

Laser optical frequency combs and their applications in optical fibre communication systems and astrophysics

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Abstract. This paper examines the generation and applications of laser optical frequency combs (LOFCs). The unique properties of LOFCs are widely used in spectroscopy and metrology, in particular for making ultra-accurate optical clocks. The use of LOFCs contributes to advances in optical fibre communication systems and studies pertaining to fundamental problems of astrophysics, such as the search for and investigation of exoplanets and verification of the hypothesis that physical constants vary with time.

Keywords: optical frequency comb, multiplexing, Doppler spectroscopy, calibration of astronomical spectrographs, exoplanets.

1. Introduction

Beginning immediately after their advent, the research of lasers progressed in two key directions, corresponding to different laser operation modes. In the case of pulsed lasers, the main purpose was, and still is, to obtain high beam peak power and intensity by reducing the pulse duration. For cw lasers, the main purpose is to obtain a narrow line in their emission spectrum. Both directions matured quite independently, using different approaches and engineering solutions, which culminated in important results.

In particular, the aim of obtaining short, high-power laser pulses led to the advent of *Q*-switched lasers capable of generating high-power nanosecond pulses. Further reduction in pulse duration, to the picosecond range, was ensured by mode locking. Subsequently, lasers were made that generated femtosecond pulses of extreme peak power.

On the other hand, the aim of minimising the width of the emission spectrum led to the advent of cw lasers having a very stable emission spectrum in the form of extremely sharp spectral lines. This was achieved to a significant degree by obviating spectral (Doppler, impact and other) broadening effects. In this context, a key issue was laser frequency measurements. For this purpose, it was necessary to compare the frequency of a spectral line of a laser in the optical region to the frequency of a caesium frequency time standard in the RF range. To this end, a complex chain of lasers and masers was used, covering the frequency range from the frequency of the laser under study to that of the caesium standard. Using nonlinear

optical methods, the output of one oscillator was converted into its harmonics, whose frequencies were compared to those of the other oscillator's harmonics. Since there are nonlinear conversion efficiency limits, it was necessary to use about ten such oscillators. The result was extremely complex, unique systems, which were made in just a few leading laboratories offering the highest scientific level. The purpose of those studies was improvements in precision metrology, in particular, the development of an ultra-accurate optical clock. Studies in this area, with the use of achievements in fibre optics, laser physics and nonlinear optics, have now made it possible to produce measurement systems with an accuracy at a level of a few parts in 10^{18} [1, 2].

The development of laser physics caused different research directions to merge. Advances in passively mode-locked lasers enabled cw operation of such lasers. They generate a continuous, strictly periodic train of identical femtosecond pulses coherent with each other. This circumstance is of critical importance for experimental studies of femtosecond pulses. To reduce the pulse duration, it was necessary to improve the temporal resolution of detection electronics. Unique systems based on electron-optical converters were designed for this purpose. Great effort was needed to reach a resolution at a level of hundreds of femtoseconds. At the same time, there is no need to investigate an individual pulse in the case of a strictly periodic pulse train. Studies of continuous pulse trains allow methods based on measurements of intensity correlation functions to be employed with great success. As a result, rather simple and affordable devices were obtained, which made it possible not only to measure the duration of femtosecond (or even attosecond) pulses but also to determine their shape.

Another extremely important feature is the emission spectrum. As a result of the coherence of light in all pulses of a continuous train, the spectrum, whose total width corresponds to the pulse duration, consists of a series (comb) of extremely sharp spectral lines, and the spacing between them is determined by the pulse separation in the train. Remarkably, the position of these lines can be controlled by fine-tuning laser parameters. Thus, a cw femtosecond laser (translator's note: the term was proposed by the author) is not only a source of high-power pulses but also a source of extremely sharp spectral lines of a laser optical frequency comb (LOFC). The facts that LOFCs can be obtained using femtosecond lasers and that the optical frequencies of their spectral lines can be tuned with high accuracy led to a paramount integration of these research directions. This groundbreaking achievement was recognised by awarding T.W. Hänsch and J.L. Hall the 2005 Nobel Prize in Physics.

It has now been well documented that LOFCs can be obtained using not only cw femtosecond lasers but also non-

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linear optical conversion of cw laser light having a sharp spectral line. Thus, various LOFC generation methods have been proposed, having particular advantages and drawbacks, which is very important for various applications. This paper considers applications of LOFCs in optical fibre communication systems (OFCS's) and astrophysics, based on different LOFC generation methods.

2. Laser optical fibre comb

2.1. Femtosecond laser-based LOFC

A key part of every laser is its cavity. The optical length of the cavity determines mode frequencies and the pulse repetition rate f_{rep} in the case of a cw femtosecond laser. As mentioned above, f_{rep} is equal to the line spacing in its spectrum. Note that the entire spectrum of a comb, consisting of equidistant lines (f_n), is offset by a certain frequency f_0 (Fig. 1). Thus, the frequency of the n th spectral line can be represented as $f_n = nf_{\text{rep}} + f_0$, where both frequencies fall in the RF range. Remarkably, these frequencies can be fine-tuned and measured via comparison with a microwave (Cs or Rb) frequency standard. An important issue is the technique for controlling and measuring f_0 . It is based on measuring the frequency difference between the high-frequency end of the LOFC and the doubled low-frequency end of the comb in the microwave range. The total width of the spectrum of the LOFC is determined by the gain linewidth of the active medium.

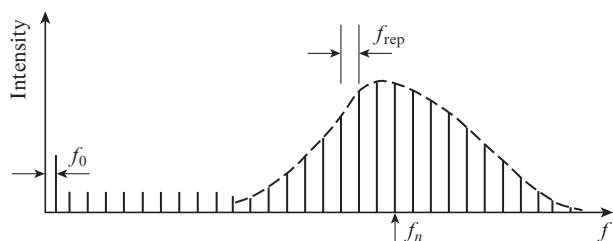


Figure 1. Emission spectrum of a cw femtosecond laser (for clarity, a small number of lines are shown; actually, the number of lines can reach a million).

Since the laser output consists of femtosecond pulses, they have high peak power and intensity. It is well documented that the propagation of such light in single-mode microstructured (photonic crystal) fibres is accompanied by a giant extension of their spectrum on account of supercontinuum generation, a nonlinear optical effect, but the line structure of the spectrum persists. Thus, using femtosecond lasers, one can obtain LOFCs with a spectral width spanning a considerable part of the optical region. This means that there is conceptual feasibility of having an extremely sharp laser line with an accurately measured frequency, which has determined the key applications of LOFCs in precision spectroscopy and metrology [3–7].

An inherent feature of and drawback to this LOFC generation method are the way in which the comb line spacing is controlled: by varying the optical length of the cavity. To this end, use is made of precision translation of one of the laser mirrors by a piezoelectric device. For operation in the optical region, the line spacing of the LOFC should exceed the resolution range of optical spectrometers, which is typically above

10–15 GHz. However, this requires that the optical length of the cavity be reduced to a few centimetres, which is extremely difficult to do. This drawback can be avoided by filtering the spectrum of the LOFC using Fabry–Perot (FP) etalons [6, 7]. An FP etalon has sharp spectral lines and offers high transmission at its resonance frequencies (wavelengths), which are determined by its optical thickness. If it is smaller than the optical length of the cavity of a femtosecond laser by a factor of m (where m is an integer), it will transmit LOFC frequencies decimated by a factor of m . Like the cavity length, the thickness of the FP etalon can be controlled using a piezoelectric system. Such mechanical systems are rather complex and difficult to run. Nevertheless, using such a technique, one can obtain an LOFC with a line spacing in the range 15–20 GHz, which is in effect a limit for this type of femtosecond LOFC generation system.

2.2. LOFC based on nonlinear optical effects in optical fibre

As shown earlier, LOFCs can be obtained not only using a cw femtosecond laser but also via nonlinear optical conversion of light from one or two cw lasers with sharp emission lines in optical fibre. This LOFC generation method relies on cascaded four-wave parametric conversion, a nonlinear optical effect during the propagation of high-intensity light in a single-mode fibre having Kerr nonlinearity and anomalous group velocity dispersion. Note that, to obtain an LOFC, input light should be modulated at the frequency of the LOFC line spacing. If such light of high intensity propagates in a nonlinear medium with appropriate dispersion, the periodic variation of intensity with time gradually transforms into a pulse train in the form of ultrashort solitons with a corresponding spectrum of the LOFC. The process is schematised in Fig. 2.

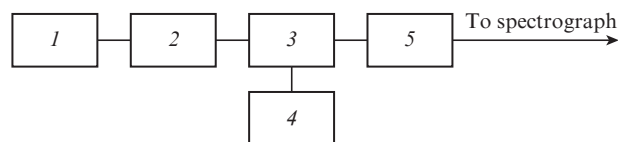


Figure 2. Block diagram of LOFC generation through four-wave parametric conversion:

(1) light source (two cw single-frequency lasers or one single-frequency laser with an electro-optical modulator); (2) section of single-mode fibre with anomalous dispersion; (3) section of fibre with gain; (4) pump diode laser; (5) section of a photonic crystal nonlinear optical fibre.

There are two approaches for modulating input light at the frequency of the LOFC spacing: using an electro-optical modulator (in such a case, the modulator frequency should be matched to a frequency standard) or two lasers having equal intensities, but differing in frequency by a certain LOFC spacing. The sum output of the lasers leads to beating at a frequency determined by the frequency difference between the lasers, which can be adjusted so as to obtain the required frequency of the LOFC spacing. Specifically, use is made of two single-mode cw lasers with sharp emission lines. One of the lasers operates at a strictly constant frequency, which should be accurately stabilised to an optical frequency standard, and the other has a tunable frequency. The difference between their frequencies determines the line spacing of the LOFC and

should be set using a frequency standard. The result is light with a beat frequency that can be tuned so as to set its exact value (frequencies of the order of hundreds of gigahertz can be obtained).

Next, the modulated light passes through a single-mode fibre section with anomalous dispersion, where a comb is performed. At an operating wavelength of $\sim 1.5 \mu\text{m}$, use is made of a standard telecom fibre, such as SMF-28, hundreds of metres long. Since comb generation is due to a nonlinear optical effect, the light should have a necessary intensity. Therefore, the light should be amplified, and the gain should be higher at a larger LOFC spacing, because this spacing is equal to the pulse repetition rate. The required gain can be achieved using amplifying fibre. Erbium-doped fibre is used at $\lambda = 1.5 \mu\text{m}$, and ytterbium-doped fibre, at $\lambda \approx 1.1 \mu\text{m}$. Diode lasers are used to pump amplifying fibre. Such a configuration was successfully demonstrated in a number of studies [8–10]. It is worth noting that this configuration was first studied with the aim of obtaining a regular train of ultrashort laser pulses with a high repetition rate, but of course in this process the possibility of LOFC generation was demonstrated as well.

Even though tuning the frequency difference between cw lasers can ensure very high frequencies of the LOFC spacing (up to hundreds of gigahertz), there is a certain difficulty. A high frequency of the LOFC spacing means a high repetition rate of the ultrashort pulses in the corresponding train, so that at a given average laser output power the peak pulse power decreases with increasing pulse repetition rate in the train. However, high peak power is needed to obtain a nonlinear effect, so a very important part of a laser system is its amplifying section, which ensures a sufficient average power. In this method, reaching a necessary average power in the amplifying section presents a certain difficulty.

One advantage of this method over the other LOFC generation methods is that it uses an all-fibre design, including well-tested, commercially available fibre-optic devices. Another important benefit is the possibility of obtaining LOFC spacings of up to hundreds of gigahertz. To monitor and tune the LOFC spacing, it is necessary to monitor and tune the emission frequencies of the two cw lasers. The use of an electro-optical modulator allows one to have only one master oscillator, with precise tuning of the emission frequency and, obviously, similar tuning of the modulator. In such a case, the maximum LOFC spacing is limited by characteristics of the electro-optical modulator. The approach involving electro-optical modulation is currently thought to be the most attractive.

2.3. Microresonator-based LOFC

Yet another promising approach to LOFC generation is to use a monolithic microresonator [11–14] in the form of a millimetre-size torus or ring made of a transparent material with an extremely small absorption coefficient for laser light propagating in it (Fig. 3). In this process, the light experiences total internal reflection from the wall. The result is multiple light propagation through the microresonator, which is referred to as the ‘whispering gallery’ effect by analogy with that in architectural acoustics. The interference of waves during multiple round trips means the formation of modes with frequencies determined by the ring round-trip time, which correspond to an ultrahigh quality factor ($Q \sim 10^8$). Radiation from a cw laser with a sharp line and required intensity is coupled into a microresonator using optical fibre (Fig. 4). The distance between the output fibre end and microresonator surface is such

that some of the light enters the microresonator owing to frustrated total internal reflection. The light is outcoupled from the microresonator in the form of an LOFC in the same way.

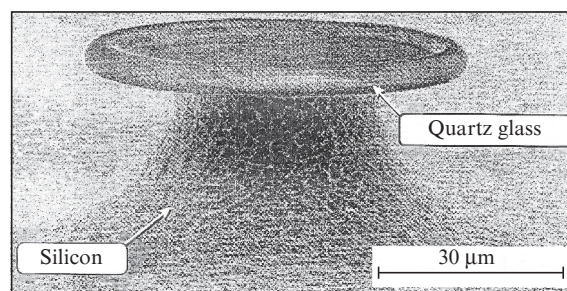


Figure 3. Microresonator made of silicon and quartz glass.

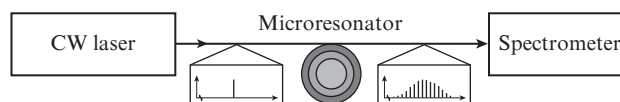


Figure 4. Coupling single-mode laser radiation into a microresonator and outcoupling the LOFC radiation.

If the microresonator material has appropriate Kerr non-linearity (Kerr microresonator), a certain light intensity leads to nonlinear parametric interaction between the pump laser output and microresonator modes. The result is light in the form of an LOFC, which is also outcoupled into optical fibre due to the frustrated total internal reflection from the surface-fibre end interface. The microresonator size, of the order of a few millimetres, means a large mode spacing (up to hundreds of gigahertz), which in turn determines the frequency range of the LOFC. Possible microresonator materials include glass, fused silica, fluorite (CaF_2) and silicon nitride. A single-frequency fibre laser with a wavelength near $1.5 \mu\text{m}$ can be used as a pump source. One advantage of the method in question is the miniature design of the device, which is consistent with current trends in microelectronics. Besides, it offers the possibility of obtaining LOFC spacings up to hundreds of gigahertz. At the same time, the fabrication of Kerr microresonators requires a special epitaxial process. Moreover, since the LOFC spacing is determined by the microresonator size, the spacing is extremely difficult to fine-tune, which is an inherent drawback.

3. Applications of LOFCs

Extremely important and advanced applications of LOFCs are multiplexing systems in OFCS's and applications in astrophysics, related to precise measurements in astronomical spectroscopy.

3.1. Applications in OFCS's

The development of OFCS's is related to the aim of increasing the volume and rate of information transfer. To transmit information with the use of advanced electronics, a channel should have a transmission bandwidth at a level of several gigahertz. OFCS's employ optical radiation with a wavelength of $\sim 1.5 \mu\text{m}$, and the gain bandwidth of fibre amplifiers

is ~ 50 nm, i. e. $\sim 5 \times 10^{12}$ Hz. Thus, single-mode fibre in an OFCS can in principle transmit information through hundreds of channels with a bandwidth (50-GHz standard) required for ultra-high speed communication systems. Since each channel has a separate laser transmitter, many such lasers, each operating at a particular frequency of a channel, will be required. An attractive idea is to replace them by a system based on a single laser generating many frequencies with wavelength division multiplexing of communication channels using LOFCs [15]. In principle, high accuracy of comb lines is unnecessary. It is sufficient if they fall in desired channels, so the microresonator-based system seems to be the best suited. An important feature is the conceptual feasibility of integrating chips into the system, with realisation of modern principles of photonics. A major problem is a scheme for launching many beams into the core of single-mode fibre and separating the output into separate channels after the beam passed through the OFCS. Proposed solutions to this problem include the use of fibre Bragg gratings and optical fibre Mach–Zehnder interferometers.

Pfeifle et al. [16] reported the fabrication and investigation of a multiplexing system using an LOFC. They used a microresonator made of SiN and demonstrated data transmission at a rate of 1.44 Tbit s^{-1} over 300 km in 20 channels. Multiplexing and demultiplexing systems were described in detail by Liu et al. [17]. An important feature is the miniature design of the LOFC generator. Results of an analogous study were reported by Fulöp et al. [18]. It is important to note that the pump laser power at $\lambda = 1.5 \mu\text{m}$ did not exceed 400 mW. They demonstrated 80-km data transmission at 4.4 Tbit s^{-1} .

3.2. Applications in astrophysics

Optical spectroscopy is an effective tool for astrophysical studies. Light from distant cosmic objects is collected by a telescope and analysed by an astronomical spectrograph. The intensity of the light collected at the focus should exceed the sensitivity of the spectrograph. It is, therefore, necessary to increase the telescope mirror area. This leads to making giant telescope systems capable of precisely tracking the position of the object of interest, which requires mechanical translation of the system with high accuracy. On the other hand, to ensure high resolution and sensitivity of a spectrograph, it is necessary to produce complex spectroscopic systems employing echelle gratings tens of centimetres in size, with high thermal and mechanical stability. Since these are bulky stationary devices, naturally there is a need to transmit light of extremely low intensity collected at the focus of a telescope to the input of a stationary spectrograph. To this end, use is made of specially designed multimode optical fibres with a core diameter of up to hundreds of microns, having the minimum possible losses and completely precluding bend-induced spectral distortions. Since images at the focus of a telescope are micron-sized, it is necessary to ensure matching to the fibre input and between the fibre output and spectrograph entrance using microlenses. As shown in special studies, a certain cross-sectional fibre microstructure is needed. The result is a state-of-the-art configuration of a fibre-fed astronomical spectrograph [19].

One of the most important issues in astronomical spectroscopy is the ability to accurately measure Doppler shifts of spectra, which allows the radial velocity (RV) of an object to be determined. This is astronomical Doppler spectroscopy. Its outstanding achievement was the discovery of the redshift of spectra of distant galaxies, which was shown to be propor-

tional to the distance to them (Hubble's law). The Doppler shift in such cases is enormous and the RV reaches a fraction of the speed of light. At the same time, small Doppler shifts also allow rather important results to be obtained. The RV of some stars was found to exhibit small, but strictly periodic variations with time [20]. This was interpreted as due to the effect that a planet orbiting a star has on its motion. In this way, an indirect, but highly reliable method made it possible to discover planets around stars beyond the Solar System (exoplanets).

The Doppler shift and the corresponding change in RV depend on the relative masses of the star and planet. In particular, the rotation of Earth around the Sun causes a maximum change in RV of 9 cm s^{-1} , with a one-year period. This suggests that prolonged spectroscopic measurements with an accuracy at a level of a few parts in 10^{10} are needed for Earth-like exoplanets orbiting Sun-like stars to be detected by Doppler spectroscopy. Thus, precision calibration of a spectrograph is needed. Conventional calibration methods with the use of spectral lamps or iodine cells are incapable of ensuring measurement accuracy better than a few parts in 10^7 . Immediately after the advent of LOFCs, their unique capabilities attracted researchers' attention. However, for practical application of such a calibrator, the frequency range of the comb should exceed the resolution range of the astronomical spectrograph. This means that the line spacing of the LOFC should be at least 10 GHz.

To design a precision calibrator for astronomical spectrographs, researchers at the Max Planck Institute for Quantum Optics (Germany) and Menlo Systems GmbH (Germany) chose a classic LOFC configuration based on a cw femtosecond laser, with subsequent light filtering by Fabry–Perot etalons [21]. The master oscillator used was a femtosecond Yb-doped fibre laser (wavelength, $\sim 1 \mu\text{m}$; pulse repetition rate, 250 MHz), with subsequent light filtering to 18 GHz. Using harmonic and supercontinuum generation, they obtained an LOFC spectrum spanning the entire visible range. The instrument thus created (FC1000-250, Menlo Systems) was successfully tested at the La Silla Observatory (Chile) in a system that included a 3-m telescope and the unique spectrograph HARPS (High Accuracy Radial Velocity Planet Searcher).

However, this instrument is rather difficult to operate, is extremely expensive and is only suitable for calibrating astronomical spectrographs with a resolution of at least tens of thousands. Because of this, it was necessary to find other approaches. Researchers at the Leibniz Institute for Astrophysics Potsdam (Germany) proposed and implemented a fibre-optic LOFC [22, 23] using commercially available fibre-optic components. Along with a reduced cost and complexity, an important feature of this calibrator is the possibility of producing LOFCs with line spacings of up to hundreds of gigahertz. As an illustration, Fig. 5 shows a portion of the spectrum of the calibrator and (for comparison) the emission spectrum of a spectral lamp. Considerable line spacings of LOFCs mean that such an instrument can be used for calibrating not only unique spectrographs but also spectrographs with medium and even relatively low resolution. This is very important because it allows the search for Earth-like exoplanets to be considerably extended. A novel version of this type of astronomical calibrator was used with a 3.5-m telescope (Calar Alto telescope), and the performance of the system was assessed via spectral measurements for stars HD3765 and HD219538 [24]. In this version of LOFC generation, two cw lasers with equal emission intensities and a certain frequency difference

were used as a master oscillator. At the same time, as shown above, one can use one cw laser in combination with an electro-optical modulator, which significantly simplifies the design of the system. Clearly, high accuracy of the modulator frequency is required in such a case. This type of calibrator for astronomical spectrographs was successfully investigated by Yi et al. [25]. The operation of an analogous system was demonstrated in an observatory through RV measurements for the standard star HD221354 [26]. An important feature was a compact design of the system. The entire device was accommodated in three boxes 45×45 cm in dimensions. This is of great importance for practical application of the astronomical calibrator in observatories.

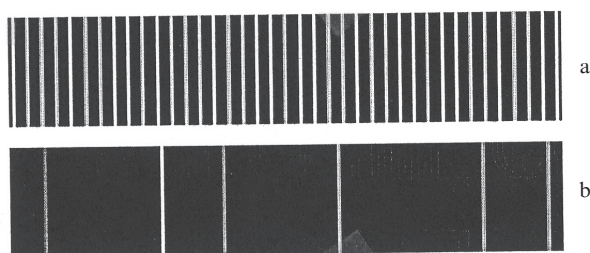


Figure 5. Line spectra of (a) the fibre-optic calibrator and (b) (for comparison) a spectral lamp.

It is worth noting that the discovery and characterisation of exoplanets is driven by inherently scientific, sacramental questions about the uniqueness of Earth and our solar system. The existence of a huge number of stars causes a natural desire to find out whether life exists on exoplanets. A critical condition for life is the presence of liquid water. This means that, among the many exoplanets, one should focus on those whose surface temperature falls in a certain range (so-called habitable zone). Clearly, this depends on the star temperature. Because of this, preferable objects are M-type (red dwarf) stars. As mentioned above, the Doppler shift depends on the mass of the star. One advantage of the red dwarf stars is that their habitable zone exoplanets cause large Doppler shifts (typical values are of the order of metres per second, against centimetres per second in the case of Earth). The orbital period of exoplanets does not exceed a year. Because of the reduced temperature of the red dwarf stars, their spectrum is redshifted, so the corresponding spectral apparatus should operate in the red and near-IR ranges. Finally, it should be kept in mind that a vast majority of stars are of this type.

These considerations suggest that, to detect Earth-like habitable zone planets, it is important to develop astronomical Doppler spectroscopy techniques. Such studies are presently under way and novel results have already been obtained [27, 28]. An electro-optical LOFC system based on achievements in fibre optics has been developed for calibration of spectrographs. The output of a 1064-nm cw laser is modulated by a frequency and a phase modulator at a frequency of 30 GHz. A fibre-optic system employing different types of optical fibre generates an ~ 300 -THz LOFC (800–1350 nm). The entire system is 60×152 cm in dimensions, which allows for transportation. The spectrograph proper, of the HPF (Habitable Zone Planet Finder) type, has high mechanical and thermal stability and operates at cryogenic temperatures under vacuum. Particular astronomical measurements were made on a 10-m telescope (Hobby-Eberly telescope) and demonstrated

the possibility of determining RVs at a level of 1 m s^{-1} with a measurement accuracy better than 10 cm s^{-1} for an M-type star with a habitable zone exoplanet.

In addition to the search for and investigation of exoplanets, precise spectral measurements allow other important issues in astrophysics to be addressed, including the possibility of direct measurements of the expansion of the universe. To this end, one should perform precise redshift measurements for a considerable length of time. This idea was put forward as early as 1962 [29], but estimates showed that, with the accuracy of spectral measurements achieved by that time, the measurements in question would take many hundred years. The discovery of LOFCs radically changed the situation. There are plans to make RV measurements for quasars using the latest achievements of precision spectroscopy. To this end, the CODEX (COsmic Dynamics EXperiment) international project is under development [30, 31]. There are plans to use giant telescopes with perfect spectrographs equipped with LOFC calibrators.

Yet another important application of LOFCs in precision spectroscopy is related to absolute measurements of the spectrum of hydrogen and determination of the Rydberg constant and fine-structure constant α . In particular, researchers are examining the feasibility of experimental verification of Dirac's hypothesis that the fundamental physical constants vary with time. Studies in this area, including astronomical ones, were reviewed by Kolachevsky [32].

4. Conclusions

Among many applications of LOFCs, an important one is their use for multiplexing in OFCS's. This is related to the need for an increase in data transmission rate up to several petabit per second. Studies in this area pertain to photonics, a new scientific area.

Another very important application of LOFCs pertains to basic research, namely, to astrophysics. It is no exaggeration to say that one of the main achievements of astrophysical studies in the past 25 years is the discovery of planets beyond our solar system (exoplanets). An advanced method for the detection and characterisation of exoplanets is Doppler measurements of small variations in RV. There is convincing evidence that modern requirements for the calibration of astronomical spectrographs are successfully met by LOFC generation techniques.

Examination and analysis of reports in this area of research shows that interdisciplinary interaction of physicists, engineers and astronomers is of great importance. There is an obvious need for mutually beneficial cooperation of experts in the fields of laser physics and nonlinear and fibre optics, experts in precise astronomical engineering and astronomers carrying out direct studies in observatories.

References

1. Holly L. et al. *Optica*, **4**, 979 (2017).
2. Rolland A. et al. *Optica*, **4**, 1070 (2017).
3. Jones D.J. et al. *Science*, **288**, 635 (2000).
4. Udem T. et al. *Nature*, **416**, 233 (2002).
5. Cundiff S.T. et al. *Rev. Mod. Phys.*, **75**, 325 (2003).
6. Steinmetz T. et al. *Science*, **321**, 1335 (2008).
7. Li C-H. et al. *Nature*, **452**, 610 (2008).
8. Dianov E.M. et al. *Opt. Lett.*, **14**, 1008 (1989).
9. Mamyshev P.M. et al. *IEEE J. Quantum Electron.*, **27**, 2347 (1991).
10. Chernikov C.V. et al. *Electron. Lett.*, **28**, 931 (1992).

11. Armani D.K. et al. *Nature*, **421**, 925 (2003).
12. Kippenberg T.J. et al. *Phys. Rev. Lett.*, **93**, 083904 (2004).
13. Savchenkov A.A. et al. *Phys. Rev. Lett.*, **93**, 243905 (2004).
14. Del'Haye P. et al. *Phys. Rev. Lett.*, **107**, 063901 (2011).
15. Takara H. et al. *Electron. Lett.*, **36**, 2089 (2000).
16. Pfeifle J. et al. *Nat. Photonics*, **8**, 375 (2014).
17. Liu A. et al. *IEEE J. Sel. Top. Quantum Electron.*, **16**, 23 (2010).
18. Fulöp A. et al. *Nat. Commun.*, **9**, 1598 (2018).
19. Panchuk V.E. et al. *Astrophys. Bull.*, **66**, 355 (2011) [*Astrofiz.. Byull.*, **66**, 382 (2011)].
20. Mayor M., Queloz D. *Nature*, **378**, 355 (1995).
21. Wilken T. et al. *Nature*, **485**, 611 (2012).
22. Boggio J.M.C. et al. *Proc. SPIE*, **8434**, 84340Y (2012).
23. Zajnulina M. et al. *Appl. Phys. B*, **121**, 171 (2015).
24. Boggio J.M. et al. *Opt. Commun.*, **415**, 186 (2018).
25. Yi X. et al. *Nat. Commun.*, **7**, 10436 (2016).
26. Obrzud E. et al. *Opt. Express*, **26**, 34830 (2018).
27. Metcalf A.J. et al. *Optica*, **6**, 233 (2019).
28. Metcalf A.J. et al. *Opt. Lett.*, **44**, 2673 (2019).
29. Santage A. *Astron. J.*, **136**, 319 (1962).
30. Liske J. et al. *Mon. Not. R. Astron. Soc.*, **386**, 1192 (2008).
31. Pasquini L. et al. *Proc. SPIE*, **7735**, 77352F (2010).
32. Kolachevsky N.N. *Phys. Usp.*, **51**, 1180 (2008) [*Usp. Phys. Nauk*, **178**, 1225 (2008)].