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Simulation of quantum logic by linear recording of superimposed Fourier holograms: Linda phenomenon

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Abstract. **The classical mechanism of the quantum-like cognitive phenomenon of 'Linda' is demonstrated, which operates by linear recording of Fourier superimposed holograms in a phase-conjugate 4*f*-scheme. The results of numerical experiments are presented.**

Keywords: *superimposed holograms, Fourier holography, holographic associative memory, quantum logic, quantum-like phenomena, correlation.*

Presently, the issue of the manifestation of quantum properties by classical objects, in particular, by systems of different nature (optical, cognitive, social, etc.) characterised by strong classical coherence, is being actively discussed [1 – 5]. To explain the possible mechanism of one of the quantum-like phenomena recognised in such systems, i.e. the phenomenon of Linda [6], the authors of Refs [7, 8] proposed using quantum logic. The noncommutativity of operators plays a key role in this formalism; however, the quantum nature itself, as well as quantum phenomena and mechanisms generating the logic, was not shown in [7, 8].

Pavlov and Orlov [9] considered a possible classical mechanism of the Linda phenomenon without the quantum formalism [7, 8], based on the nonlinearity of the exposure characteristics (EC) of holographic recording media (HRM) under recording Fourier superimposed holograms (SHs) in 4*f*-scheme with a phase-conjugate mirror – holographic autoassociative memory (AAM) [10, 11]. In the present work, one more classical mechanism of the phenomenon is shown, which operates under linear EC HRM, that is, without the requirement for noncommutativity.

The essence of the Linda phenomenon [6] is as follows. Respondents were told about a fictional person named Linda and offered to choose from the list of answers – who is Linda: 'Feminist' (F), 'Teller' (T) or 'Feminist Working as A Bank Teller'' ($F\&T$)? The story was constructed in such a way as to evoke obvious associations with the answer 'Feminist' and none with the 'Teller'. According to the results of the statistical processing of the responses, their probabilities are arranged in the following order: $p(F) > p(F\&T) > p(T)$, which contradicts classical logic and probability theory, i.e. the probability of conjunction of independent events cannot exceed the probability of each event.

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To implement this experiment, a holographic AAM scheme was used (Fig. 1). A multiplex hologram as a sum of two SHs recorded in the linear range of the EC HRM, stores the references F and T:

$$H_{\rm M}(\nu) = [R_F\exp(\mathrm{j}2\pi\nu x_F) + \mathrm{F}(F(x))][R_F\exp(\mathrm{j}2\pi\nu x_F) + \mathrm{F}(F(x))]^* + [R_T\exp(\mathrm{j}2\pi\nu x_T) + \mathrm{F}(T(x))] \times [R_T\exp(\mathrm{j}2\pi\nu x_T) + \mathrm{F}(T(x))]^*, \quad (1)$$

where ν is the spatial frequency; j is the imaginary unit; R_F and R_T are the amplitudes of the wavefronts from point sources $\delta_F(x_F)$ and $\delta_T(x_T)$, shifted by x_F and x_T from the main optical axis; and F and * are the Fourier transform and complex conjugation symbols.

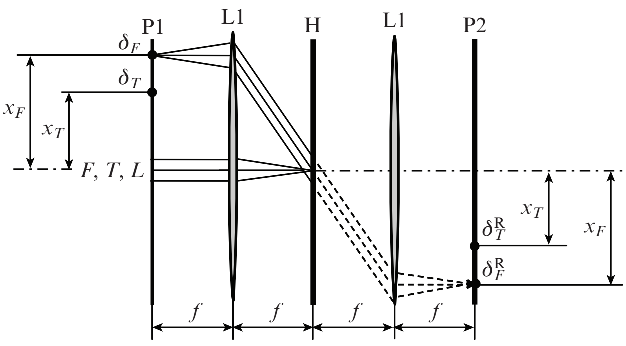

Figure 1. Holographic AAM based on Fourier holographic 4*f*-schemes: F, T are the reference images in recording superimposed holograms; L is the input image; $\delta_{\rm F}$ and $\delta_{\rm T}$ are reference point sources; L1, L2 are Fourier transform lenses; P1, H, and P2 are the planes of the images, holograms, and correlation, respectively (the phase-conjugate mirror is placed in the latter, not shown in the scheme).

Upon presentation of the image L in the input plane, the multiplex hologram (1) in the P2 plane forms the amplitude distributions described by the cross-correlation functions:

$$E = E_F(\Delta) + E_T(\Delta) = [L(x)\otimes F(x)] + [L(x)\otimes T(x)], \quad (2)$$

where $\otimes$ is the correlation operation symbol.

A phase-conjugate mirror (PCM) selects from field (2) only global maxima with amplitudes

$$E_F(\Delta)|_{\Delta=\delta_F} = k_{LF}[F(x)\otimes F(x)]\big|_{\Delta=\delta_F} = k_{LF}\mu_F^2\mathcal{S}_F = k_{LF}P_F,$$
$$E_T(\Delta)|_{\Delta=\delta_T} = k_{LT}[T(x)\otimes T(x)]\big|_{\Delta=\delta_T} = k_{LT}\mu_T^2\mathcal{S}_T = k_{LT}P_T, \quad (3)$$

where k is the correlation coefficients; μ^2 are the initial moments of the second order; S is the area; and P is the power of the corresponding images. Upon phase conjugation in the P2 plane, hologram (1) in the reverse direction of the rays in the P1 plane restores the reference images:

$$\begin{aligned} F^{\mathrm{R}}(x) &= k_{LF}\mu_F^2 S_F F(x) = k_{LF} P_F F(x), \\ T^{\mathrm{R}}(x) &= k_{LT}\mu_T^2 S_T T(x) = k_{LT} P_T T(x). \end{aligned} \tag{4}$$

The ratio of the powers of the registered reconstructed images, taking into account the generally nonlinear dependence of the sensitivity of the sensor on the power $S(P)$, has the form

$$V_{FT} = \frac{S\langle F^{\mathrm{R}}(x), F^{\mathrm{R}}(x)\rangle}{S\langle T^{\mathrm{R}}(x), T^{\mathrm{R}}(x)\rangle} = \frac{S\langle [k_{LF} P_F F(x)], [k_{LF} P_F F(x)]\rangle}{S\langle [k_{LT} P_T T(x)], [k_{LT} P_T T(x)]\rangle}, \tag{5}$$

where the angle brackets are the scalar product symbol.

Consider (4) and (5) in terms of the experiment [6]. The probability in quantum physics is defined as the scalar product of wave functions, that is, as a quadratic norm, and is mathematically equivalent to intensity (power P). The field restored by the AAM depends on the properties and the input image of Linda: correlation coefficients with the references and their powers. The correlation coefficients depend on the purely individual characteristics of internal representations of the reasoning agent and the image of Linda, given by the story [6], and the references. For many sensors, including neurons, the sigmoidal type of $S(P)$ is typical, often with an inverse region. If the relation for V_{FT} (5) falls within the relative dynamic range of the straight section of $S(P)$, then we have a conjunction ($F\&T$); otherwise, the sensor registers only one image of the two references restored by the AAM (4).

Holographic auto-associative memory (see Fig. 1) was numerically simulated by linear recording of a hologram (1) for a number of characteristics of the images and the sensor. Figure 2 shows the results when changing only one parameter, i.e. type of $S(P)$. The images contain 256×256 pixels each: the inscriptions on a black background are filled with realisations of two-dimensional fractal Brownian motion (Hurst parameter $H = 0.1$). The power ratio of the references P_F/P_T is equal to 1.943, and the ratio of the correlation coefficients k_{LF}/k_{LT} is equal to 2. Three types of $S(P)$ dependences were used: sigmoidal $S(P, a, b, d) = \{1 + \exp[a(-P/b + 1)]\}^{-1} - d$, where a, b and d are parameters, with b specifying the point $S(P) = P$, and d specifying the value of $S(0)$ (Fig. 2a); linear (Fig. 2b); sigmoidal with the inverse section $S(P, a, c, d) = P^c \exp[-(P + d)^2/(2a^2)]$ (Fig. 2c).

a b c

Figure 2. Restored images and their powers: P^{R} = (a) 2.469×10^8, (b) 1.328×10^8, and (c) 9.642×10^7. In the latter case, only a few pixels were restored from the F image, and so it cannot be recognised.

Figure 2b shows the $F\&T$ conjunction. Thus, $P^{\mathrm{R}}(F) > P^{\mathrm{R}}(F\&T) > P^{\mathrm{R}}(T)$, which fully coincides with the result of [6], i.e., we clearly see the realisation of the Linda phenomenon by the classical method, without reference not only to quantum physics in essence, but also to the quantum formalism. Note that the holographic AAP model is biologically motivated. We also emphasise that the presented result should not be interpreted as a negation, in principle, of the very possibility of the presence of quantum phenomena and mechanisms in macrosystems, including cognitive and social ones, but only as a search for the simplest explanation of experimental results adequate to physical reality.

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