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Development and numerical simulation of spherical microresonators based on SiO_2–GeO_2 germanosilicate glasses for generation of optical frequency combs

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Abstract. **We propose and theoretically investigate the possibility of using spherical, whispering gallery mode microresonators based on germanosilicate glasses with different GeO_2 contents for generating optical frequency combs in the dissipative soliton regime under pumping at a wavelength of 1.55 or 2 μm. The dispersion and nonlinearity of microspheres of different radii are calculated and analysed, and their optimal characteristics and the expected parameters of the output radiation are determined. It is shown that the spectral widths of optical frequency combs formed in $0.8SiO_2$–$0.2GeO_2$ and GeO_2 glass microspheres pumped at λ = 1.55 and 2 μm can be ~200 and ~300 nm, respectively. In these cases, in addition to the dissipative soliton with a duration of ~100 fs, the generation of dispersive waves is also observed.**

Keywords: *microresonator, microsphere, whispering gallery modes, optical frequency combs, dispersion, Kerr nonlinearity, germanosilicate glasses.*

1. Introduction

Optical frequency combs (OFCs), periodic trains of ultrashort laser pulses with equidistantly spaced spectral lines, are used in various applications, including metrology, spectroscopy, probing, remote diagnostics, and have a huge impact on science and technology [1]. OFCs formed on the basis of mode-locked lasers are employed used in scientific laboratories. Discovered in 2007, Kerr frequency combs generated in high-quality optical whispering gallery mode (WGM) microresonators [2] can significantly reduce the size and power consumption of optical devices and design new types of devices with unprecedented characteristics (resolution, speed, compactness, and power consumption). In recent years, significant progress has been made in the development of compact microresonators on microchips for generating OFCs in the dissipative soliton regime. For the existence of dissipative solitons, a double balance must be satisfied – the Kerr nonlinearity and anomalous dispersion, as well as dissipation and amplification [3]. In this connection, in developing microresonators for generating dissipative solitons, special attention is paid to dispersion, to which both the material and the geometric components contribute [4]. As materials for microresonators, use can be made, for example, of crystalline materials [5] or various glasses, the simplest of which is silica [6].

In the present work, for the first time, as far as we know, we theoretically investigate the possibility of using germanosilicate glass microspheres to generate optical combs (in the dissipative soliton regime). Special attention is paid to the study of the nonlinearity and dispersion of such microresonators, since these parameters have a strong influence on the nonlinear dynamics of the system. Note that the use of microspheres based on $(1-x)SiO_2-xGeO_2$ with $x \approx 0.2$ was previously demonstrated experimentally for a precision frequency shift under the action of UV radiation [7] and for narrowband filters [8]. The currently existing technologies allow the production of high-quality $(1-x)SiO_2-xGeO_2$ glasses with any germanium-dioxide content x ($0 \leqslant x \leqslant 1$) [9, 10]. Germanosilicate glasses have a higher nonlinear refractive index than that of silica and zero dispersion shifted to the long wavelength region. At the same time, according to the thermophysical properties, germanosilicate glasses are close to silica ones, which makes it possible to apply to them well developed technologies for the manufacture of glass microresonators, for example, by heating the end of the optical fibre. Note also that so far, germanosilicate microresonators have been manufactured by applying a film of germanate glass on the surface of a silica microsphere [7]. In silica glasses at wavelengths greater than 2.2 μm, optical losses are large, which limits the range of their use. The transparency region of $(1-x)SiO_2-xGeO_2$ glasses for large values of x goes beyond 3 μm [9], which determines their potential applicability both in the telecommunication range and in the wavelength range of 2–3 μm. An encouraging fact is that currently the generation of tunable solitons in the range of 2–3 μm [11] and supercontinuums with a long wavelength boundary of 3 μm and more [12–16] has been demonstrated in germanosilicate fibres.

In Section 2 of this work, we numerically investigate the dispersion and nonlinear properties of spherical WGP microresonators based on germanosilicate glasses with different contents of germanium dioxide ($0 \leqslant x \leqslant 1$) and different radii, and in Section 3 we model the generation of dissipative solitons pumped at wavelengths 1.55 and 2 μm.

2. Calculation of dispersion and nonlinear characteristics

The eigenfrequencies of the WGM microresonators can be found by numerically solving the characteristic equation, which is obtained on the basis of Maxwell's equations for a

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given system [1]. The characteristic equation for spherical microresonators for TM modes is [1, 17]

$$\frac{[(kR)^{1/2}J_{l+1/2}(kR)]'}{(kR)^{1/2}J_{l+1/2}(kR)} = n\frac{[(k_0R)^{1/2}H^{(1)}_{l+1/2}(k_0R)]'}{(kR)^{1/2}H^{(1)}_{l+1/2}(k_0R)}, \quad (1)$$

and for TE-type waves it has the form

$$\frac{[(kR)^{1/2}J_{l+1/2}(kR)]'}{(kR)^{1/2}J_{l+1/2}(kR)} = \frac{1}{n}\frac{[(k_0R)^{1/2}H^{(1)}_{l+1/2}(k_0R)]'}{(k_0R)^{1/2}H^{(1)}_{l+1/2}(k_0R)}, \quad (2)$$

where the prime means the total derivative with respect to the argument; $k = k_0n$ is the propagation constant in the medium; $k_0 = 2\pi\nu/c$ is the propagation constant in vacuum; c is the speed of light; ν is the radiation frequency; λ is the wavelength ($\lambda = c/\nu$); $J_{l+1/2}$ is the Bessel function of order $(l + 1/2)$; $H^{(1)}_{l+1/2}$ is the Hankel function of the first kind of order $(l + 1/2)$; R is the radius of the microsphere; n is the refractive index; and l is the mode index, which coincides with the azimuthal index for the WGM. An index q is also introduced, indicating the root index of the characteristic equation. We considered fundamental modes with $q = 1$.

The dependence of the refractive index n of $(1 - x)SiO_2{-}xGeO_2$ germanosilicate glasses on the wavelength λ was determined according to the model [18]:

$$n^2 = 1 + \sum_1^3 \frac{[SA_i + x(GA_i - SA_i)]\lambda^2}{\lambda^2 - [Sl_i + x(Gl_i - Sl_i)]^2}, \quad (3)$$

where x is the mole fraction of GeO_2; $i = 1{-}3$; SA_i, Sl_i are the Sellmeier coefficients for SiO_2, and GA_i, Gl_i are the Sellmeier coefficients for GeO_2: $SA_1 = 0.6961663$ μm, $Sl_1 = 0.0684043$ μm, $SA_2 = 0.4079426$ μm, $Sl_2 = 0.1162414$ μm, $SA_3 = 0.8974794$ μm, $Sl_3 = 9.896161$ μm, $GA_1 = 0.80686642$ μm, $Gl_1 = 0.068972606$ μm, $GA_2 = 0.71815848$ μm, $Gl_2 = 0.15396605$ μm, $GA_3 = 0.85416831$ μm, and $Gl_3 = 11.841931$ μm [18].

We developed a numerical code to find the roots of the characteristic equation and determine the eigenfrequencies ν_l. The roots were localised by using approximation formulas for the eigenfrequencies of TM modes [17],

$$\nu_l = \frac{c}{2\pi Rn}\left[(l + 1/2) + 1.85576(l + 1/2)^{1/3} - \frac{1}{n^2}\left(\frac{n^2}{n^2 - 1}\right)^{1/2}\right], \quad (4)$$

and TE modes:

$$\nu_l = \frac{c}{2\pi Rn}\left[(l + 1/2) + 1.85576(l + 1/2)^{1/3} - \left(\frac{n^2}{n^2 - 1}\right)^{1/2}\right]. \quad (5)$$

Approximate values of ν_l were used as the initiators of the algorithm for finding the roots of the equation (the modified Powell method). The dependence of the refractive index on frequency was taken into account iteratively.

It was found that the WGM eigenfrequencies for the TE and TM modes differ slightly; therefore, all calculations are given below for the TM modes.

The coefficient of quadratic dispersion β_2 taking into account the material and geometric contributions was calculated by the formula

$$\beta_2 = -\frac{1}{4\pi^2R}\frac{\Delta(\Delta\nu_l)}{(\Delta\nu_l)^3}, \quad (6)$$

where

$$\Delta\nu_l = \tfrac{1}{2}(\nu_{l+1} - \nu_{l-1}) \text{ and } \Delta(\Delta\nu_l) = \nu_{l+1} - 2\nu_l + \nu_{l-1}. \quad (7)$$

We calculated the dispersion of $(1 - x)SiO_2{-}xGeO_2$glass microspheres with different radii R for $0 \leqslant x \leqslant 1$. Figure 1 shows the dependences of the zero dispersion wavelength λ_0, and Fig. 2 displays dependences of the coefficient β_2 on x and R at the expected pump radiation wavelengths $\lambda_p \approx 1.55$ and 2 μm. Here and below, we found the mode index corresponding to the wavelength closest to λ_p, and assumed that the wavelength of the pump laser could be adjusted as needed. We were interested in the regions of small anomalous dispersion that is necessary for the potential generation of dissipative solitons. For the fundamental mode in the WGM microresonators, the dispersion zero is shifted to the long wavelength region relative to the zero of the material dispersion, which is due to the waveguide contribution. The smaller the

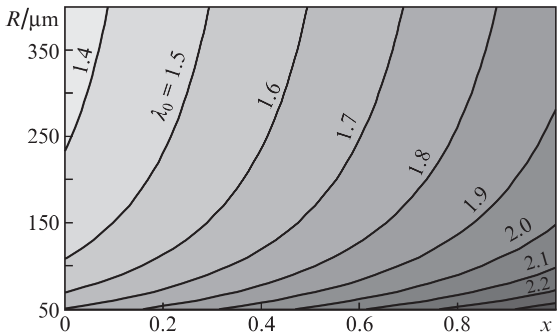

Figure 1. Zero dispersion wavelengths λ_0 (μm) as functions of the molar fraction x of germanium dioxide in glass and the radius R of the microsphere based on this glass.

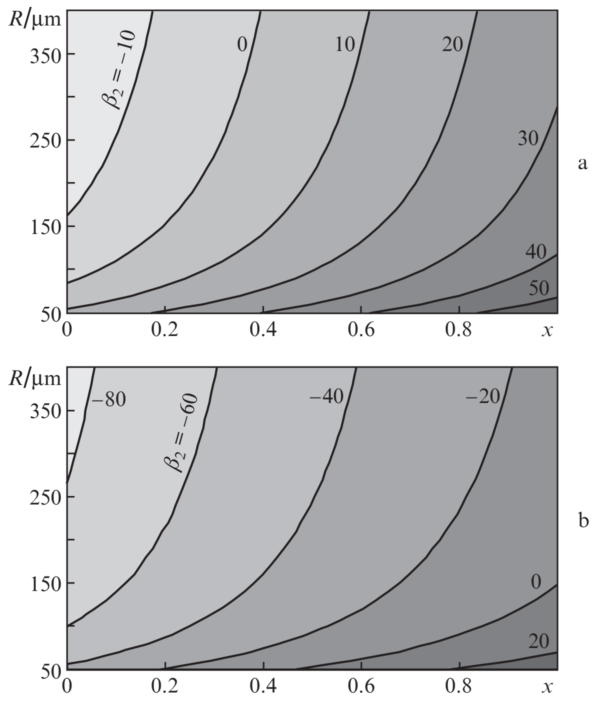

Figure 2. Quadratic dispersion coefficient β_2 (ps^2 km^{-1}) as a function of the molar fraction x of germanium dioxide in glass and the radius R of the microsphere based on this glass at a wavelength of (a) 1.55 and (b) 2 μm.

size of the microsphere, the greater the waveguide contribution to the WGM dispersion and the stronger its difference from the glass dispersion. Figures 1 and 2 show that under pumping at λ_p = 1.55 μm, microspheres should be chosen from silica glass or germanosilicate glass with a low content of germanium dioxide ($x \leqslant 0.2$), and under pumping at λ_p = 2 μm, microspheres, on the contrary, should be made of germanate glass (GeO_2) or germanosilicate glass with a high content of germanium dioxide ($x \geqslant 0.7$). In both cases, it is preferable to use microspheres with $R \approx 150-250$ μm. Figure 3 shows the calculated dispersion dependences of WGM microspheres with a radius of 200 μm for various x. Also for comparison, we present the material dispersion dependences of silica and germanate glasses (SiO_2 and GeO_2). Silica microspheres at a wavelength of 2 μm have a large anomalous dispersion, which is a factor limiting their use.

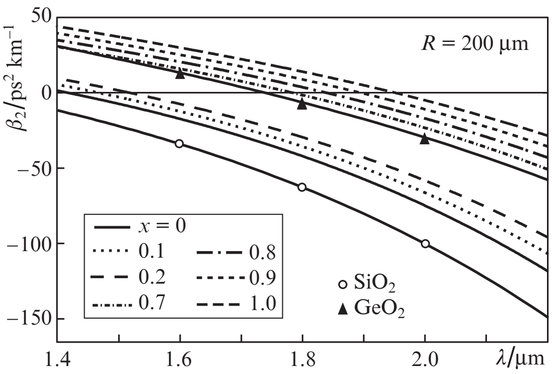

Figure 3. Spectral dependences of the quadratic dispersion coefficient β_2 for microspheres with a radius of 200 μm based on $(1-x)SiO_2-xGeO_2$ glasses with $0 \leqslant x \leqslant 1$ and SiO_2 and GeO_2 glasses.

To estimate the effective WGM volumes, we used an approximation formula [19]:

$$V_{\mathrm{eff}} \approx 3.4\pi^{3/2}\left(\frac{\lambda}{2\pi n}\right)^3 l^{11/6}. \quad (8)$$

The calculated effective WGM volumes at λ_p = 1.55 and 2 μm are shown in Fig. 4. We should note the weak dependence of V_{eff} on x.

The nonlinear coefficients γ of microspheres were determined by the formula [5]

$$\gamma = \frac{2\pi}{\lambda}\frac{n_2}{V_{\mathrm{eff}}/(2\pi R)}. \quad (9)$$

In calculating γ, we used the experimentally measured values of the nonlinear refractive index n_2 of $(1-x)SiO_2-xGeO_2$ glasses for various x [20] and approximated them with a linear function:

$$n_2 = n_2(\mathrm{SiO_2}) + xK(\mathrm{GeO_2}), \quad (10)$$

where $n_2(\mathrm{SiO_2}) = 2.2 \times 10^{-20}$ m^2 W^{-1} and $K(\mathrm{GeO_2}) = 2.9 \times 10^{-20}$ m^2 W^{-1}.

Figure 5 shows the calculated dependences of nonlinear coefficients γ on x and R at pump wavelengths λ_p = 1.55 and 2 μm. The greater the x, the greater the γ, which is caused by the dependence $n_2(x)$.

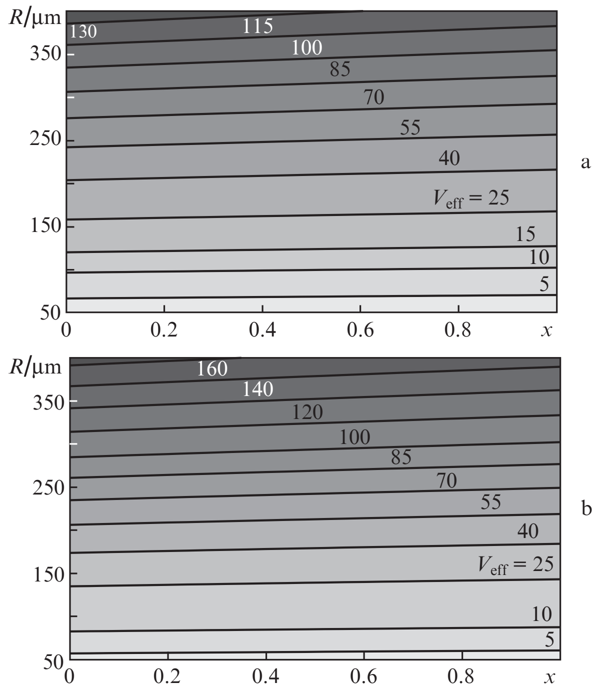

Figure 4. Effective WGM volumes V_{eff} (10^3 μm^3) as functions of the molar fraction x of germanium dioxide in glass and the radius R of the microsphere based on this glass at λ_p = (a) 1.55 and (b) 2 μm.

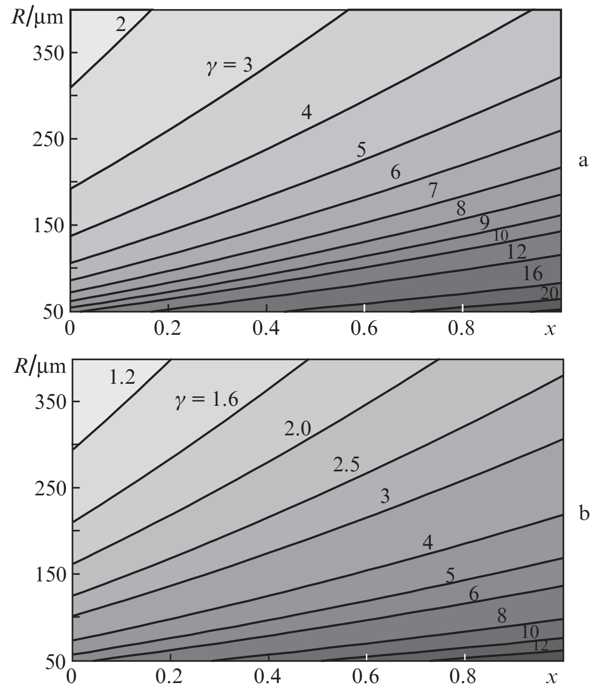

Figure 5. Nonlinear coefficients γ (W^{-1} km^{-1}) of WGM resonators as functions of the molar fraction x of germanium dioxide in glass and the radius R of the microsphere based on this glass at λ_p = (a) 1.55 and (b) 2 mm.

3. Simulation of optical frequency combs

The dynamics of the formation of optical frequency combs in microspheres based on germanosilicate glasses was modelled using the Lugiato–Lefever equation [21, 22]:

$$t_R \frac{\partial E(t,\tau)}{\partial t} = \sqrt{\theta}\, E_{\rm in} + \left[-\alpha - {\rm i}\delta_0 + {\rm i}2\pi R \sum_{k\geqslant 2} \frac{\beta_k}{k!} \left({\rm i}\frac{\partial}{\partial \tau} \right)^k + {\rm i}\gamma 2\pi R\, |E|^2 \right] E, \quad (11)$$

where $E(t,\tau)$ is the complex envelope of the field inside the microresonator; t and τ are the slow and fast times; $t_R = 2\pi Rn/c$ is the microresonator roundtrip time; $t = mt_R$; m is the microresonator roundtrip number; δ_0 is the frequency detuning of the pump field $E_{\rm in}$ from the nearest resonance; θ is the coupling coefficient; and α is the loss coefficient as the sum of intrinsic and coupling losses. In the calculations, we used the Q value of 10^7 to demonstrate the possibility of generating optical frequency combs. The loss coefficient at the pump wavelength is related with the parameters of the microsphere as follows: $\alpha = (2\pi)^2 R/(Q\lambda_{\rm p})$. We neglected the wavelength dependence of losses. The power of narrow-band pump sources both at $\lambda_{\rm p} = 1.55$ and 2 μm was assumed to be 100 mW. Characteristic detuning from the resonance, δ_0, was $\sim 10^{-3}$, and the radius R of the microspheres was 200 μm. The coupling coefficient was equal to the loss coefficient [23]: $\theta = \alpha = 5\times 10^{-4}$ and 4×10^{-4} at $\lambda_{\rm p} = 1.55$ and 2 μm, respectively. Under pumping at a wavelength of 1.55 μm, we considered the compositions of glasses with a molar fraction of germanium dioxide $x = 0-0.2$, and under pumping at 2 μm, the molar fraction of germanium dioxide in the glasses amounted to $x = 0.7-1$, in accordance with the requirements for the parameters of the microspheres, formulated in Section 2 in the analysis of dispersion properties.

In modelling equation (11) with the help of a specially developed numerical code, we used the split-step Fourier method (SSFM) using the fast Fourier transform [24].

It is known that the dynamics of the formation of optical combs can be very complex, i.e. depending on the parameters of the system, various scenarios can be implemented [3, 5, 25–27]. Here, the objects to be analysed are dissipative solitons [3, 5, 23, 28]. We are looking for stationary solutions;

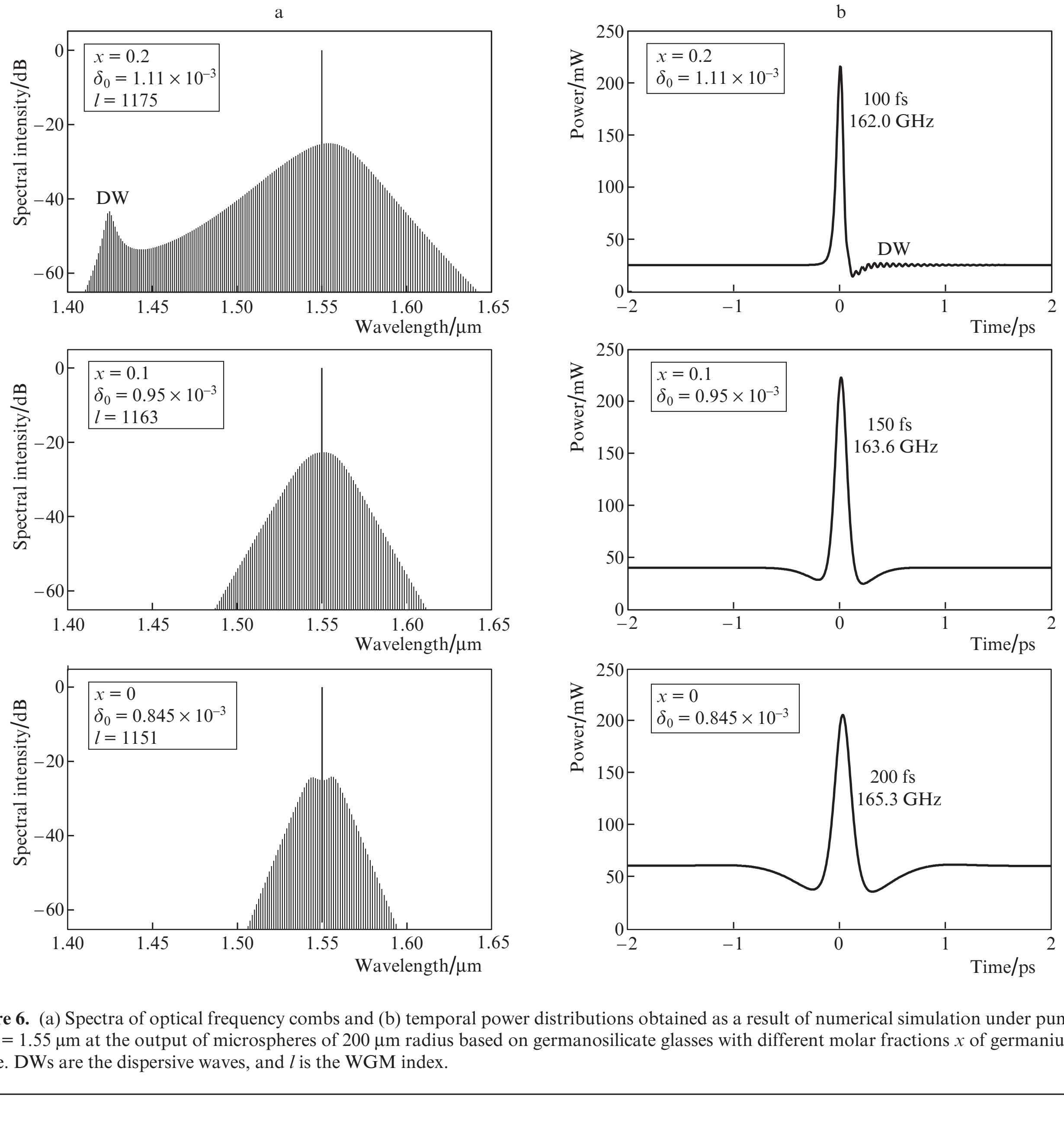

Figure 6. (a) Spectra of optical frequency combs and (b) temporal power distributions obtained as a result of numerical simulation under pumping at $\lambda_{\rm p} = 1.55$ μm at the output of microspheres of 200 μm radius based on germanosilicate glasses with different molar fractions x of germanium dioxide. DWs are the dispersive waves, and l is the WGM index.

therefore, the approximate analytical expressions given in [5] are given as initial conditions for equation (11). In the numerical simulation, we used dispersion dependences shown in Fig. 3. The calculated OFC spectra and the temporal power distribution for λ_p = 1.55 μm at x = 0.2, 0.1, 0 and for λ_p = 2 μm at x = 1, 0.9, 0.8, 0.7 are shown in Figs 6 and 7. Figures 6a and 7a show the pump wavelength and the index l of the corresponding WGM, and Figs 6b and 7b display the duration and repetition rate ($1/t_R$) of dissipative solitons in the time domain. The smaller the absolute value of the anomalous dispersion at the pump wavelength, the wider the spectrum of the dissipative soliton and the shorter its duration. Thus, the spectral width of an OFC generated in a $0.8SiO_2-0.2GeO_2$ glass microsphere with $\beta_2 = -3$ ps^2 km^{-1} at a wavelength of 1.55 μm is ~200 nm with a soliton duration of ~100 fs, while the spectral width of the OFC generated in

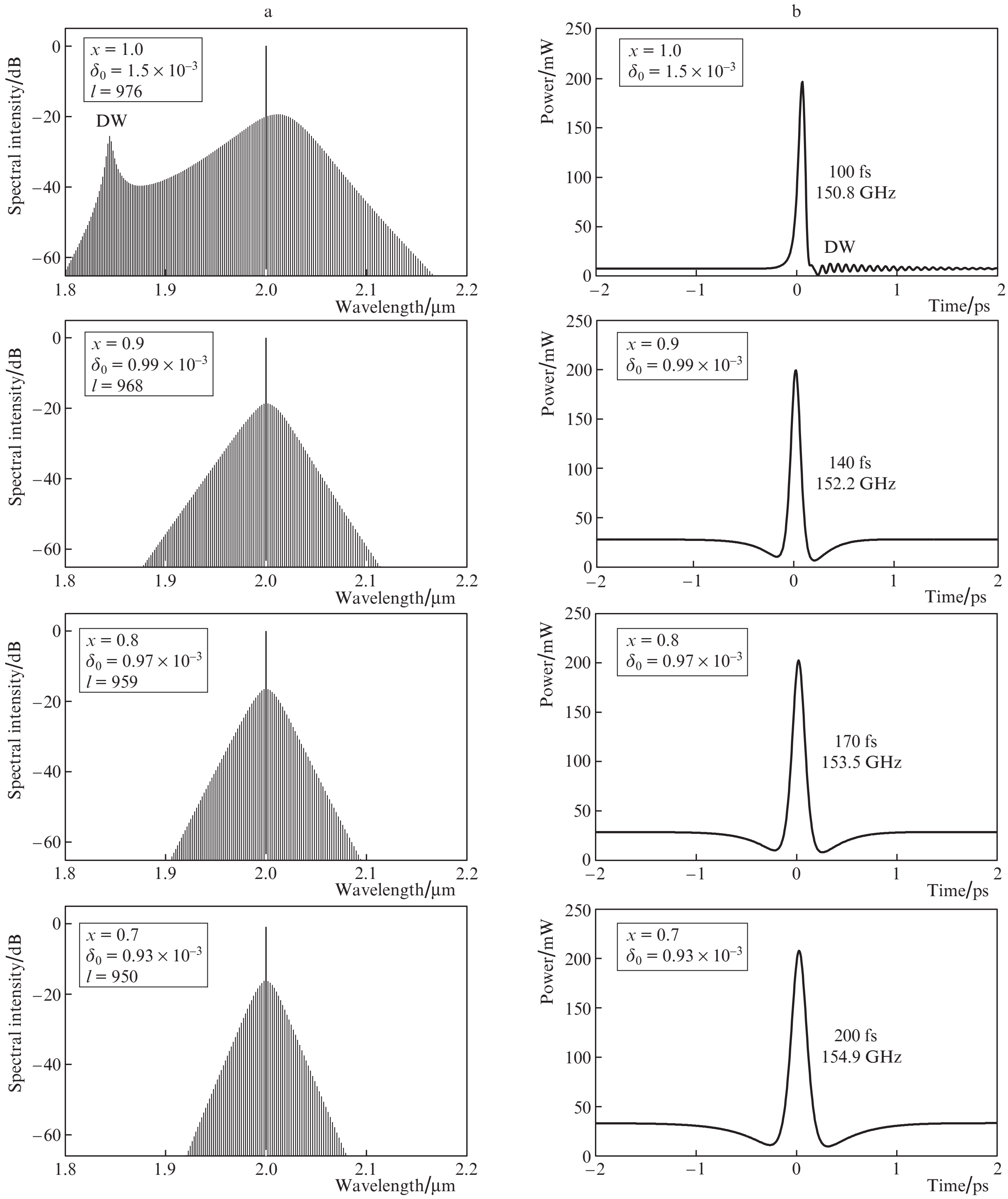

Figure 7. (a) Spectra of optical frequency combs and (b) temporal power distributions obtained as a result of numerical simulation under pumping at λ_p = 2 μm at the output of microspheres of 200 μm radius based on germanosilicate glasses with different molar fractions x of germanium dioxide. DWs are the dispersive waves, and l is the WGM index.

a silica glass microsphere with $\beta_2 = -12$ ps^2 km^{-1} at the same wavelength is less than 100 nm with a soliton duration of ~200 fs. At $\lambda_p = 2$ μm, the spectral widths of the optical frequency combs are ~300 and ~150 nm with soliton durations of ~100 and ~200 fs, the solitons being generated in microspheres made of germanate glass with $\beta_2 = -5$ ps^2 km^{-1} and of $0.3SiO_2-0.7GeO_2$ glass with $\beta_2 = -23$ ps^2 km^{-1}, respectively.

Note an interesting feature of optical frequency combs generated at $\lambda_p = 1.55$ μm, $x = 0.2$ and at $\lambda_p = 2$ μm, $x = 1$, i.e. the presence of local maxima of spectral intensities on short-wavelength wings (Figs 6 and 7). This feature has a simple explanation. It is known that if in the course of pulse evolution in a medium with Kerr nonlinearity and anomalous dispersion in the presence of cubic dispersion, the broadened wing of the spectrum falls into the region of normal dispersion, then dispersive waves are generated near the point of synchronism with a soliton. Note that this effect was first discovered and studied for optical fibres [24, 29], and later investigated for microresonators [23, 30]. In the time domain, the short-wavelength dispersive waves are located on the back front of the dissipative soliton, which is also noted in Figs 6 and 7.

4. Conclusions

We have proposed and theoretically studied the possibility of using spherical, whispering gallery mode microresonators based on $(1-x)SiO_2-xGeO_2$ germanosilicate glasses with different GeO_2 contents ($0 \leqslant x \leqslant 1$) for generating optical frequency combs in the dissipative soliton regime under pumping at a wavelength of 1.55 or 2 μm. We have calculated and analysed the dispersion and nonlinearity of microspheres of different radii and determined their optimal characteristics and the expected parameters of the output radiation. It has been shown that under pumping at a wavelength of 1.55 μm, one should choose microspheres made of silica or germanosilicate glass with a low content of germanium dioxide ($x \leqslant 0.2$), and under pumping at a wavelength of 2 μm, on the contrary, one should choose germanosilicate glass with a high content of germanium dioxide ($x \geqslant 0.7$). In both cases, it is preferable to use microspheres with a radius of 150 to 250 μm. It has been shown that, at a laser pump power of 100 mW, in optimal cases, the spectral widths of optical frequency combs generated in microspheres made of $0.8SiO_2-0.2GeO_2$ and GeO_2 glasses at a centre wavelength of 1.55 and 2 μm can be ~200 and ~300 nm, respectively. In these cases, in addition to the dissipative soliton with a duration of ~100 fs, the generation of dispersive waves is also observed. The use of microspheres based on germanosilicate glasses with optimal compositions allows one to obtain optical frequency combs with a larger spectral width than those generated by using silica microspheres.

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