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Propagation of surface waves along a dielectric layer in a photorefractive crystal with a diffusion mechanism for the nonlinearity formation

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Abstract. **It is shown that TM-polarised surface waves, which exhibit a different character of their attenuation, can propagate along a dielectric layer separating photorefractive crystals with a diffusion mechanism of the nonlinearity formation. The wave profiles can be symmetric and antisymmetric with respect to the centre of the three-layer structure. For all considered wave types, dispersion equations are derived. The dependences of the propagation constant on the characteristics of the layered structure are found for long-wavelength surface waves in an explicit analytical form, and the conditions for their existence are revealed. The influence of the temperature and thickness of the dielectric layer on the propagation regimes of surface waves and their characteristics is analysed.**

Keywords: *surface waves, photorefractive crystal, photorefractive diffraction grating, propagation constant, attenuation coefficient, layered media.*

1. Introduction

Many optical devices (sensors, triggers and waveguides) enjoy the use of both waveguide and control properties of media interfaces in multilayer structures that allow light beams with wavelengths of specified ranges to be transmitted or blocked [1–4]. Among the materials used in such devices, photorefractive crystals are of particular importance due to a number of their specific properties [5–9]. Strontium barium niobate (SBN) crystals, sillenite family crystals ($Bi_{12}SiO_{20}$, $Bi_{12}TiO_{20}$, $Bi_{12}GeO_{20}$), $LiTaO_3$, $BaTiO_3$, $KNbO_3$, $Fe:LiNbO_3$ and other crystals are usually used as materials with a photorefractive effect [10–12].

In many optoelectronic devices based on the use of such materials, controlled localisation along the interfaces of the layers of optical radiation energy and mechanisms for varying its spatial and spectral characteristics play an important role. The possibilities of controlling such nonlinear optical phenomena and the prospects of practical application for optical information processing give rise to an interest in the theoretical study of the unique properties of surface waves (SWs) [13–15] propagating along the interfaces of the crystal, one of which exhibts a photorefractive effect.

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A theoretical study of the laws governing the propagation of SWs excited under various conditions at the photonic crystal–dielectric interface has been carried out repeatedly [16–23]. Nevertheless, a detailed analysis is required of the peculiarities of the SW propagation in heterostructures with alternating nonlinear optical effects in the layers. In this regard, the present work proposes a theoretical description of new types of SWs propagating along a three-layer structure, which is a photonic crystal with a dielectric layer of finite thickness.

Significant differences in the electro-optical responses of the media of neighbouring layers, caused, for example, by the induction of a field in one of the layers due to the redistribution of charge density, lead to the possibility of the appearance of fundamentally new features of the formation of the SW profile, the amplitude of which can oscillatory decay when moving away from the interface into the outer layers.

In this work, we study only a symmetric three-layer structure, the outer layers of which consist of a photonic crystal with the same optical characteristics. By virtue of symmetry, such a system can produce SWs with a field profile distribution characterised by a certain symmetry with respect to the layer interfaces, in particular in-phase and out-of-phase, described by even and odd solutions, respectively. Such SWs will differ from the waves described in [21], propagating in a combined waveguide of a photonic crystal with a dielectric film on its surface. In contrast to [23], the present work focuses on the analysis of the existence conditions and characteristics of TM-polarised SWs attenuating in two different regimes for the case of a finite layer thickness, and also on obtaining the dispersion laws for a thin layer in an explicit analytical form.

2. Model equations

We consider the propagation of nonlinear, extraordinarily polarised SWs (TM-, or p-polarised waves), for which $E_y = 0$, $H_x = H_z = 0$, in a photonic crystal with an unperturbed refractive index n_P, a dielectric plate of finite thickness $2a$ being located inside the crystal. Let the diffusion regime of redistribution of photoinduced electric charge carriers, which form an intracrystalline field determining the nonlinear dependence of the refractive index, be realised in the photonic crystal.

We choose the coordinate system so that the middle of the dielectric layer passes through the coordinate origin, and its interface with the parts of the photorefractive crysal lies in the planes $x = \pm a$ perpendicular to the x axis. Then the parts of the photonic crystal occupy the half-spaces $|x| > a$, and the dielectric layer is in the region $|x| < a$. The polar photorefractive crystal axes in both half-spaces are considered oppositely directed along the x axis.

We will consider only the stationary distribution of the SW field. In this case, from the system of Maxwell's equations we obtain the equation for the non-zero component of the magnetic field vector of the TM wave [19–21]:

$$\frac{\partial^2 H_y}{\partial x^2} + \frac{\partial^2 H_y}{\partial z^2} + k_0^2 n^2(x) H_y = 0, \tag{1}$$

where $k_0 = 2\pi/\lambda_0$; λ_0 is the wavelength of light in vacuum; and $n(x)$ is the spatial distribution of the refractive index of light in the direction perpendicular to the photonic crystal–dielectric layer interface. We represent the dependence of $n(x)$ in the form

$$n(x) = \begin{cases} n_P + \Delta n_P(x), & |x| > a, \\ n_L, & |x| < a, \end{cases} \tag{2}$$

where n_L is the refractive index of the dielectric layer, which is considered constant; and Δn_P is a small nonlinear addition to the unperturbed refractive index n_P in the photonic crystal. Hereinafter, the subscript P corresponds to the quantities describing the properties of the photorefractive crystal in the region $|x| > a$, and the subscript L corresponds to the quantities describing the properties of the dielectric layer in the region $|x| < a$.

We assume that a nonlinear addition to the unperturbed refractive index in a photonic crystal is formed as a result of only the diffusion mechanism of nonlinearity [11]. We also assume that the dark intensity I_d (the intensity of dark illumination) is negligible compared to the intensity of light in the surface wave $I \propto |H_y|^2$. Then, in the approximation $I_d \ll I$, the nonlinear addition to the refractive index of the photonic crystal can be represented as

$$\Delta n_P(x) = \frac{1}{2} n_P^3 r_{\text{eff}} \frac{k_B T}{e} \frac{I'}{I}, \tag{3}$$

where the primes hereinafter denote the derivatives with respect to the x coordinate; r_{eff} is the effective electro-optical coefficient; k_B is the Boltzmann constant; T is the temperature; and e is the electron charge modulus. Nonlinear addition (3) is small compared to the unperturbed refractive indices of both the photonic crystal and the dielectric interlayer.

Let the resulting distribution of the wave propagating along the z axis have the form

$$H_y(x,z) = \exp(i\beta k_0 z) \begin{cases} H_1(x), & x < -a, \\ H_L(x), & |x| < a, \\ H_2(x), & x > a \end{cases} \tag{4}$$

(β is the propagation constant). Then, taking into account $I_d \ll I$ and $\Delta n_P \ll n_{P,L}$ from (1)–(4) we obtain equations

$$H''_{1,2} + \mu H'_{1,2} + (n_P^2 - \beta^2) k_0^2 H_{1,2} = 0, \tag{5}$$

$$H''_L + (n_L^2 - \beta^2) k_0^2 H_L = 0, \tag{6}$$

where the wave attenuation coefficient in the photonic crystal has the form

$$\mu = 2k_0^2 n_P^4 r_{\text{eff}} k_B T/e. \tag{7}$$

The continuity conditions for the field components at the layer interfaces imply the boundary conditions

$$H_1(-a) = H_L(-a), \quad H_2(a) = H_L(a), \tag{8}$$

$$\frac{1}{n_P^2} H'_1(-a) = \frac{1}{n_L^2} H'_L(-a), \quad \frac{1}{n_P^2} H'_2(a) = \frac{1}{n_L^2} H'_L(a). \tag{9}$$

Thus, the mathematical formulation of the model for describing the SW propagation along the dielectric layer in a photorefractive crystal reduces to equations (6) and (7) with boundary conditions (8) and (9). The field distribution in various types of SWs in the model in question is described by solutions of the contact-boundary value problem (6)–(9) satisfying the boundedness conditions and $|H_y| \to 0$ for $x \to \infty$.

Since this model assumeы that all the optical characteristics of the parts of the photonic crystal to the left and to the right of the dielectric interlayer are identical, surface states with symmetry of the field distribution can occur in such a symmetric three-layer structure, which are described by even $[H_1(x) = H_2(-x)]$ and odd $[H_1(x) = -H_2(-x)]$ solutions. In the present work, only surface states of this kind are considered; therefore, in what follows we will use the notation for the magnetic field strength in the photorefractive crystal, $H_P(x) = H_1(x)$, for $x < -a$, and in the region $x > a$ the field $H_P(x)$ will continue in an even or odd way.

Equation (5) has two types of solutions that disappear at infinity, depending on the relationship between the values of the propagation constant, attenuation coefficient, and unperturbed refractive index in the photonic crystal. The amplitude of the wave of the first type decays non-oscillatory when moving away from the dielectric layer in the depth of the photonic crystal, and the amplitude of the wave of the second type oscillates.

The solutions of the linear equation (6) are determined by the sign of the difference $n_L^2 - \beta^2$. Inside the layer, several types of the field distribution can arise, described either by nonperiodic or periodic solutions that are expressed in terms of hyperbolic and trigonometric functions (sines and cosines), respectively. In each of the sets of SWs generated by such solutions, in addition to the type of symmetry of the surface state, there are various forms of field attenuation arising with distance from the dielectric layer to the depth of the photonic crystal.

3. Main types of surface waves

1. Under the condition $\max(n_L, \sqrt{n_P^2 - \mu^2/4k_0^2}) < \beta < n_P$, there are two types of SWs, the field amplitude of which non-oscillatory decreases in the photonic crystal and is aperiodically distributed inside the dielectric layer.

Waves with a field profile distribution symmetric with respect to the centre of the dielectric layer in the region $x < -a$ are described by the solution of equation (5), which can be expressed as

$$H_P(x) = H_a \exp[\mu(x+a)/2] \Big\{ \cosh[\nu(x+a)] - \frac{\kappa_e + \mu/2}{\nu} \sinh[\nu(x+a)] \Big\}, \tag{10}$$

where

$$\nu^2 = \frac{1}{4}\mu^2 - k_0^2(n_P^2 - \beta^2); \tag{11}$$

$$\kappa_e = \frac{n_P^2}{n_L^2} q \tanh(qa); \tag{12}$$

$$q^2 = k_0^2(\beta^2 - n_L^2); \quad (13)$$

and $H_a = H(-a)$ is the field amplitude on the left boundary of the photonic crystal with the dielectric layer. Expression (10) extends to the region $x > a$ in an even way.

Inside the dielectric layer, the profile of the SW field is described by a non-periodic even solution of equation (6) for $\beta > n_L$:

$$H_L(x) = H_a \cosh(qx)/\cosh(qa). \quad (14)$$

Solutions (10) and (14) satisfy the boundary conditions (8) and (9). The field amplitude H_a plays the role of a control parameter.

Waves with the distribution of the field profile antisymmetric with respect to the centre of the dielectric layer in the region $x < -a$ are described by the solution of equation (5), which takes the form

$$H_P(x) = H_a \exp[\mu(x+a)/2]\left\{\cosh[\nu(x+a)] - \frac{\kappa_o + \mu/2}{\nu}\sinh[\nu(x+a)]\right\}, \quad (15)$$

where

$$\kappa_o = \frac{n_P^2}{n_L^2} q \coth(qa). \quad (16)$$

Solution (15) extends to the region $x > a$ in an odd way.

Inside the dielectric layer, the profile of the SW field is described by a non-periodic odd solution of equation (6) for $\beta > n_L$:

$$H_L(x) = -H_a \sinh(qx)/\sinh(qa). \quad (17)$$

Solutions (15) and (17) satisfy the boundary conditions (8) and (9).

2. Under the condition $\sqrt{n_P^2 - \mu^2/4k_0^2} < \beta < \min(n_L, n_P)$, there are two more types of SWs, the field amplitude of which non-oscillatory decreases in the photonic crystal and is periodically distributed inside the dielectric layer.

Waves with a field profile distribution symmetric with respect to the centre of the dielectric layer in the region $x < -a$ are described by the solution of equation (5) in the form

$$H_P(x) = H_a \exp[\mu(x+a)/2]\left\{\cosh[\nu(x+a)] - \frac{\mu/2 - \gamma_e}{\nu}\sinh[\nu(x+a)]\right\}, \quad (18)$$

where

$$\gamma_e = \frac{n_P^2}{n_L^2} k \tan(ka), \quad (19)$$

$$k^2 = -q^2 = k_0^2(n_L^2 - \beta^2). \quad (20)$$

Solution (18) extends to the region $x > a$ in an even way.

Inside the dielectric interlayer, the profile of the SW field is described by a periodic even solution of equation (6) for $\beta < n_L$:

$$H_L(x) = H_a \cos(kx)/\cos(ka). \quad (21)$$

Solutions (18) and (21) satisfy the boundary conditions (8) and (9).

Waves with a field profile distribution asymmetric with respect to the centre of the dielectric layer in the region $x < -a$ are described by the solution of equation (5) in the form

$$H_P(x) = H_a \exp[\mu(x+a)/2]\left\{\cosh[\nu(x+a)] - \frac{\mu/2 - \gamma_o}{\nu}\sinh[\nu(x+a)]\right\}, \quad (22)$$

where

$$\gamma_o = \frac{n_P^2}{n_L^2} k \cot(ka). \quad (23)$$

Solution (22) extends to the region $x > a$ in an odd way.

Inside the dielectric layer, the profile of the SW field is described by a periodic odd solution of equation (7) for $\beta < n_L$:

$$H_L(x) = -H_a \sin(kx)/\sin(ka). \quad (24)$$

Solutions (22) and (24) satisfy the boundary conditions (8) and (9).

3. Under the condition $n_L < \beta < \sqrt{n_P^2 - \mu^2/4k_0^2}$, there are two types of SWs, the field amplitude of which oscillatory decreases in the photonic crystal and is aperiodically distributed inside the dielectric layer.

The symmetric distribution of the profile of the field oscillatory decaying in the photonic crystal in the region $x < -a$ is described by the solution of equation (5), which for $\beta < \sqrt{n_P^2 - \mu^2/4k_0^2}$ can be represented as

$$H_P(x) = H_a \exp[\mu(x+a)/2]\cos[p(x+a) - \varphi]/\cos\varphi, \quad (25)$$

where the wave number

$$p^2 = -\nu^2 = k_0^2(n_P^2 - \beta^2) - \mu^2/4. \quad (26)$$

Solution (25) extends to the region $x > a$ in an even way. Inside the dielectric layer for $|x| < a$, the field profile in a symmetric SW is described by expression (14). Substituting solutions (14) and (25) into the boundary conditions (8) and (9) leads to the dispersion relation

$$\frac{\mu}{2} + p\tan\varphi + \kappa_e = 0, \quad (27)$$

where κ_e is defined by expression (12). The phase φ here plays the role of a control parameter.

The antisymmetric distribution of the profile of the field oscillatory decaying in the photonic crystal in the region $x < -a$ is described by expression (25), which extends to the region $x > a$ in an odd way. Inside the dielectric layer for $|x| < a$, the field profile in the antisymmetric SW is described by expression (17). Substituting solutions (17) and (25) into the boundary conditions (8) and (9) leads to the dispersion relation

$$\frac{\mu}{2} + p\tan\varphi + \kappa_o = 0, \qquad (28)$$

where κ_o is determined by expression (16).

4. Under the condition $\beta < \min[n_L, \sqrt{n_P^2 - \mu^2/4k_0^2}]$, there are two more types of SWs, the field amplitude of which oscillatory decreases in the photorefractive crystal and is periodically distributed inside the dielectric layer.

The symmetric distribution of the profile of the field oscillatory decaying in the photonic crystal in the region $x < -a$ is described by expression (25), which extends to the region $x > a$ (25) in an even way. Inside the dielectric layer for $|x| < a$, the field profile in a symmetric SW is described by expression (21). Substituting solutions (21) and (25) into the boundary conditions (8) and (9) leads to the dispersion relation

$$\frac{\mu}{2} + p\tan\varphi - \gamma_e = 0, \qquad (29)$$

where γ_e is defined by expression (19).

The antisymmetric distribution of the profile of the field oscillatory decaying in the photonic crystal in the region $x < -a$ is described by expression (25), which continues to the region $x > a$ in an odd way. Inside the dielectric interlayer for $|x| < a$, the field profile in the antisymmetric PV is described by expression (24). Substituting solutions (24) and (25) into the boundary conditions (8) and (9) leads to the dispersion relation

$$\frac{\mu}{2} + p\tan\varphi + \gamma_o = 0, \qquad (30)$$

where γ_o is defined by expression (23).

4. Results and discussion

4.1. Variation in the ranges of surface-wave existence

In each of these ranges of variation in the propagation constant, there are two types of SWs, differing in the symmetry of the field distribution profile relative to the centre of the dielectric layer. In the case of symmetric SWs, the field amplitudes at the left and right boundaries of the dielectric layer are identical: $H(a) = H(-a) = H_a$, and in the case of antisymmetric SWs they are identical in absolute value, but opposite in sign: $H(a) = -H(-a) = -H_a$.

It is important to note that in order to change such ranges of SW existence, for example, when moving from the region $\sqrt{n_P^2 - \mu^2/4k_0^2} < \beta < \min(n_L, n_P)$, where there are non-oscillatory decaying SWs, to the region $n_L < \beta < \sqrt{n_P^2 - \mu^2/4k_0^2}$, where there are oscillatory decaying SWs, it is not necessary to rigidly set the relation between the refractive indices of the photonic crystal and the dielectric layer, i.e., to change the types of materials of this layered structure. Such a transition can be made by changing only the damping coefficient μ, the variation of which, in turn, can be carried out by changing the temperature. Consequently, in the layered structure, it becomes possible to regulate the attenuation form (with or without oscillations) of the SW excited in it by changing its temperature.

4.2. Characteristics of aperiodically decaying surface waves

Symmetric and antisymmetric, non-oscillatory decaying SWs exist in two ranges: for $\max(n_L, \sqrt{n_P^2 - \mu^2/4k_0^2}) < \beta < n_P$ and for $\sqrt{n_P^2 - \mu^2/4k_0^2} < \beta < \min(n_L, n_P)$. In both cases, the field distributions in the photonic crystal have the same character [see (10), (15) and (18), (22)]; however, they fundamentally differ within the dielectric layer. If in the first range, regardless of the type of symmetry, the field distribution is aperiodic [see (14) and (17)], then in the second range, the field has a periodic distribution over the width of the dielectric layer [see (21) and (24)].

The period of spatial field oscillations in the dielectric layer for symmetric and antisymmetric SWs in both ranges is defined as

$$\Lambda_L = 2\pi/k = 2\pi/[k_0(n_L^2 - \beta^2)^{1/2}]. \qquad (31)$$

The maxima of the field amplitude of symmetric and antisymmetric SWs and their positions in both ranges may not coincide with the field amplitude at the photonic crystal–dielectric layer interface. The maximum amplitude of the SW field can be reached inside the photorefractive crystal at a distance from the interface with the layer

$$x_{\mathrm{me,o}} = \frac{1}{2v}\ln F_{\mathrm{e,o}}, \qquad (32)$$

where in the first range

$$F_{\mathrm{e,o}} = \frac{(v - \mu/2)(v + \mu/2 + \kappa_{\mathrm{e,o}})}{(v + \mu/2)(v - \mu/2 - \kappa_{\mathrm{e,o}})},$$

and in the second range

$$F_{\mathrm{e,o}} = \frac{(v - \mu/2)(v + \mu/2 \mp \gamma_{\mathrm{e,o}})}{(v + \mu/2)(v - \mu/2 \pm \gamma_{\mathrm{e,o}})};$$

here, the upper sign is chosen for symmetric SWs (subscript e), and the lower one – for antisymmetric SWs (subscript o).

The fundamental difference between the SWs of these ranges lies in the fact that the SWs only in the second range can decay strictly monotonously into the depth of the photonic crystal with a certain relation between the propagation constant and the characteristics of the photonic crystal and the dielectric layer.

4.3. Monotonically decaying surface waves

The wave monotonically decays into the depth of the photonic crystal when the dispersion equations

$$\gamma_{\mathrm{e,o}} = \mu/2 \pm v \qquad (33)$$

are satisfied.

In the case of a symmetric SW, which monotonically decays into the photonic crystal in the region $|x| > a$, the field distribution takes the form

$$H_P(x) = H_a \exp[\pm\gamma_e(x \pm a)], \qquad (34)$$

and for an antisymmetric SW we have

$$H_P(x) = \pm H_a \exp[\pm\gamma_o(x \pm a)]. \tag{35}$$

The values of $\gamma_{e,o}$ determined by expressions (19) and (23) for symmetric and antisymmetric SWs, respectively, have the meaning of attenuation coefficients. Since the attenuation coefficient must be positive, restrictions arise for the propagation constant during a monotonic decay of a SW. In this case, the requirement $\tan(ka) > 0$ must be satisfied for symmetric SWs, which leads to a limitation of the wavenumber variation range, for example, in the interval $0 < k < \pi/2a$, and the requirement $\cot(ka) > 0$ should be satisfied for antisymmetric SWs, which leads to a limitation of the wavenumber variation range, in particular, in the interval $\pi/2a < k < \pi/a$.

Thus, dispersion equations (33) determine such relations of the propagation constant with the physical characteristics of the layered structure for which a monotonic decrease in the amplitude of the SW field is observed with distance from the interface with the dielectric layer to the depth of the photonic crystal.

In the approximation $ka \ll 1$, which can be called long-wavelength, from dispersion equations (33) with allowance for (19) for symmetric SWs, we explicitly obtain the dependence of the propagation constant on the physical characteristics of the layered structure (dispersion law):

$$\beta^2 = \frac{n_P^2 n_L^2 (1 - a\mu)}{n_L^2 - n_P^2 a\mu}. \tag{36}$$

The attenuation coefficient of symmetric long-wavelength SWs is expresses as

$$\gamma_e = \frac{a k_0^2 n_P^2 (n_L^2 - n_P^2)}{n_L^2 - n_P^2 a\mu}. \tag{37}$$

For the existence of monotonically decaying symmetric SWs in the long-wavelength range, the refractive index of the dielectric layer must be less than the unperturbed refractive index of the photonic crystal ($n_L < n_P$), and the width of the dielectric layer must be related to the attenuation coefficient in the photonic crystal and the refractive indices by the condition $a > n_L^2/(\mu n_P^2)$.

In the main approximation, for $a\mu \ll 1$, the dispersion law (36) takes a simpler form: $\beta^2 = n_P^2(1 - a\mu)$.

For the existence of a SW with such a dependence of the propagation constant, the thickness of the dielectric layer should be less than the depth of the SW penetration into the photonic crystal. With a fixed thickness of the dielectric layer, the fulfilment of this condition can be achieved by reducing its temperature (because $\mu \backsim T$).

The period of spatial oscillations inside the dielectric layer (31) in the limiting case is determined by the expression

$$\Lambda_L = 2\pi/[k_0\sqrt{n_L^2 - n_P^2(1 - a\mu)}]. \tag{38}$$

With increasing temperature, the Λ_L period increases. In the main order, it will be determined only by the refractive index of the dielectric layer.

The dispersion law of long-wavelength monotonically decaying antisymmetric SWs can also be obtained in explicit form from (33) with allowance for (23):

$$\beta^2 = n_P^2\left[1 + \frac{1}{a^2 k_0^2 n_L^2}\left(\frac{n_P^2}{n_L^2} - a\mu\right)\right]. \tag{39}$$

For the attenuation coefficient of antisymmetric long-wavelength SWs with allowance for (23), we have

$$\gamma_o = n_P^2/(a n_L^2). \tag{40}$$

It follows that the depth of penetration of the antisymmetric SW of the long-wavelength range, $l_o = 1/\gamma_o = a n_L^2/n_P^2$, in the photonic crystal can be directly controlled by choosing the thickness of the dielectric layer a and the ratio between the unperturbed refractive indices of the layered structure. Consequently, near the dielectric layer of small thickness, the antisymmetric SW will propagate with the localisation of a larger fraction of the energy in narrower near-surface layers. The same effect of narrowing the spatial localisation of the SW energy can be obtained by selecting the ratio of the refractive index of the dielectric ilayer to the unperturbed refractive index of the photonic crystal.

4.4. Characteristics of oscillatory decaying surface waves

Symmetric and antisymmetric, oscillatory decaying SWs also exist in two ranges: for $n_L < \beta < \sqrt{n_P^2 - \mu^2/4k_0^2}$ and for $\beta < \min(n_L, \sqrt{n_P^2 - \mu^2/4k_0^2})$. In both cases, the field distributions in the photonic crystal have the same character [see (25)], and inside the dielectric layer, as for aperiodically decaying SWs, regardless of the type of symmetry, the field distribution in the first range will be aperiodic [see (14) and (17)], while in the second range the field has a periodic distribution over the width of the dielectric layer [see (21) and (24)].

The period of spatial oscillations of the field attenuation in a photonic crystal with a symmetric (antisymmetric) profile of the first range is determined by the expression

$$\Lambda_P = 2\pi/p = -2\pi\tan\varphi/(\kappa_{e,o} + \mu/2). \tag{41}$$

In this case, the wave penetrates into the crystal at a distance of $l = 2/\mu = e/(k_0^2 n_P^4 r_{\text{eff}} k_B T)$. With increasing temperature, the depth of energy localisation of oscillatory decaying SWs decreases. The Λ_P period actually determines the period of the photorefractive diffraction grating formed by the SW field due to the photorefractive effect, when the change in the refractive index distribution in such a diffraction grating has the form $\Delta n_P \backsim E^{sc}(x)$, where E^{sc} is the induced intracrystalline field associated with the SW field (25) [11]. In particular, in the case where the diffusion mechanism of nonlinear response formation prevails, we used expression (3) to estimate Δn_P.

It follows from dispersion equations (27) and (28) that the oscillatory decaying SWs of the first range can exist only for such values of phase for which $\tan\varphi < 0$. For the oscillatory decaying SWs to exist in the second range, this condition is not required , which follows from dispersion equations (29) and (30).

In the approximation $qa \ll 1$, from the dispersion equation (27), we obtain in explicit form the dispersion law of symmetric oscillatory decaying SWs of the first range:

$$\beta^2 = n_L^2 \frac{k_0^2 n_P^2 (a\mu + \tan^2\varphi) - \mu^2(1 + \tan^2\varphi)/4}{k_0^2(n_P^2 a\mu + n_L^2 \tan^2\varphi)}. \tag{42}$$

The existence of such a wave relies on an additional condition

$$n_P^2 < \frac{\mu^2(\tan^2\varphi + 1)}{4k_0^2(\tan^2\varphi + a\mu)}.$$

In the limiting case $n_L \gg n_P$ in the main approximation, the dispersion law (42) takes a simpler form:

$$\beta^2 = n_P^2 - \frac{\mu^2}{4k_0^2}(1 + \cot^2\varphi). \quad (43)$$

The period of spatial oscillations of the field attenuation in the photorefractive crystal in this limiting case is related to the attenuation coefficient: $\Lambda_P = -4\pi\tan\varphi/\mu$. It follows that with increasing temperature of the layered structure, both the period of spatial oscillations of the field attenuation in the photonic crystal and its penetration depth decrease.

From the dispersion equation (28) in the approximation $qa \ll 1$, we explicitly obtain the dispersion law of antisymmetric oscillatory decaying SWs of the first range:

$$\beta^2 = n_P^2 - \frac{\mu^2}{4k_0^2} - \left(\frac{\mu}{2} + \frac{n_P^2}{an_L^2}\right)^2 \cot^2\varphi. \quad (44)$$

For the existence of antisymmetric SWs, taking into account the dispersion law (44), it is sufficient to satisfy the condition $\tan\varphi < 0$.

The period of spatial oscillations of the field attenuation in a photonic crystal with a symmetric SW profile of the second range, taking into account (29), has the form

$$\Lambda_P = 2\pi\tan\varphi/(\gamma_e - \mu/2). \quad (45)$$

The phase $\varphi = 0$ corresponds to the case when the attenuation coefficients are related ($\gamma_e = \mu/2$) and the oscillation period is 2π. From this, taking into account (19) in the long-wavelength approximation for $ka \ll 1$, we obtain explicitly the expression for the dispersion law of symmetric SWs of the second range:

$$\beta^2 = n_L^2\left(1 - \frac{\mu}{2ak_0^2 n_P^2}\right). \quad (46)$$

It should be noted that the SWs with a zero phase in the first range and antisymmetric long-wavelength SWs in the second range do not exist.

From (31) and (46) we find the period of spatial field oscillations in the dielectric layer for long-wavelength symmetric SWs of the second range:

$$\Lambda_L = \pi \frac{n_P}{n_L}\sqrt{\frac{2a}{\mu}}. \quad (47)$$

As follows from (47), this period can be reduced by increasing the temperature at fixed refractive indices and fixed thickness of the dielectric layer. It also follows from expression (47) that the thicker the dielectric layer, the longer the period of field oscillations inside it.

For the period of spatial oscillations of the field attenuation in a photonic crystal with an antisymmetric SW profile of the second range with allowance for (30) we have

$$\Lambda_P = -2\pi\tan\varphi/(\gamma_o + \mu/2). \quad (48)$$

For such long-wavelength SWs, the period takes the form

$$\Lambda_P = -\frac{4\pi a n_L^2 \tan\varphi}{2n_P^2 + a\mu n_P^2}. \quad (49)$$

For these waves, as well as for oscillating SWs of the first range, the condition $\tan\varphi < 0$ should be met. It follows from (49) that the period of spatial oscillations can be controlled by changing the temperature and the ratio of the refractive index of the dielectric interlayer to the unperturbed refractive index of the photonic crystal. At fixed refractive indices, an increase in temperature leads to a decrease in the period of spatial oscillations of the SW amplitude attenuation in the photonic crystal.

5. Conclusions

We have found that TM-polarised surface waves, which differ in the character of attenuation and the symmetry of the field profile, can propagate along the dielectric layer inside the photorefractive crystal. The waves of one type decay without oscillations when moving away from the interface into the depth of the photorefractive crystal, while the waves of the other type decay with oscillations. The waves of the first type can decay monotonously under certain conditions of relation between the propagation constant, refractive indices, and other physical characteristics of the layers.

Surface TM waves can exist in four different ranges of the propagation constant β:

1) for $\max(n_L, \sqrt{n_P^2 - \mu^2/4k_0^2}) < \beta < n_P$, the SW field decreases without oscillations in the photonic crystal and is aperiodically distributed inside the dielectric layer;

2) for $\sqrt{n_P^2 - \mu^2/4k_0^2} < \beta < \min(n_L, n_P)$, the SW field decreases without oscillations in the photonic crystal and has a periodic distribution inside the dielectric layer;

3) for $n_L < \beta < \sqrt{n_P^2 - \mu^2/4k_0^2}$, the SW field decreases with oscillations in the photonic crystal, and its distribution inside the dielectric layer is aperiodic; and

4) for $\beta < \min(n_L, \sqrt{n_P^2 - \mu^2/4k_0^2})$, the SW field decreases with oscillations in the photonic crystal and is periodically distributed inside the dielectric layer.

In each of the indicated ranges, there are symmetric (relative to the centre of the dielectric layer) SWs with coinciding amplitudes at the left and right interfaces and antisymmetric SWs with amplitudes identical in absolute value and opposite in sign. In fact, such types of SWs correspond to in-phase and out-of-phase field oscillations of the field at the layer boundaries.

The required propagation regime of TM-polarised SWs can be implemented as a result of the transition of β from one range to another by changing the temperature of the layered structure in question.

For all types of SWs considered in the work, their characteristics are analytically determined and dispersion equations are obtained. The dependences of the propagation constant on the characteristics of the photonic crystal and the dielectric layer for the 'long-wavelength' regime of SW propagation are found, and the conditions for their existence are indicated.

The penetration depth of the SW into the photorefractive crystal and the periods of their oscillations in the photorefractive crystal and inside the dielectric layer for the considered types of the waves are determined. It is shown that an increase in temperature leads to a decrease in the depth of the energy localisation of oscillatory decaying SWs, as well as to a decrease in the period of spatial oscillations of the field attenuation in the photonic crystal and in the dielectric layer.

The estimates of the periods of oscillations obtained in this work allow us to take into account the effect of the size of the dielectric layer in order to control the parameters of the formed photorefractive diffraction grating. These estimates should be taken into account when designing optical elements from layered crystal structures with a photorefractive effect.

The possibility of the existence of the waves with the oscillatory decaying field amplitude fundamentally distinguishes heterostructures based on optical media with a photorefractive effect from layered structures consisting of other optical media. The methods for suppressing oscillations and adjusting the depth of the field energy localisation along the layers indicated in this work can be useful in developing various optical devices based on the use of the photorefractive properties of crystals in multilayer structures.

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