 0.53 rad T^{-1} m^{-1}.

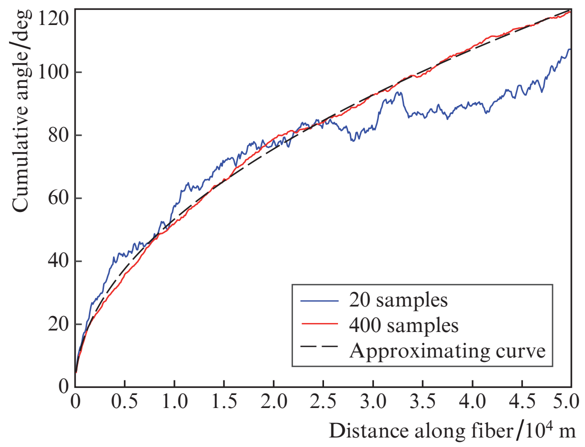

Figure 2. Model dependence of the cumulative angle on the fibre length in accordance with formula (13) (approximating curve) and averaged dependences obtained by numerical simulation.

Averaging over a larger number of samples gives an approximation dependence, which at $z \gg L_B, L_C$ can be described with a high degree of accuracy by the formula

$$\theta_{app} = \frac{1}{2\pi} VBL_B \left(\frac{z}{L_C}\right)^{1/2}. \qquad (13)$$

The root-like dependence on the coordinate z is characteristic of processes with random variation of parameters. For example, polarisation mode dispersion behaves in a similar way [9]. In a numerical experiment, it was found that for fixed values of VB, L_B, and L_C, the dependences of the cumulative angle on z can be accurately described by the expression $\theta_{app} = f(VB, L_B, L_C)\sqrt{z}$. The dependence of the function f on the parameters VB, L_B, and L_C was assumed to be a power law: $f(VB, L_B, L_C) = a(VB)^{\alpha} L_B^{\beta} L_C^{\gamma}$. As a result of numerical simulation, four fitting parameters were determined: $a = 1/2\pi$, $\alpha = 1, \beta = 1$, and $\gamma = -1/2$. The value of the coefficient a coincides with the value of $1/2\pi$ with an error of 10^{-3}.

3. Experimental setup and discussion of the results

To study the effect of an external longitudinal magnetic field on the state of the light polarisation at the fibre output, we assembled a setup, the schematic of which is shown in Fig. 3. Continuous-wave optical radiation (λ = 1550 nm) of a laser (ITLA 1.5 μm, NeoPhotonics) was fed through a segment of a polarisation-maintaining (PM) fibre and an optical connector C into the coil of a standard single-mode SSMF fibre with a length of 50 km. A longitudinal magnetic field in the fibre was formed using a solenoidal coil S (the number of turns, N = 30), through which a constant electric current was passed from a power source (PS). The current varied from zero to the

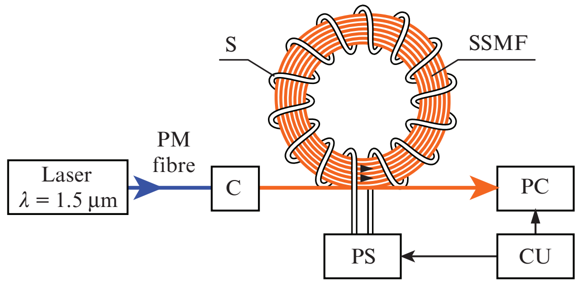

Figure 3. Scheme of the experimental setup.

maximum value I_{max} = 80 A with a step of 5 A and a time step of 5 s. The current of 80 A corresponded to the field B = 4.6 mT. The components of the Stokes vector of the output were measured using a polarisation controller (PC) (New Ridge Technologies, NRT 2550 model). The control unit (CU) set the coil current and controlled the polarisation controller operation, i.e. turned on and recorded the readings at each step of the current change. Thus, the polarisation state dependence of the output light on the coil current (longitudinal magnetic field in the fibre) was recorded.

The evolution of polarisation of the output radiation with a change in the magnetic field in both physical and numerical experiments can be conveniently represented as a trajectory on the Poincaré sphere. For each trajectory, successive rotation angles of the Stokes vectors $\Delta\theta$ were calculated by formula (11).

Figure 4 presents the experimentally and numerically obtained trajectories of the state of polarisation (SOP) of light at the output of a 50 km long fibre on the Poincaré sphere with increasing magnetic field. Five trajectories are shown, designated for the experiment and numerical simulation. To determine the cumulative angle, in physical experiment and numerical simulation averaging was performed over 1000 trajectories.

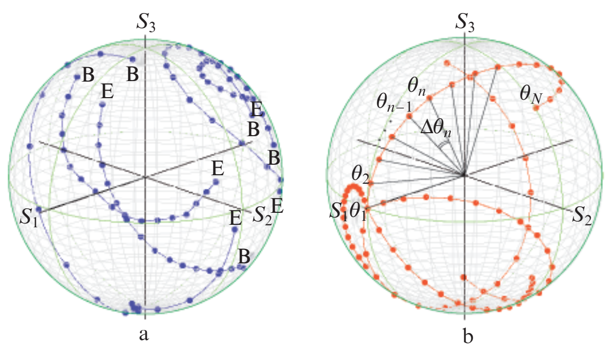

Figure 4. Mapping of SOP trajectories on the Poincaré sphere: (a) experiment [points of the zero field B (begin) and the maximum field E (end) are marked]; (b) numerical simulation (all curves start from point S_1).

Figure 5 shows the processed data of the experiment and numerical simulation. Vertical bars show the standard mean-square deviation σ_θ obtained during averaging. It should be noted that σ_θ does not tend to zero as the number of averaging samples increases. In this case, a linear dependence of the cumulative rotation angle on the current/field is observed.

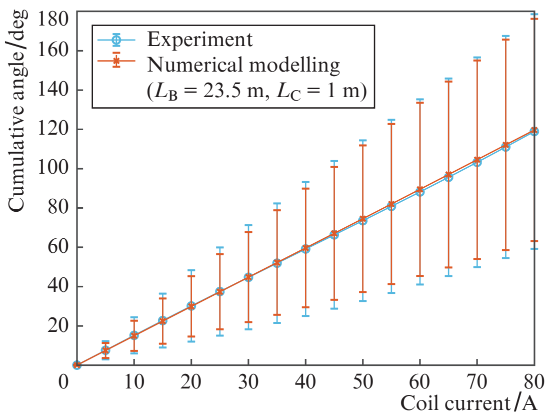

Figure 5. (Colour online) Experimental and model dependences of the average cumulative angle and its standard deviation on the coil current.

One can see a good agreement between the experimental and numerical results both in the behaviour of the dependence itself and in terms of the dispersion value.

Thus, for the first time, the effect of the magnetic field on the polarisation of light propagating in a telecommunication fibre has been studied, and a phenomenological analytical expression for the cumulative angle has been obtained.

The analytical expression is obtained as a result of statistical processing of a set of numerical simulation data on the polarisation characteristics of a telecommunication fibre with random anisotropy. According to the expression obtained, the polarisation state's rotation angle linearly depends on the magnetic field, while the length dependence is root-like. It was established and confirmed experimentally that in a long (50 km) telecommunication fibre, the cumulative rotation angle of the polarisation state is two orders of magnitude less than the corresponding angle for an isotropic fibre of the same length and material.

Appendix. Depolarisation length determination

According to work [10, 11, 13], the depolarisation length L_E is the distance at which information about the initial polarisation of a light wave is partially lost. Numerically, L_E is defined as the length at which the relation

$$\langle p_x(L_E)\rangle - \langle p_y(L_E)\rangle = 1/e^2 \tag{14}$$

is satisfied. Here, $\langle p_x(z)\rangle$ and $\langle p_y(z)\rangle$ are the fibre-ensemble averaged dependences of the normalised powers of two orthogonal linearly polarised components of a light wave along the z length. Normalisation is performed to the total power in both polarisations, so that $p_x(z) + p_y(z) = 1$. At the fibre input ($z = 0$), polarisation is determinate: the light is linearly polarised along the x: axis: $p_x(0) = 1$, $p_y(0) = 0$ (Fig. 6). At all other points with the $z \neq 0$ coordinates, polarisation is not completely determinate and is characterised by an average value. This definition of the depolarisation length was introduced by Kaminow in 1981 and has since been widely used to describe the polarisation characteristics of a telecommunication fibre with a random distribution of linear birefringence [10].

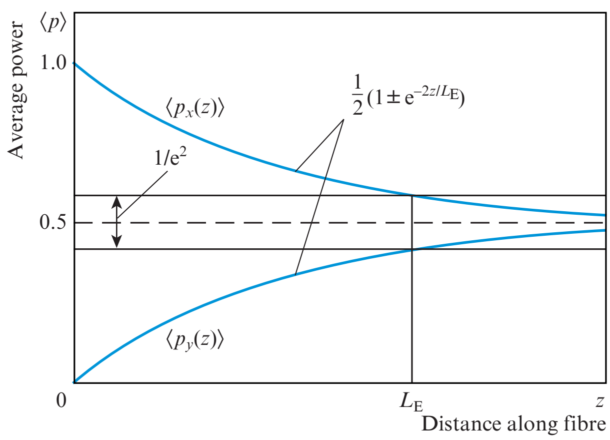

Figure 6. Determination of the depolarisation length of optical fibre [11] according to formula (14).

References

1. Maxwell J.C. *A Treatise on Electricity and Magnetism* (Oxford: Clarendon Press, 1873) Vols 1, 2.
2. Verdet E. *Recherches sur les propriétés optiques développées dans les corps transparents par l'action du magnétisme* (Paris: Mallet-Bachelier, 1854).
3. Aftab A., Salman R., Anwar S. *Phase Sensitive Faraday Rotation* (Lahore: LUMS School of Science and Engineering, 2016).
4. Mansuripur M. *Field, Force, Energy and Momentum in Classical Electrodynamics* (Sharjah, UAE: Bentham e-books, 2017).
5. Zapasskii V.S., Kozlov G.G. *Usp. Fiz. Nauk*, **169** (8), 909 (1999).
6. Waddy D.S. et al. *IEEE Photon. Technol. Lett.*, **13** (9), 1035 (2001).
7. Pietralunga S.M. et al. *IEEE Photon. Technol. Lett.*, **16** (11), 2583 (2004).
8. Krummrich P.M. et al. *Opt. Express*, **24** (11), 12442 (2016).
9. Wai P., Menyak C. *J. Lightwave Technol.*, **14** (2), 148 (1996).
10. Kaminow I. *IEEE J. Quantum Electron.*, **17** (1), 15 (1981).
11. Kogelnik H., Jopson R.M., Nelson L.E., in *Optical Fiber Telecommunications IV-B* (San Diego: Elsevier, 2002) pp 725–861.
12. Galtarossa A. et al. *Opt. Lett.*, **26** (13), 962 (2001).
13. Wai P., Menyuk C. *Opt. Lett.*, **19** (19), 1517 (1994).
14. Agrawal G.P. *Nonlinear Fiber Optics* (Oxford: Academic Press, 2013).
15. Saleh B.E., Teich M.C., Saleh B.E. *Fundamentals of Photonics* (New York: Wiley, 1991) Vol. 22.