Quantum Electronics **49** (5) 443–448 (2019)

https://doi.org/10.1070/QEL16999

Investigation of the possibility of ultra-deep laser cooling using a quadrupole transition

A.A. Kirpichnikova, O.N. Prudnikov, D. Wilkowski

Abstract. **We consider the kinetics of atoms in nonuniform spatially polarised light fields resonant to the quadrupole optical transition with $F_g \to F_e = F_g + 2$ ($F_{g,e}$ is the total angular momentum in the ground and excited states). The lowest possible temperatures of laser cooling of atoms are analysed numerically and the results are compared with the data obtained for sub-Doppler cooling using light waves resonant to electric dipole optical transitions.**

Keywords: *ultra-deep laser cooling, light fields, sub-Doppler cooling.*

1. Introduction

Since the mid-1980s, laser cooling of atoms has been a rapidly developing area at the intersection of laser and atomic physics. Currently, laser-cooled atoms are ideal candidates for precision spectroscopy; they are also used to develop quantum frequency standards [1–3], to achieve Bose–Einstein condensation [4, 5], to simulate quantum effects in condensed media, to study interatomic collisions, etc. [6, 7].

To date, there have been developed various methods and approaches for describing laser cooling. At the initial stage of research, semi-classical approaches have become widespread, which make it possible to describe kinetics of atoms in terms of the forces acting on atoms from the resonant electromagnetic field and in terms of diffusion resulting from abrupt absorption/emission of photons of the field [8–15]. The main reasons for limiting the application of the semi-classical theory are the smallness of the momentum Δp transmitted to the atoms interacting with the photons of the field as compared to the photon momentum $\hbar k/\Delta p \ll 1$, and also the smallness of the semi-classicality parameter $\varepsilon_r = \omega_r/\gamma \ll 1$, i.e. the ratio of recoil energy $\hbar\omega_r = \hbar^2k^2/(2M)$, obtained by a fixed atom with a mass M as a result of absorption/emission of photons of the field, to the natural width of the level γ. The presence of these parameters allows us to separate the evolution of internal and translational degrees of freedom of atoms and to reduce the solution of the complex quantum mechanical problem to the semi-classical Fokker–Planck equation for the distribution function of atoms in the phase space with a release of forces and diffusion coefficients for atoms in light fields [11–14]. It is worth noting the importance of developing semi-classical approaches, since they allowed the basic mechanisms of laser cooling of atoms to be described, including Doppler [8–10, 16] and sub-Doppler laser cooling mechanisms [10, 17–19] in optical molasses, i.e. in the field of counterpropagating light waves.

An alternative approach to the description of laser cooling problems in optical molasses was the development of quantum approaches (see, for example, [20–25]), which, for atoms with optical transitions characterised by extremely small semi-classicality parameters ($\varepsilon_r \ll 1$), leads to the results comparable with the data of the semi-classical theory [21, 25]. For atoms with narrow optical transitions, when the parameter ε_r is insufficiently small, recoil effects play an important role, leading to a significant difference in the results of the semi-classical and quantum approaches [26–30]. For example, for atoms with the ground state nondegenerate in the angular momentum projection, quantum approaches predict laser cooling of atoms to energies comparable to the recoil energy of the atom, $E \sim \hbar\omega_r$ [26, 28], in contrast to the semi-classical approaches, where the minimum temperature of laser cooling is determined by the natural width $k_BT \sim \hbar\gamma/2$ [8–10, 16], i.e., the Doppler limit.

Note that for atoms with levels degenerate in the angular momentum projection, an insufficiently small parameter ε_r results in a decrease in the efficiency of the sub-Doppler friction mechanisms and in an increase in the hotter fraction of atoms with temperatures of the order of the Doppler limit [29, 30], which may result in the impossibility of achieving sub-Doppler temperatures, i.e., in Doppler laser cooling [29].

The prospects for reaching ultra-deep temperatures (several recoil energies) make it important to study the possibility of laser cooling of atoms using narrow optical transitions. In this paper, we investigated the possibility of ultra-deep laser cooling of alkali atoms using a narrow quadrupole optical transition ${}^2S_{1/2}-{}^2D_{5/2}$ between the hyperfine structure levels $F_g = F \to F_e = F + 2$ with the total angular momenta F_g and F_e in the ground and excited states. A comparison was also made of the minimum temperatures for laser cooling with those obtained by standard methods of sub-Doppler laser cooling (for example, Cs atoms using the ${}^2S_{1/2}-{}^2P_{3/2}$ dipole optical transition in nonuniform spatially polarised fields).

A.A. Kirpichnikova Institute of Laser Physics, Siberian Branch, Russian Academy of Sciences, prosp. Akad. Lavrent'eva 15B, 630090 Novosibirsk, Russia; e-mail: kirpichnikovaaa@gmail.com;
O.N. Prudnikov Institute of Laser Physics, Siberian Branch, Russian Academy of Sciences, prosp. Akad. Lavrent'eva 15B, 630090 Novosibirsk, Russia; Novosibirsk State University, ul. Pirogova 2, 630090 Novosibirsk, Russia; e-mail: oleg.nsu@gmail.com;
D. Wilkowski School of Physical and Mathematical Sciences, Nanyang Technological University, 637371 Singapore, Singapore; Center for Quantum Technologies, National University of Singapore, 117543 Singapore, Singapore; MajuLab, CNRS-UCA-SU-NUS-NTU International Joint Research Unit, Singapore

Received 12 March 2019
Kvantovaya Elektronika **49** (5) 443–448 (2019)
Translated by I.A. Ulitkin

2. Statement of the problem

Let us consider the one-dimensional motion of atoms (along the z axis) having a closed optical transition with $F_g \to F_e$ in a resonant monochromatic field formed by opposing light waves of equal intensity:

$$\boldsymbol{E}(z,t) = E_0[\boldsymbol{e}_1\exp(\mathrm{i}kz) + \boldsymbol{e}_2\exp(-\mathrm{i}kz)]\exp(-\mathrm{i}\omega t) + \text{c.c.}, \quad (1)$$

where E_0 is the complex amplitude of the light waves; ω is the field frequency; and $k = \omega/c$ is the wave vector. The polarisations of the opposite waves, e_1 and e_2, in the Cartesian basis e_x, e_y and e_z can be expressed through the components of the vectors $e_{x,y,z}$ in the cyclic basis

$$\boldsymbol{e}_n = \sum_{\sigma=0,\pm1} e_n^{\sigma}\boldsymbol{e}_{\sigma}, \quad n = 1,2. \quad (2)$$

Here, $\boldsymbol{e}_{\pm1} = \mp(\boldsymbol{e}_x \pm \boldsymbol{e}_-)/\sqrt{2}$ and $\boldsymbol{e}_0 = \boldsymbol{e}_z$ are the unit vectors in a cyclic basis. Note that the components e_n^0 are equal to zero due to the orthogonality of the vectors $\boldsymbol{e}_n$ and $\boldsymbol{k}$. In particular, the counterpropagating orthogonally polarised waves form well-known configurations of the light fields: lin $\perp$ lin configuration of the light field with $\boldsymbol{e}_1 = \boldsymbol{e}_x$ and $\boldsymbol{e}_2 = \boldsymbol{e}_y$; and $\sigma_+ - \sigma_-$ configuration of the light field with $\boldsymbol{e}_1 = \boldsymbol{e}_+$ and $\boldsymbol{e}_2 = \boldsymbol{e}_-$.

The evolution of an ensemble of low-density atoms, when the interatomic interaction can be neglected, is determined by the quantum kinetic equation for the atomic density matrix

$$\frac{\partial}{\partial t}\hat{\rho} = -\frac{\mathrm{i}}{\hbar}[\hat{H},\hat{\rho}] - \hat{\Gamma}\{\hat{\rho}\}, \quad (3)$$

where $\hat{H}$ is the Hamiltonian, and $\hat{\Gamma}\{\hat{\rho}\}$ describes the relaxation of atomic levels during spontaneous decay. The Hamiltonian $\hat{H}$ of the atom is divided into the sum of the contributions:

$$\hat{H} = \frac{\hat{p}^2}{2M} + \hat{H}_0 + \hat{V}, \quad (4)$$

where the first term is the kinetic energy operator; $\hat{H}_0 = -\hbar\delta\hat{P}_e$ is the Hamiltonian of the free atom in the rotating wave approximation (RWA); $\delta = \omega - \omega_0$ is the detuning of the optical frequency ω from the atomic transition frequency ω_0; and

$$\hat{P}_e = \sum_{\mu} |F_e,\mu\rangle\langle F_e,\mu| \quad (5)$$

is the projection operator to the excited state levels $|F_e,\mu\rangle$, characterised by the total angular momentum F_e and the angular momentum projection μ on the quantisation axis. The last term $\hat{V}$ describes the atom–field interaction (1). Note that in the interaction of an atom with a field resonant to the electric dipole transition E1, the interaction operator takes the form

$$\hat{V} = \hat{V}_1\exp(\mathrm{i}kz) + \hat{V}_2\exp(-\mathrm{i}kz),$$
$$\hat{V}_n = \hbar\Omega(\hat{D}\boldsymbol{e}_n) = \hbar\Omega\sum\hat{D}_{\sigma}e_n^{\sigma}, \quad n = 1,2 \quad (6)$$

(Ω is the Rabi frequency of the electric dipole transition) and is determined by the polarisation vectors of the counterpropagating waves and the vector operator $\hat{D}$. Its matrix components $\hat{D}_{\sigma}$ in the circular basis are expressed via the Clebsch–Gordan coefficients:

$$\hat{D}_{\sigma} = \sum_{\mu,m} C^{F_e,\mu}_{1,\sigma;F_g,m}|F_e,\mu\rangle\langle F_g,m|. \quad (7)$$

The operator $\hat{V}$ specifies induced transitions with a change in the angular momentum projection $\sigma = \pm1$ for the field configuration in question (Fig. 1a). The last member of the kinetic equation (3), which describes the relaxation of the atomic density matrix, has a known form (see, for example, [25]):

$$\hat{\Gamma}\{\hat{\rho}\} = \frac{\gamma}{2}(\hat{P}_e\hat{\rho} + \hat{\rho}\hat{P}_e)$$
$$-\gamma\frac{3}{2}\left\langle\sum_{\xi=1,2}(\hat{D}\boldsymbol{e}_{\xi}(\boldsymbol{k}))^{\dagger}\exp(-\mathrm{i}\boldsymbol{k}\hat{\boldsymbol{r}})\hat{\rho}\exp(-\mathrm{i}\boldsymbol{k}\hat{\boldsymbol{r}})(\hat{D}\boldsymbol{e}_{\xi}(\boldsymbol{k}))\right\rangle, \quad (8)$$

where $\langle\ldots\rangle_{\Omega_k}$ means averaging over the directions of emission of a spontaneous photon having a momentum $\hbar\boldsymbol{k}$ with two orthogonal polarisations $e_{\xi}(\boldsymbol{k})$.

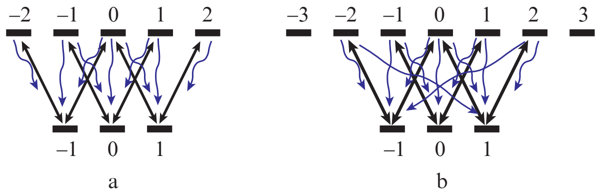

Figure 1. Scheme of spontaneous (wavy arrows) and light-induced (straight arrows) transitions for (a) the electric dipole transition with $F_g = 1 \to F_e = 2$ and (b) the quadrupole transition with $F_g = 1 \to F_e = 3$.

Spontaneous decays in electric dipole transitions result in relaxation of the atomic density matrix with a change in the angular momentum projection $\Delta\mu = 0, \pm1$ (Fig. 1a). For the one-dimensional problem, the relaxation operator (8) is reduced to the form

$$\hat{\Gamma}\{\hat{\rho}\} = \frac{\gamma}{2}(\hat{P}_e\hat{\rho} + \hat{\rho}\hat{P}_e)$$
$$-\gamma\sum_{\sigma=0,\pm1}\int_{-1}^{1}\hat{D}_{\sigma}^{\dagger}\exp(-\mathrm{i}ks\hat{z})\hat{\rho}\exp(\mathrm{i}ks\hat{z})\hat{D}_0 K_{\sigma}(s)\,\mathrm{d}s, \quad (9)$$

where the functions

$$K_{\pm1}(s) = \frac{3}{8}(1+s^2), \quad K_0 = \frac{3}{4}(1-s^2) \quad (10)$$

are determined by the probability of spontaneous emission of a photon in the direction making an angle θ with the z axis; and $s = \cos\theta$. To solve the kinetic problem of laser cooling, it is convenient to use the coordinate representation for the atomic density matrix [25], in which the spontaneous relaxation operator in the electric dipole approximation (9) takes the simplest form:

$$\hat{\Gamma}\{\hat{\rho}(z_1,z_2)\} = \frac{\gamma}{2}(\hat{P}_e\hat{\rho}(z_1,z_2) + \hat{\rho}(z_1,z_2)\hat{P}_e)$$
$$-\gamma\sum_{\sigma=0,\pm1}\kappa_{\sigma}(kq)\hat{D}_{\sigma}^{\dagger}\hat{\rho}(z_1,z_2)\hat{D}_{\sigma}, \quad (11)$$

where $q = z_1 - z_2$; and

$$\kappa_0 = 3\left(\frac{\sin(kq)}{(kq)^3} - \frac{\cos(kq)}{(kq)^2}\right);$$

$$\kappa_{\pm 1} = \frac{3}{2}\left(\frac{\cos(kq)}{(kq)^2} + \frac{\sin(kq)}{kq} - \frac{\sin(kq)}{(kq)^3}\right). \tag{12}$$

For the quadrupole optical transition E2, the operator of interaction with field (1) is similar to (6),

$$\hat{V} = \hat{V}_1 \exp(\mathrm{i}kz) + \hat{V}_2 \exp(-\mathrm{i}kz),$$

$$\hat{V}_n = \hbar\Omega^{(2)}(\hat{Q}\mathcal{E}_n), \quad n = 1,2, \tag{13}$$

but is given by the tensor operator $\hat{Q}$ and the second rank tensor

$$\mathcal{E}_n = \{k_n \otimes e_n\}_2. \tag{14}$$

The matrix components of the operator $\hat{Q}_\sigma$ in the circular basis are expressed via the Clebsch–Gordan coefficients:

$$\hat{Q}_\sigma = \sum_{\mu,m} C^{F_e,\mu}_{2,\sigma;F_g,m} |F_e,\mu\rangle\langle F_g,m|. \tag{15}$$

Despite the fact that in the general form, the quadrupole transition operator allows transitions from the ground to the excited state with a change in the angular momentum projection on $\Delta\mu = 0, \pm1, \pm2$ in the considered configuration of the light field formed by counterpropagating light waves (1), only induced transitions with $\Delta\mu = \pm1$ (Fig. 1b) are possible, which allows the mechanisms of laser cooling of atoms to be realised, similar to those known in fields resonant to electric dipole transitions. For the spontaneous relaxation operator, an expression similar to (11) can be obtained:

$$\hat{\Gamma}\{\hat{\rho}(z_1,z_2)\} = \frac{\gamma}{2}(\hat{P}_e\hat{\rho}(z_1,z_2) + \hat{\rho}(z_1,z_2)\hat{P}_e)$$

$$-\gamma \sum_{\sigma=0,\pm1,\pm2} \tilde{\kappa}_\sigma(kq)\hat{Q}^\dagger_\sigma\hat{\rho}(z_1,z_2)\hat{Q}_\sigma, \tag{16}$$

where the functions

$$\tilde{\kappa}_0 = 15\left(5\frac{\sin(kq)}{(kq)^3} - \frac{\cos(kq)}{(kq)^2} + 12\frac{\cos(kq)}{(kq)^4} - 12\frac{\sin(kq)}{(kq)^5}\right),$$

$$\tilde{\kappa}_{\pm1} = 5\left(\frac{1}{2}\frac{\sin(kq)}{kq} + \frac{5}{2}\frac{\cos(kq)}{(kq)^2} - \frac{21}{2}\frac{\sin(kq)}{(kq)^3}\right.$$

$$\left. - 24\frac{\cos(kq)}{(kq)^4} + 24\frac{\sin(kq)}{(kq)^5}\right), \tag{17}$$

$$\tilde{\kappa}_{\pm2} = 5\left(3\frac{\sin(kq)}{(kq)^3} - \frac{\cos(kq)}{(kq)^2} + 6\frac{\cos(kq)}{(kq)^4} - 6\frac{\sin(kq)}{(kq)^5}\right)$$

describe the rate of spontaneous decays taking into account the recoil effects when changing the angular momentum projection $\Delta\mu = 0, \pm1, \pm2$ (Fig. 1b).

3. Results

The problem of laser cooling of atoms in a light field (1) resonant to electric dipole (6) or quadrupole (13) transitions can be analysed on the basis of a numerical solution of the quantum-kinetic equation for the atomic density matrix (3) using the methods proposed in [23–25]. As a concrete example, we consider laser cooling of Cs atoms in a field resonant to the $6^2S_{1/2}$ quadrupole transition ($F = 4$) → $5^2D_{5/2}$ ($F = 6$) ($\lambda = 685$ nm) with a natural width $\gamma/2\pi \simeq 124$ kHz [31]. The spontaneous decay of the $5^2D_{5/2}$ level ($F = 6$) occurs predominantly in the cascade scheme $5^2D_{5/2}$ ($F = 6$) → $6^2P_{3/2}$ ($F = 5$) → $6^2S_{1/2}$ ($F = 4$) (Fig. 2), which somewhat modifies the relaxation operator in equation (3), but, nevertheless, as well as (16) leads to transitions with a change in the angular momentum projection $\Delta\mu = 0, \pm1, \pm2$.

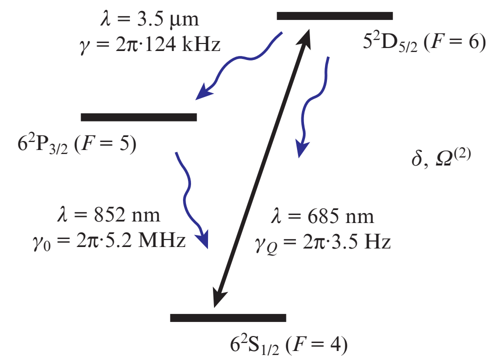

Figure 2. Diagram of the levels involved in the laser cooling of Cs atoms in a field resonant to the quadrupole optical transition $6^2S_{1/2}$ ($F = 4$) → $5^2D_{5/2}$ ($F = 6$). The straight line indicates induced transitions, the wavy arrows show spontaneous transitions.

For the scheme using the E2 transition, the semi-classicality parameter is $\varepsilon_r \simeq 0.026$, which is several orders of magnitude higher than the semi-classicality parameter for the standard laser cooling scheme using the E1 transition $6^2S_{1/2}$ ($F = 4$) → $6^2P_{3/2}$ ($F = 5$) ($\varepsilon_r \simeq 0.0004$, and the natural width is $\gamma_0/2\pi \simeq 5.2$ MHz). It should be noted that in the case of atoms with levels degenerate in the angular momentum projection, the solution of the quantum mechanical equation for the atomic density matrix with full account for the recoil effects is a very resource-consuming task. Therefore, in the present work, we restricted ourselves to the model of an atom with an angular momentum $F = 1$ in the ground state and, accordingly, transitions with $F_g = 1 \to F_e = 3$ and $F_g = 1 \to F_{e1} = 2$ for quadrupole and electric dipole optical transitions, respectively.

Before considering the results of the analysis of laser cooling of atoms, we note that the problem of laser cooling in nonuniform polarised fields resonant to optical transitions E1 was examined by many authors, which made it possible to describe the main mechanisms of sub-Doppler laser cooling [12–14, 17, 19, 20, 22–25].

To study the limits of laser cooling, we should analyse this problem within the framework of quantum approaches. Note also that the stationary solution for the atomic density matrix (3) is determined by the chosen spatial configuration of the light field and depends on its parameters, i.e. detuning δ and field intensity (Rabi frequency Ω). In the limit of the low intensity of the cooling field and the secular approximation considered in [20], the stationary solution of equation (3) is characterised by only one parameter, namely, the depth of the optical shift of the levels U,

$$\frac{U}{\hbar\omega_r} \sim u = \frac{|\delta|}{\omega_r}\frac{|\Omega|^2}{\delta^2 + \gamma^2/4}, \tag{18}$$

proportional to the dimensionless parameter u. Castin and Dalibard [20] analysed sub-Doppler cooling within the framework of the atom model with the $F_g = 1/2 \to F_e = 3/2$ levels in the lin $\perp$ lin configuration of the field degenerate in the angular momentum projection, the minimum kinetic energy of the laser cooling of atoms being $E_{kin} \simeq 30\hbar\omega_r$. Outside the framework of the secular approximation $\sqrt{u} \ll |\delta|/\gamma$ (see work [20]), laser cooling also depends on the detuning of the light field and on the semi-classicality parameter ε_r [25]. However, at extremely small values $\varepsilon_r \ll 1$ and sufficiently large 'red' detunings, when the kinetic energy of cold atoms reaches its minimum, the results agree quite well with those obtained in [20].

Following the definitions of papers [20, 25], we presented an analysis of laser cooling of atoms, considering the light shift parameter u (18) for various detunings δ. The results are given for two polarisation configurations of the light field, $\sigma_+ - \sigma_-$ and lin $\perp$ lin.

3.1. Laser cooling using a E2 transition in the $\sigma_+ - \sigma_-$ configuration of the field

Figure 3 shows the results of calculating the kinetic energy of cold atoms in the spatial configuration $\sigma_+ - \sigma_-$ of the field for various detunings δ of the light field as functions of the dimensionless parameter u (18). The attainable kinetic energy of atoms is represented in units of recoil energy. Note that the interaction of atoms with photons of the field resonant to the E1 or E2 transition is characterised by different recoil energies: $\hbar\omega_{0r} = 0.1$ μK for the E1 transition and 0.15 μK for the E2 transition (see Fig. 2). For the E1 transition in the $\sigma_+ - \sigma_-$ configuration of the field (Fig. 3b) the minimum value of the kinetic energy of cold atoms is reached at large red detunings $|\delta| \geqslant 10\gamma_0$, when the solution tends to a universal dependence on one parameter u, which was also observed with laser cooling in the lin $\perp$ lin configuration of the field [20, 25]. In this case, the minimum kinetic energy of cold atoms is $E_{kin} \simeq 100\hbar\omega_{0r}$ (which corresponds to 10 μK in temperature units).

For laser cooling using a light field resonant to the E2 transition, the dependence of the attainable kinetic energy of cold atoms on the light field parameters significantly changes. The minimum energy is reached at small detunings ($\delta \simeq -\gamma$) and is $E_{kin} \simeq 10\hbar\omega_r$ (~1.5 μK in temperature units). Note that at detunings $|\delta| < \gamma$, it is possible to attain slightly lower

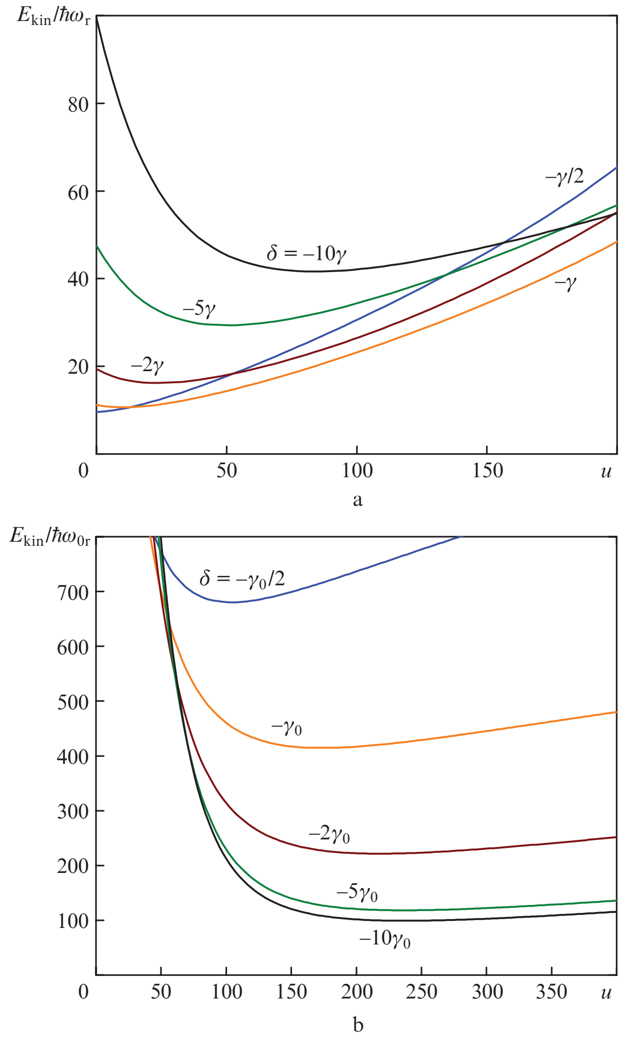

Figure 3. Kinetic energy of cold atoms in units of recoil energy in the $\sigma_+ - \sigma_-$ configuration of the field resonant to (a) quadrupole and (b) electric dipole transitions as a function of the dimensionless parameter u (18).

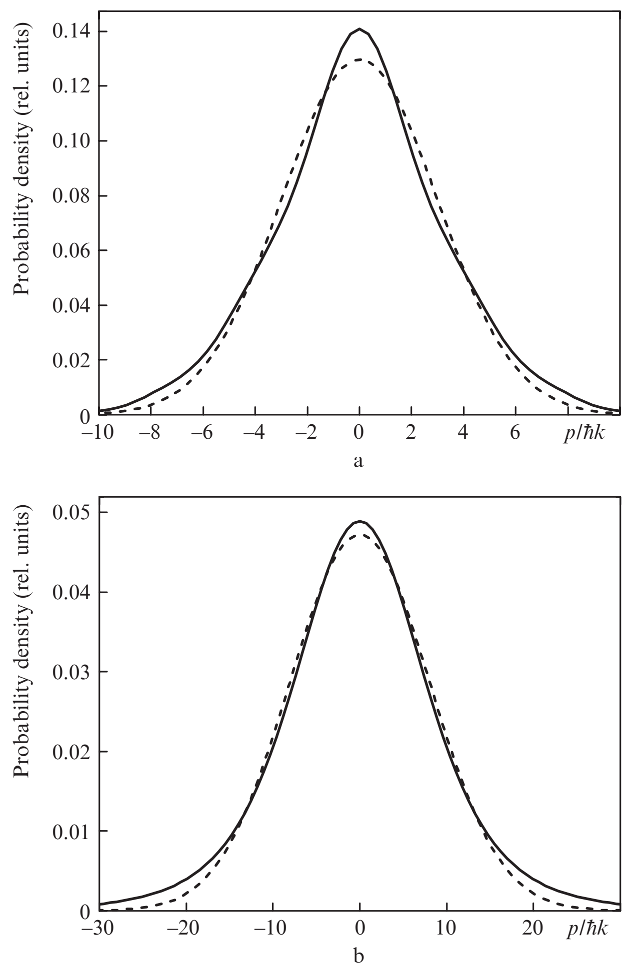

Figure 4. Momentum distribution of cold atoms in the $\sigma_+ - \sigma_-$ configuration of the field in units of the recoil momentum $\hbar k$ in fields resonant to (a) quadrupole and (b) electric dipole transitions for the light field parameters corresponding to the minimum kinetic energy of the atoms (Fig. 3): (solid curves) $\delta = -\gamma$, $\Omega^{(2)} \simeq 0.7\gamma$ for cooling using the E2 transition and $\delta = -10 \simeq \gamma_0$, $\Omega \simeq \gamma_0$ for cooling using the E1 transition, as well as (dashed curves) an approximation by the Gaussian function.

energies in the limit of low intensities (small u), but at given small detunings, the energy of cold atoms increases rapidly with increasing intensity of the light field (Fig. 3a).

Figure 4 shows the momentum distribution of cold atoms with light field parameters corresponding to the minimum values of kinetic energy: $E_{\rm kin} \simeq 10\hbar\omega_{\rm r}$ for laser cooling by a field resonant to the E2 transition, and $E_{\rm kin} \simeq 100\hbar\omega_{0{\rm r}}$ for laser cooling by a field resonant to the E1 transition. The momentum distribution of cold atoms is nonequilibrium; therefore, it cannot be characterised in terms of temperature. The dashed lines show the optimal approximation of the momentum distribution by the Gaussian function with $T \simeq 2.7$ μK for the E2 transition and $T \simeq 13$ μK for the E1 transition. In general, the use of the E2 transition for laser cooling in the $\sigma_+ - \sigma_-$ spatial configuration of the field can lead to a deeper cooling of the atoms.

3.2. Laser cooling using a E2 transition in the lin ⊥ lin configuration of the field

Figure 5 shows the results of calculating the kinetic energy of cold atoms in the lin ⊥ lin spatial configuration of the field for various detunings of the light field as functions the dimensionless parameter u (18). In accordance with the results of work [20, 25], it can be seen that for the considered E1 transition with $F_{\rm g} = 1 \to F_{\rm e} = 2$, laser cooling at large detunings is characterised only by the parameter u (18) and depends little on detuning when the secular approximation condition $\sqrt{u} \ll |\delta|/\gamma_0$ is met [20]. The minimum kinetic energy of cold atoms is $E_{\rm kin} \simeq 20\hbar\omega_{0{\rm r}}$, and the momentum distribution (Fig. 6b) is well approximated by a Gaussian function with $T \simeq 3.5$ μK.

The dependence of the kinetic energy of cold atoms using the E2 transition in the lin ⊥ lin configuration of the field (Fig. 5a) is similar to that obtained in the $\sigma_+ - \sigma_-$ configuration of the field (Fig. 3a). The minimum kinetic energy of cold atoms, $E_{\rm kin} \simeq 7.6\hbar\omega_{0{\rm r}}$, is reached in the region of small detunings ($\delta = -\gamma$). In this case, the temperature obtained by approximating the momentum distribution by the Gaussian function (Fig. 6a) is ~2 μK, which is only slightly lower than that for the E1 transition.

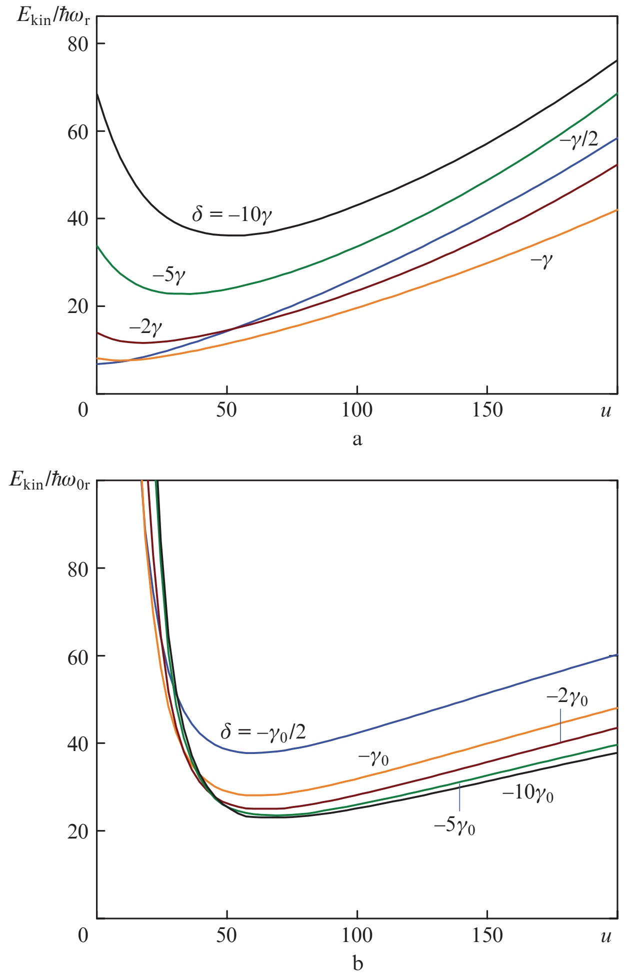

Figure 5. Kinetic energy of cold atoms in units of recoil energy in the lin ⊥ lin configuration of the field resonant to (a) quadrupole and (b) electric dipole transitions as a function of the dimensionless parameter u (18).

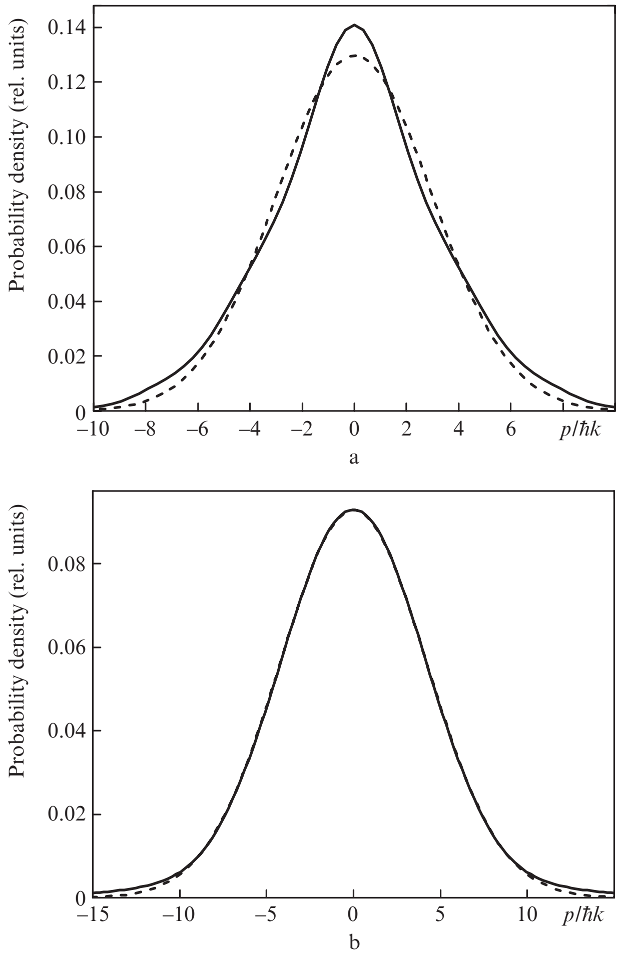

Figure 6. Momentum distribution of cold atoms in the lin ⊥ lin configuration of the field in units of the recoil momentum $\hbar k$ in fields resonant to (a) quadrupole and (b) electric dipole transitions for the light field parameters corresponding to the minimum kinetic energy of the atoms (Fig. 5): (solid curves) $\delta = -\gamma$, $\Omega^{(2)} \simeq 0.5\gamma$ for cooling using the E2 transition and $\delta = -10\gamma_0$, $\Omega \simeq 0.5\gamma_0$ for cooling using the E1 transition, as well as (dashed curves) an approximation by the Gaussian function.

4. Conclusions

We have examined the possibility of ultra-deep laser cooling of alkali atoms using a narrow quadrupole optical transition $^2{\rm S}_{1/2} - {}^2{\rm D}_{5/2}$ between the hyperfine structure levels with

$F_g = F \rightarrow F_e = F + 2$. The attainable energies of an ensemble of cold atoms are analysed on the basis of a numerical solution of the equation for an atomic density matrix with full allowance for quantum recoil effects in the interaction of atoms with photons of a light field. Based on the one-dimensional problem, for the lin ⊥ lin and $\sigma_+ - \sigma_-$ configurations of the light fields formed by orthogonally polarised counter-propagating waves, we have employed for caesium atoms a simplified model of optical transitions with $F_g = 1 \rightarrow F_e = 2$ (E1) and $F_g = 1 \rightarrow F_e = 3$ (E2) to calculate the minimum attainable values of kinetic energies and to estimate the temperatures.

It is shown that the use of a quadrupole transition allows one to cool atoms to energies equivalent to several recoil energies. Note that the recoil energy $\hbar\omega_r$ produced by an atom when a photon is absorbed by a field resonant to the E2 transition is slightly higher than the recoil energy $\hbar\omega_{0r}$ in a field resonant to the E1 transition. This makes it impossible to obtain significantly lower laser cooling temperatures than those reached by standard methods using an electric dipole E1 transition in the lin ⊥ lin configuration of the field.

Note that the attainable kinetic energy of cold atoms during laser cooling using the E2 quadrupole transition in the lin ⊥ lin and $\sigma_+ - \sigma_-$ configurations of the fields has similar dependences on the parameters of the light fields. Its smallest value is reached at small ($\delta \simeq -\gamma$) detunings, in contrast to laser cooling in fields resonant to the E1 transition. Also during cooling using the E1 transition, there is a significant dependence of the results on the selected polarisation configuration of the light field, which is explained by the manifestation of various polarisation mechanisms of sub-Doppler cooling in the lin ⊥ lin and $\sigma_+ - \sigma_-$ configurations of the fields. Slight differences in the results of laser cooling using the E2 transition in fields with different polarisation configurations, as well as the failure to reach temperatures below the Doppler limit (determined by the natural width of $^2D_{5/2}$), demonstrate the low efficiency of the sub-Doppler polarisation mechanisms of laser cooling in fields resonant to the E2 quadrupole transition.

A significant difference between the studied laser cooling schemes using light fields resonant to the E1 or E2 transitions is observed in the $\sigma_+ - \sigma_-$ spatial configuration of the fields. Such configurations of light fields are used for laser cooling of neutral atoms in a magneto-optical trap, where the lin ⊥ lin configuration of the field does not lead to the formation of a magneto-optical potential. In this connection, laser cooling of alkali atoms using a quadrupole transition can be promising as the second stage for deeper laser cooling in a magneto-optical trap.

Acknowledgements. The work was supported the Russian Foundation for Basic Research and the Government of the Novosibirsk Region (Grant No. 18-42-540003) and by the Ministry of Education and Science of the Russian Federation (Project No. 8.1326.2017/4.6).

References

1. Ludlow A.D., Boyd M.M., Ye J., Peik E., Schmidt P.O. *Rev. Mod. Phys.*, **87**, 637 (2015).
2. Taichenachev A.V., Yudin V.I., Bagayev S.N. *Usp. Fiz. Nauk*, **186**, 193 (2016).
3. Marti G.E., Hutson R.B., Goban A., Campbell S.L., Poli N., Ye J. *Phys. Rev. Lett.*, **120**, 103201 (2018).
4. Cornell E.A., Wieman C.E. *Rev. Mod. Phys.*, **74**, 875 (2002).
5. Ketterle W. *Rev. Mod. Phys.*, **74**, 1131 (2002).
6. Garraway B.M., Perrin H. *J. Phys. B: At. Mol. Opt. Phys.*, **49**, 172001 (2016).
7. Bloch I., Dalibard J., Zwerger W. *Rev. Mod. Phys.*, **80**, 885 (2008).
8. Kazantsev A.P., Surdutovich G.I., Yakovlev V.P. *The Mechanical Action of Light on Atoms* (Singapore, London: World Scientific, 1990; Moscow: Nauka, 1991).
9. Minogin V.G., Letokhov V.S. *Laser Light Pressure on Atoms* (New York: Gordon and Breach Science Publ., 1987; Moscow: Nauka, 1986).
10. Metcalf H.J., van der Straten P. *Laser Cooling and Trapping* (New York: Springer-Verlag, 1999).
11. Dalibard J., Cohen-Tannoudji C. *J. Phys. B: At. Mol. Phys.*, **18**, 1661 (1985).
12. Javanainen J. *Phys. Rev. A*, **44**, 5857 (1991).
13. Prudnikov O.N., Taichenachev A.V., Tumaikin A.M., Yudin V.I. *JETP*, **88**, 433 (1999) [*Zh. Eksp. Teor. Fiz.*, **115**, 791 (1999)].
14. Bezverbnyi A.V., Prudnikov O.N., Taichenachev A.V., Tumaikin A.M., Yudin V.I. *JETP*, **96**, 383 (2003) [*Zh. Eksp. Teor. Fiz.*, **123**, 437 (2003)].
15. Prudnikov O.N., Arimondo E. *J. Opt. B: Quant. Semiclass. Opt.*, **6**, 336 (2004).
16. Wineland D., Itano W. *Phys. Rev. A*, **20**, 1521 (1979).
17. Dalibard J., Cohen-Tannoudji C. *J. Opt. Soc. Am. B*, **6**, 2023 (1989).
18. Weiss D., Ungar P.J., Chu S. *J. Opt. Soc. Am. B*, **6**, 2072 (1989).
19. Prudnikov O.N., Taichenachev A.V., Tumaikin A.M., Yudin V.I. *JETP Lett.*, **70**, 443 (1999) [*Pis'ma Zh. Eksp. Teor. Fiz.*, **115**, 439 (1999)].
20. Castin Y., Dalibard J. *Europhys. Lett.*, **14**, 761 (1991).
21. Castin Y., Dalibard J., Cohen-Tannoudji C., in *Light Indiced Kinetic Effects on Atoms, Ions and Molecules* (Pisa: ETS, 1991) p. 5.
22. Castin Y., Molmer K. *Phys. Rev. A*, **50**, 5275 (1996).
23. Prudnikov O.N., Taichenachev A.V., Tumaikin A.M., Yudin V.I. *Phys. Rev. A*, **75**, 023413 (2007).
24. Prudnikov O.N., Taichenachev A.V., Tumaikin A.M., Yudin V.I. *JETP*, **104**, 839 (2007) [*Zh. Eksp. Teor. Fiz.*, **131**, 963 (2007)].
25. Prudnikov O.N., Il'enkov R.Ya., Taichenachev A.V., Tumaikin A.M., Yudin V.I. *JETP*, **112**, 939 (2011) [*Zh. Eksp. Teor. Fiz.*, **139**, 1074 (2011)].
26. Castin Y., Wallis H., Dalibard J. *J.Opt. Soc. Am. B*, **6**, 2046 (1989).
27. Chalony M., Kastberg A., Klappauf B., Wilkowski D. *Phys. Rev. Lett.*, **107**, 243002 (2011).
28. Prudnikov O.N., Ilenkov R.Ya., Taichenachev A.V., Yudin V.I. *Phys. Rev. A*, **99**, 023427 (2019).
29. Prudnikov O.N., Brazhnikov D.V., Taichenachev A.V., Yudin V.I., Bonert A.E., Il'enkov R.Ya., Goncharov A.N. *Phys. Rev. A*, **92**, 063413 (2015).
30. Kalganova E., Prudnikov O., Vishnyakova G., Golovizin A., Tregubov D., Sukachev D., Khabarova K., Sorokin V., Kolachevsky N. *Phys. Rev. A*, **96**, 033418 (2017).
31. Chan E.A., Aljunid S.A., Zheludev N.I., Wilkowski D., Ducloy M. *Opt. Lett.*, **41**, 2005 (2016).