Quantum Electronics **49** (8) 728–734 (2019)

https://doi.org/10.1070/QEL17017

Rate equations for the diode laser and their applicability area

A.P. Bogatov

Abstract. **Taking into account spontaneous emission by adding its intensity in the form of a constant term to the laser emission intensity in rate equations is shown to be an inadequate approach. The use of rate equations without equations for phases of waves is limited to problems in which light is characterised by its total intensity, without its spectral composition. Such equations are incapable of adequately modelling the emission spectrum of diode lasers.**

Keywords: *diode lasers, spontaneous emission, rate equations.*

1. Introduction

This paper can be viewed as a continuation of a previous one [1]. I use the same notation as previously and continue to critically examine the approach to the theory of the diode laser as a laser with an 'asymptotic' lasing threshold and the ideas described in Refs [2, 3].

In the literature dealing with diode lasers, so-called rate equations are rather often used as a point of departure for analysis of their characteristics. They are very convenient owing to their clarity and simplicity. In problems where coherent properties of light are unimportant and only the total light intensity is considered, these equations rather adequately describe the real situation because the underlying physical models rely on the conservation-of-energy principle as applied to a system comprising a gain medium and an electromagnetic field in a cavity.

From early work (see e.g. Basov et al. [4] and references therein) to the present day, rate equations have been successfully used to study unsteady-state operation of diode lasers and the dynamics of laser output intensity in the case of direct high-frequency modulation by the pump current. The convenience and simplicity of using rate equations led many researchers to modify them in order to extend the range of problems that can be treated using them. This probably was one of the reasons that works began to appear where the physics underlying rate equations turned out to be insufficient for solving the problems involved. This refers to studies of the steady-state operation of diode lasers, in particular to analysis of spectral characteristics of light in this mode. Indeed, a given energy in a cavity can be concentrated in one spectral line or distributed over several lines (modes). In connection with this, it became necessary to modify rate equations with allowance for additional physical mechanisms capable of controlling the energy distribution over laser cavity modes. It turned out that, in Refs [2, 3] and other works, rate equations were modified inadequately.

Given the above, the purpose of this work is to analyse the applicability of rate equations to modelling the spectral characteristics of diode lasers.

A.P. Bogatov P.N. Lebedev Physical Institute, Russian Academy of Sciences, Leninsky prosp. 53, 119991 Moscow, Russia;
e-mail: ya.bogatov47@yandex.ru

Received 18 March 2019; revision received 29 May 2019
Kvantovaya Elektronika **49** (8) 728–734 (2019)
Translated by O.M. Tsarev

2. Analysis of rate equations

In this work, rate equations, no matter how written, are taken to mean equations in which an electromagnetic field is represented only by light intensity (absolute square of the wave amplitude) at one or several optical frequencies, whereas the phase of waves at these frequencies is completely ignored. Another feature of rate equations is that spontaneous emission is taken into account by introducing a constant term or terms for the intensity of each laser mode. The most complete and typical form of rate equations is presented in Suhara [5] [Eqns (6.29a) and (6.29b)]:

$$\frac{\mathrm{d}S_m}{\mathrm{d}t} = \Gamma_m\Big(G_m - \sum_j \xi_{mj} S_j\Big) S_m - \frac{S_m}{\tau_{\mathrm{ph}}} + \frac{C_{\mathrm{s}m} N}{\tau_{\mathrm{s}}}, \tag{1}$$

$$\frac{\mathrm{d}N}{\mathrm{d}t} = \frac{J}{qd} - \frac{N}{\tau_{\mathrm{s}}} - \sum_m \Gamma_m\Big(G_m - \sum_j \xi_{mj} S_j\Big) S_m, \tag{2}$$

where (like in the original publication) S_m and G_m are the photon density and gain coefficient of the mth mode; N is the electron concentration (inversion density); τ_{ph} is the photon lifetime in the cavity; the coefficient $C_{\mathrm{s}m}$ takes into account the contribution of spontaneous emission to the mth mode; Γ_m is the optical confinement factor of the mth mode; the coefficients ξ_{mj} take into account gain saturation; d is the thickness of the active layer; q is the electron charge; J is the pump current; and τ_{s} is the spontaneous recombination time.

It is worth noting two serious drawbacks to Eqns (1) and (2). One of them is that spontaneous emission is taken into account without proper substantiation and, as a consequence, inadequately. Indeed, physical considerations suggest that spontaneous emission 'entering' a particular cavity mode is spectrally limited by the transmission band of the cavity. This means that it will certainly interfere with ('laser') light already present in the mode. Since the transmission band of the cavity corresponds to the inverse of its response time, such dynamic interference will lead to inversion dynamics and, hence, the dynamics of the gain and other lasing parameters. The dynamic interference effect due to the addition of the amplitudes of a strong (laser) and a weak (spontaneous) field considerably exceeds the effect due to the addition of their intensities. In connection with this, taking into account spontaneous emission by adding a time-independent term to the intensity (addition of intensities), which completely ignores the interference effect, appears at least unjustified.

The other drawback stems from the fact that the equations are written not in terms of mode amplitudes but in terms of mode intensities, S_m, so they automatically ignore the mechanism of interaction between the fields of different modes through intermode intensity beating and the corresponding inversion oscillations. Even in the first studies of lasers (see e.g. Refs [6; 7, p. 315]), this mechanism was included in analysis as predominant in multimode lasing. It can only be revealed by sequentially solving Maxwell's equations together with equations for the gain medium density (inversion) matrix. In the case of diode lasers, interaction between the fields of different modes is especially strong. The corresponding induced additional gain (absorption) can exceed the spectral difference in material gain between neighbouring modes by an order of magnitude and more. This mechanism was first studied with application to diode lasers by Bogatov et al. [8]. To obtain direct evidence of whether or not the corresponding modification of rate equations is adequate for modelling the emission spectrum of a diode laser, I sequentially analyse the dynamics of lasing intensity proceeding from Maxwell's equations for each physical process involved.

3. Contribution of spontaneous emission

Consider first a rate equation of the simplest form, which was used by Suhara [5] [Eqns (6.28a) and (6.28b)] for single-mode lasing:

$$\frac{\partial N}{\partial t} = -\Gamma G S - \frac{N}{\tau_s} + \frac{J}{dq}, \tag{3a}$$

$$\frac{\mathrm{d}S}{\mathrm{d}t} = \left(\Gamma G - \frac{1}{\tau_{\mathrm{ph}}}\right)S + \frac{C_s N}{\tau_s}. \tag{3b}$$

Sequentially deriving a dynamic equation for single-mode lasing intensity involves the following: All the terms of the abbreviated equation (21) in Ref. [1] for the 'slow' mode amplitude $A(t)$ should be multiplied by the complex conjugate of the amplitude, $A^*(t)$, and then the same should be done with the complex conjugate of Eqn (21). Adding up the two equations obtained, we have

$$A^* \frac{\mathrm{d}A}{\mathrm{d}t} + A\frac{\mathrm{d}A^*}{\mathrm{d}t} = \frac{\mathrm{d}S}{\mathrm{d}t} = 2\Omega_0 \tilde{n} S + \left[2\mathrm{i}\xi \sum_j^{r_j \in V_{\mathrm{act}}} A^*(t) d_j(t) \langle \boldsymbol{e}_j \bar{u}_c(r_j)\rangle + \mathrm{c.\,c.}\right]. \tag{4}$$

Here $S = |A|^2 \omega Q$ is the mode energy, which is often referred to (not quite adequately) as 'photon density', and the coefficient $\xi \sim 1/\sqrt{Q}$ is defined by relations (22) in Ref [1]. In this way, we obtain a dynamic equation for S, where $2\Omega_0\tilde{n}$ is a so-called net gain, i.e. gain minus loss. This expression is completely analogous to the term $\Gamma G - 1/\tau_{\mathrm{ph}}$ in (3b). The second term (in the square brackets) represents the action of spontaneous emission sources. It is nothing but energy inflow [or outflow, depending on the relationship between the phases of $A(t)$ and $d_j(t)$] in the laser cavity due to the interference of the laser field with microcurrents of spontaneous emission sources. As a result, Eqn (4) describes the electromagnetic field energy balance in the laser cavity, including the term representing spontaneous emission sources. This can be imagined clearly if we take into account that harmonic time dependences of the field $E(t)$ and dipole oscillations meet the relation

$$\left(\mathrm{i}\sum_j^{r_j \in V_{\mathrm{act}}} A^*(t) d_j(t) + \mathrm{c.c.}\right) \sim E(t)\boldsymbol{j}(t),$$

where $\boldsymbol{j}(t)$ is the sum of microcurrents in the cavity due to intrinsic electron motion upon resonance transitions.

Thus, only Eqn (4) can be thought of as an adequate rate equation for single-frequency lasing with allowance for spontaneous emission sources.

The term $C_s N/\tau_s$, corresponding to spontaneous emission in (3), differs fundamentally from its analogue in (4), as a result of which the effect of spontaneous emission is inadequately taken into account in 'asymptotic threshold' models. The misinterpretation of the positive definite term $C_s N/\tau_s$, attributed to the effect of spontaneous emission and appearing in equations of the form (3), is most likely the result of intuitive views of the addition of the intensities of separate fields: 'laser' field and 'spontaneous emission' field. Actually, these fields are indistinguishable and are a single field in the cavity. What can be thought of as spontaneous emission in a cavity originates in it from microcurrents of randomly oscillating dipoles (incoherent with the laser field) at once in the form of stochastic modulation of the laser field amplitude. Gain saturation also results not from the sum of the 'laser' intensity and independently arriving 'spontaneous' emission but from a single field with an amplitude and phase modulated by the currents of incoherently emitting dipoles, as shown earlier [1]. The latter refers to the interpretation of mathematical expressions and can be understood differently. This is particularly noticeable if lasing is interpreted in terms of quantum theory.

It is also worth paying attention to the fact that, performing operations over the starting equation (21) from Ref. [1], which led to the rate equation (4) for mode intensity S, we lost some of the information present in the starting equation (21). Equation (4) is not 'closed-form' because it does not contain the laser field phase in explicit form. It describes the dynamics of only laser output energy (intensity). It is easy to see that this information can be retained. To this end, we should find the difference $A^*\mathrm{d}A/\mathrm{d}t - A\mathrm{d}A^*/\mathrm{d}t$. This will allow us to obtain another equation for the phase φ:

$$S\frac{\mathrm{d}\varphi}{\mathrm{d}t} = \Omega_0 \tilde{n} \tilde{R} S + \left[\xi \sum_j^{r_j \in V_{\mathrm{act}}} A(t) d_j^*(t) \langle \boldsymbol{e}_j \bar{u}_c(r_j)\rangle + \mathrm{c.\,c.}\right].$$

This equation together with (4) and the equation for electron concentration (3a) form a complete system similar to the system of equations (22) and capable of providing adequate solutions analogous to those to Eqns (22).

Thus, Ivanov et al. [2] and Kurnosov V.D. and Kurnosov K.V. [3] are mistaken in not only that they inadequately represent the contribution of spontaneous emission but also that they basically ignore the dynamics of the laser field phase, which makes their calculation inadequate.

Clearly, in the case of single-frequency lasing, with spontaneous emission neglected, the rate equations (3) are quite adequate. In such a case, they merely reflect the conservation-of-energy principle. However, in the case of multimode lasing, Eqns (1) and (2) written in terms of individual mode intensities become, generally speaking, inadequate, even if spontaneous emission is neglected. Such equations do not contain dynamics due to oscillations at difference mode frequencies because the oscillation amplitude depends on phase relationships between mode field amplitudes.

4. Photons in the cavity of a diode laser

Discussion of whether the presence of the term $C_s N/\tau_s$, thought to represent spontaneous emission, in a rate equation is adequate almost always evolves into discussion of whether this circumstance is obvious. As a rule, it is stated in this context that, in a steady state, the energy balance constraint should be met in the laser cavity. Next, passing to the quantum theory language, researchers put forward an argument that is convincing and obvious in their opinion: since the energy in a cavity is determined by the number of photons, the balance should be extended to include the number of photons. There are three mechanisms controlling such balance: stimulated emission, absorption and output coupling losses and spontaneous emission. Without spontaneous emission, the photon generation rate would be equal to the rate of decrease in the number of photons. However, an additional photon inflow due to spontaneous emission disturbs this equilibrium, so the gain in a laser is always lower than the loss because there is always spontaneous emission. Accordingly, lasing threshold is never reached. The gain in a laser only asymptotically approaches the loss from below as the pump current (laser output power) increases. According to a number of researchers, this is the 'asymptotic' property of the lasing threshold.

One observation follows from the doubtfulness of adding up and subtracting 'photons' like Cuisenaire rods. The concept of photon emerged in quantum theory as a minimum discrete characteristic of a change in electromagnetic field energy. As to physical quantities, including field energy, they are found in quantum theory as the trace of the product of the operators of the corresponding quantity (a^+a for the number of photons) and the density matrix or as the average of the action of an operator on the wave function of a pure state.

Quantum theory treats an electromagnetic field as a system of oscillators in which to each oscillator corresponds its spatial mode (see e.g. Ref. [7, p. 158]). In our case, this is a vector function of spatial coordinates, $\bar{u}(r)$, defined by the classical equation (11) in Ref. [1]. In such a case, quantum theory deals only with the temporal dynamics of the field amplitude (for example, of the electric field intensity), which can be expressed through a generalised canonical coordinate $\tilde{x}$ and generalised canonical momentum κ of an oscillator using secondary quantisation. To a pure state with a particular energy or (what is the same) a particular number n of quanta (photons) there corresponds a wave function in a coordinate representation, $\varphi_n(\tilde{x})$, in the form [9]

$$\varphi_n(\tilde{x},t) = \pi^{-1/4}(2^n n!)^{-1/2} H_n(\tilde{x})\exp(-\tilde{x}^2/2 - \mathrm{i}n\omega t),$$

where H_n is a Hermitian polynomial.

To this quantum state with a particular number of photons, n, there does not correspond any state of a classical oscillator. In this state, the potential and kinetic energies are equal to each other and time-independent, and the oscillation phase is not completely definite. An approximate classical analogue of this state is an ensemble of an infinite number of identical oscillators with an evenly distributed oscillation phase. For this reason, the state of the field in a laser cavity is described by other states, so-called coherent states, in which the potential and kinetic energies are oscillating functions of time, periodically transforming into each other (like in a classical oscillator), except for the zero oscillation energy. Each of these coherent unsteady electromagnetic field states, represented by the function $\psi(\tilde{x},t)$, are a superposition of an infinite number of $\varphi_n(\tilde{x},t)$ functions corresponding to steady states with a particular energy (number of photons) [7, p. 158; 9]. The function $\psi(\tilde{x},t)$ of a generalised canonical coordinate $\tilde{x}$ and time t can be written as follows:

$$\psi = \exp\left(-\frac{b^2}{4} - \frac{\mathrm{i}\omega t}{2}\right)\sum_n C_n\varphi_n(\tilde{x},t) =$$
$$= \exp\left\{-\frac{[\tilde{x} - b\cos(\omega t)]^2 - \mathrm{i}\theta}{2}\right\}, \qquad (5)$$

where

$$C_n = \frac{\exp(-b^2/4)b^n}{\sqrt{2^n n!}}; \ \theta = \frac{\omega t}{2} + b\tilde{x}\sin(\omega t) - \frac{b^2}{4}\sin(2\omega t).$$

It follows from the right-hand side of Eqn (5) that, in the case of the existing dynamics of a coherent state of the field, its 'coordinate' $\tilde{x}$ (field amplitude) behaves very similarly to the coordinate of a classical oscillator with an amplitude $\backsim b$.

The average field energy in a cavity can be expressed through the average number of photons, $\bar{n}$, in a coherent state if we take into account that the numbers C_n^2 are a Poisson distribution with an average $b^2/2$. As a result we obtain $\bar{n} = (b^2/2)\hbar\omega$. Note here that it is somewhat inconvenient to use the quantum approach because, even though functions of a coherent state form a complete set, they are not orthogonal to each other, which further complicates the mathematics involved.

Another, no less significant, argument for choosing the classical approach is that, at e.g. a laser output power of at least 10 μW, cavity Q no less than 10^3 and photon energy of $\sim$1 eV, the number of photons is $\bar{n} > 10$. It is clearly demonstrated in Schiff [10] that, even at $\bar{n} = 10$, the classical and quantum oscillator coordinate probability density distributions differ rather little. Since practically significant powers of diode lasers are well above 10 μW (the same refers to their Q), the field in their cavity can be analysed with good accuracy in terms of classical theory. In the IR and, especially, microwave regions, the critical power at which quantum effects are significant only decreases. Because of this, in the vast majority of cases, self-oscillators are analysed in terms of quantum theory and its concepts just for the love of the game and sophisticated mathematics.

Another remark should be made in connection with the notion of 'spontaneous photons' arriving at a laser cavity. It reduces to the observation that one often intuitively uses only one of the two representations of a photon, namely, corpuscular. Spontaneous emission emergence in a cavity reduces to sort of two steps. In the first step, all photons emerge. In the second, some of them enter the cavity in addition to the photons already present in the cavity. It should be kept in mind that any arithmetic operations with the number of photons are the result of a very loose interpretation of quantum theory. Quantum theory deals with amplitudes of states and operators. The creation of 'spontaneous' photons in a laser cavity is a change in the amplitude of the state of the field in the cavity as a result of its interaction with dipoles, which is represented by the dipole current operator and the vector potential of the field [7, p. 158]. The probability that a system is in a particular state is determined by the absolute square of its amplitude. As to the amplitudes of states, they can interfere with allowance for their phases. This corresponds to the notion of interference between field amplitudes in classical electrodynamics.

5. Multimode operation of a diode laser

In analysing single-frequency lasing in previous work [1], we proceed from the fact that a spatial electron distribution, $f(r)$, above threshold does not vary or varies insignificantly. This means that the electron 'burnout' due to stimulated transitions is compensated for by the pump current exceeding threshold, the compensation being uniform throughout. Clearly, this is not so in general and such compensation is only possible in rare cases. To this end, special laser cavity designs are needed. The point is that the spatial pump distribution $J(r)/ed$, inversion decay $\sim\mathrm{Im}(\delta\varepsilon)\bar{E}^2(r,t)$ and electron diffusion are different mechanisms, characterised by different functions. The question that arises in this context is to what extent the change in $f(r)$ described by Eqn (7) in Ref. [1] is critical so that it can be thought of as 'small' (in particular, from the viewpoint of the suppression of the single-frequency lasing considered above and excitation of multiple modes).

Consider this issue with application to a diode laser having a Fabry–Perot cavity and parameters similar to those for which calculations were performed and references to experimental data were given in previous work [2, 3]. The existence of modes and the possibility of exciting them are determined by Eqns (11) and (12) in Ref. [1]. In such lasers, modal losses γ_k can be thought to be equal because they are spectrally independent. In connection with this, regular spectral discrimination of modes is only due to the shape of the gain spectrum profile. It is easy to show that, in an ideal case (based on previously reported measurements [11, 12]), gain deficiency in the longitudinal below-threshold modes closest to the lasing mode with respect to it ranges from 10^{-5} to 10^{-4}. This means that lasing occurs on an essentially flat top of the spectral profile.

A slightest distortion (at a level of 10^{-4}) of such a gain spectrum profile entails either lasing switching from one mode to another or even suppression of single-frequency lasing and simultaneous excitation of several modes. Calculation of such spectral profile distortion upon electron distribution $N(r)$ 'burnout' involves finding a self-consistent solution to a complicated nonlinear problem which includes Eqns (7) and (9) and equations of the form (11). For example, in the case of a laser with a Fabry–Perot cavity and a 'horizontal' waveguide formed by amplification, the solution presented in Ref. [1] is unreasonably simplified and, hence, inadequate. Indeed, to solve Eqn (7) one should know the spatial distribution $\bar{E}^2(r) \backsim \langle \bar{u}^*(r)\bar{u}(r) \rangle$. To find $\bar{u}(r)$, it is in turn necessary to solve Eqn (11) for ω_k^2 eigenvalues and $\bar{u}_k(r)$ eigenfunctions. This requires knowledge of the complex dielectric permittivity $\varepsilon(\omega, r)$ and, hence, the spatial distribution $N(r)$, which completes the iteration cycle. The functions $\bar{u}_k(r)$, $N(r)$ and $\varepsilon(\omega, r)$ thus found prove to be interdependent. The interdependence is especially strong for all so-called gain-guided lasers, or lasers with a 'weak horizontal' waveguide (in the plane of the p–n junction), in which $\bar{u}_k(r)$ [solution to Eqn (11)] is determined by the imaginary part of the $\varepsilon(\omega, r)$ profile. It is such lasers that were modelled not quite successfully in Ref. [3].

Clearly, such a problem cannot be solved analytically and, moreover, certain difficulties are encountered in attempts to solve it numerically. A separate, nontrivial problem is the stability of the solution found and the ambiguity of solutions to nonlinear equations. Nevertheless, some estimates can be done without solving this complicated problem.

We can separate out three main physical mechanisms responsible for regular distortion of the spectral profile of the modal gain at pump currents above threshold.

One mechanism is the spatial nonuniformity of pumping and inversion decay, as considered above.

Another mechanism is related to a fundamental property of a semiconductor gain medium, namely, to the dependence of the spectral profile of a gain line on electron concentration because of the sequential filling of the corresponding band (subband) of electron states. In such a case, the increase in gain with increasing electron concentration is accompanied by not only an increase in gain in the spectral maximum but also a considerable shift of the peak to shorter wavelengths. This effect is well known and has been the subject of extensive theoretical and experimental studies (see e.g. Batrak et al. [11] and Bogatov et al. [12]). In quantum-confined heterostructure lasers, the spectral shift can be comparable to the gain linewidth.

The third mechanism is the dependence of the transverse distribution of the field amplitude $\bar{u}(r)$ (intensity) along the plane of the layers on the distribution of the gain [imaginary part of $\varepsilon(r, N, \omega)$] and, hence, wavelength (this was also considered above). If we take into account that even the fundamental transverse mode in the lasers under consideration occupies a region no smaller than the pump region (3–5 μm according to Gorlachuk et al. [13]) in a direction along the plane of the active layer, it becomes clear that the gain of this mode is determined by different shapes of spectral profiles. Each profile corresponds to its own concentration, which in turn varies from the highest value on the optic axis to zero in the peripheral parts of the active region.

Thus, spatial nonuniformity of the carrier distribution contributes to the 'inhomogeneous' gain linewidth, comparable to the quasi-homogeneous linewidth. As a result, gain saturation is nonuniform not only spatially but also spectrally. Since the difference between the unsaturated and saturated gains is of the same order as the threshold gain, The spectral distortion of the profile is comparable to its spectral width and the relative amplitude of such distortion always exceeds the above-mentioned gain deficiency (10^{-5} to 10^{-4}) by several orders of magnitude. Accordingly, single-frequency lasing is almost never observed in this type of laser. Even a fraction of a percent above the lasing threshold, excitation conditions are fulfilled for several modes. As a result, the lasing threshold of such lasers is sometimes difficult to determine from spectral measurements only. With increasing pump current, a typical experimentally observed picture for such lasers appears as a smooth transition from a few spectral lines of amplified spontaneous emission to a picture of several multimode lasing lines. Note that, as the pump current rises to above threshold, the number of excited modes sequentially increases and the laser spectrum broadens, reflecting the spectral flattening of the saturated effective modal gain profile near its top.

Such behaviour of the spectrum proves to be rather sensitive to a particular laser design and even to distinctive features of a particular sample, which can be related to deviations of the optical properties of the diode cavity from ideal properties because of its technological imperfections. All this leads to diverse behaviour of experimentally observed spectra of this type of laser.

In other words, unlike Kurnosov V.D. and Kurnosov K.V. [3], I think that the problem is not to explain multimode lasing but to formulate single-frequency lasing conditions. Assumptions about them were made previously [1]. Potential candidates for single-mode (single-frequency) lasers are diodes in which the spatial distribution of the mode amplitude $\bar{u}_k(r)$, i.e. the solution to Eqn (11) in Ref. [1], is only determined by the real part of $\varepsilon(\omega, r)$ and is independent of its imaginary part. This is possible if 'frozen-in' (set by the dielectric cavity design) spatial variations in the real part of permittivity, $\delta\mathrm{Re}(\varepsilon_0(\omega, r)$, far exceed variations in its part related to electrons, $|\delta\varepsilon(\omega, N(r), r)|$. In such a case, Eqn (11) drops out of the system and can be solved separately. Besides, gain deficiency (difference) between neighbouring modes should be sufficiently large. This can be reached e.g. by using a small cavity length or placing a spectrally selective component in the cavity.

Not only the three mechanisms considered above can be responsible for multimode lasing. In some diode laser designs, multimode lasing is a consequence of periodic self-sustained pulsations of light at a gigahertz frequency. The pulsation regime was the subject of one of the first studies dealing with diode lasers [14]. Bogatov et al. [15] demonstrated connection of the pulsation regime with multimode lasing. Later, special diode laser designs were proposed in which this regime emerged reproducibly at regular intervals slightly above the lasing threshold [16]. Such lasers have a regular spectrum (with a controlled linewidth) and are used in CD devices for suppressing speckle patterns during reading. Another important mechanism governing the formation of a multimode lasing spectrum is nonlinear interaction of modes through dynamic inversion oscillations at a difference frequency (see e.g. Refs [8, 17, 18]).

The purpose of the above is to call attention to the fact that the mechanisms responsible for multimode lasing are unrelated to spontaneous emission. Spontaneous emission does not play any role in the formation of a multimode lasing spectrum, in contrast to what is stated by a number of researchers {see e.g. formula (13) in Ref. [3] and formula (6.42) in Ref. [5]}.

In this context, it is worth mentioning work by Meller et al. [19], who rather adequately took into account the contribution of spontaneous emission to lasing on a large number of modes. Quite reasonably, they divided the problem into two independent parts: the problem of multimode lasing in the absence of spontaneous emission, whose solutions they used, assuming them to be known, and the problem of assessing the effect of spontaneous emission on each mode. The latter problem was the subject of their report. Its result differs little from that obtained in Ref. [1]. The effect of spontaneous emission reduces exclusively to spectral broadening of excited modes, for which a threshold condition is met no matter whether or not there is spontaneous emission.

6. Two-photon absorption

The use of the 'asymptotic threshold' model leads to a paradoxical result as an unavoidable consequence. At a sufficiently high pumping level, the emission spectrum should always have the form of a single, predominant line, whose intensity rises linearly with current, whereas the intensity of the other modes should saturate at some constant level. Clearly, this is absolutely inconsistent with reality because, as mentioned above, the vast majority of diode lasers operate in a multimode regime, if they do not have a specially designed cavity.

Clearly, this led to a search for 'foreign' physical mechanisms capable of modifying the model in question so as to eliminate the inconsistency between theory and experiments. According to Kurnosov V.D. and Kurnosov K.V. [3], one such mechanism is nonlinear optical losses due to two-photon absorption. Taking into account such absorption in their model, they simulated a multimode diode laser spectrum. The input rate equations were as follows:

$$\frac{\mathrm{d}S_m}{\mathrm{d}t} = \left[G_m(1-\varepsilon S_m) - \frac{1}{\tau_{\mathrm{ph}}}\right]S_m + \beta R_{\mathrm{sp}} - \gamma S_m^2, \tag{6a}$$

$$\frac{\mathrm{d}N}{\mathrm{d}t} = \frac{I}{eV_{\mathrm{act}}} - R_\Sigma - \Sigma G_m(1-\varepsilon S)S_m. \tag{6b}$$

Here S_m and G_m are the photon density and gain coefficient of the mth mode; τ_{ph} is the photon lifetime; the coefficient β takes into account the contribution of spontaneous emission to the mode; γ is the nonlinear loss due to two-photon absorption; ε is the spectral hole burning coefficient; R_{sp} is the spontaneous recombination rate; R_Σ is the total rate of radiative and nonradiative recombination; V_{act} is the volume of the active region; e is the electron charge; and I is the pump current. This form of rate equations, used in Ref. [3], coincides with the typical form of such equations used by other researchers, except for the last term on the right-hand side of (6a).

That Eqns (6) are inadequate for analysing spectral characteristics of diode lasers was pointed out above. Kurnosov V.D. and Kurnosov K.V. [3] went even further. Using numerical simulation, they allegedly showed that introducing a negative term, $-\gamma S_m^2$, into (6a) leads to a transition from single-mode to multimode lasing. Below, it will be shown that this is another erroneous result.

Note first of all that, in the few-mode regime, the total intensity $S(t)$ is an inherently dynamic quantity, because there is field intensity beating at different frequencies. Let us represent the complex field intensity amplitude of multimode lasing, $\mathcal{E}(t)$, in the form of an expansion in terms of modes:

$$\mathcal{E}(t) = \sum_k A_k(t)\exp(-\mathrm{i}\omega_k t).$$

The total intensity $S(t)$ can then be written as the average ($\tilde{S}$) and a variable part $a(t)$, whose average is zero in the case of quasi-steady-state multimode lasing:

$$S(t) = \tilde{S} + a(t), \tag{7}$$

$$a(t) = \frac{1}{2}\sum_{k\neq j} A_k(t)A_j^*(t)\exp[-\mathrm{i}(\omega_k-\omega_j)t], \tag{8}$$

where

$$\tilde{S} = \frac{1}{2}\sum_m |A_m(t)|^2 = \sum_m S_m(t).$$

It is seen from (8) that, if the variation in the $A_k(t)$ amplitudes is sufficiently slow, the dynamics of the $a(t)$ intensity are represented by a quasi-periodic function with a period $T_0 = 2\pi/\Omega = 2Ln_{gr}/c$, which corresponds to the cavity round-trip time, where $\Omega = |\omega_j - \omega_{j+1}|$ is the intermode intensity beat frequency.

Like in (6), in Eqn (8) the coordinate dependence, which corresponds to the spatial intensity distribution in the cavity, is neglected. This is an inherent drawback and property of rate equations, but in the case under consideration a different issue is of importance. Equations (6) do not contain a dynamic term $\backsim a(t)$ at all. This makes them initially inadequate because two-photon absorption is a 'fast' process, which responds to instantaneous intensity, and one should take this into account when performing any averaging. Summing (6a) over m and averaging over 'slow' time $T \geqslant T_0$, we obtain for multimode lasing

$$\frac{d\tilde{S}}{dt} = [G(N) - \alpha]\tilde{S}(T) - \gamma\tilde{S}^2(T) - \gamma b(T), \qquad (9)$$

where $b(T) = \overline{a^2(T)} > 0$.

The steady-state equation for single-frequency intensity S_0 has the form

$$\frac{dS_0}{dT} = 0 = [G(n_0) - \alpha]S_0 - \gamma S_0^2. \qquad (10)$$

Let $S(0) = S_0$ at time $T = 0$. Using (10) and the fact that $\gamma b(t) \geqslant 0$ is a positive quantity, we obtain $d\tilde{S}/dt = -\gamma b(t) \leqslant 0$. Given that $\tilde{S}$ can be only positive or zero, we find that two-photon absorption leads to further stabilisation of single-frequency lasing and suppression of multimode lasing. In contrast, single-frequency lasing suppression and a transition to multimode lasing can be caused by the nonlinear absorption mechanism, which has the opposite sign ($\gamma < 0$), for example, saturable absorption. This mechanism is well-known in laser physics and employed in passively Q-switched and passively mode-locked lasers [20]. We have to conclude that Kurnosov V.D. and Kurnosov K.V. [3] seem to be unaware that two-photon absorption was proposed previously (almost 50 years ago), not quite properly, by Popov and Shuikin [21] as a mechanism of multimode operation of diode lasers.

7. Conclusions

Thus, the present results demonstrate that the applicability area of rate equations for diode lasers is limited to simulation of the dynamics of their total output intensity and does not include details of their emission spectrum.

Attempts to take into account spontaneous emission in the framework of rate equations are likely to lead to erroneous (inadequate) results. The reason for this is that rate equations completely ignore one of the main characteristics of lasers: coherence of their output. Rate equations can be modified further by adding equations for phases, but this will hardly simplify analysis compared to the use of equations for amplitudes derived directly from Maxwell's equations.

Modelling a steady-state emission spectrum of a diode laser using rate equations, in particular, that in Ref. [3], should be considered erroneous.

At present, it seems questionable from the viewpoint of both practical application and the development of theory whether modelling multimode operation of diode lasers is a topical issue. There is only one example of a practical problem in which reduced laser coherency was needed. It was mentioned by Miftakhutdinov et al. [16] in the context of the development of CD devices. The problem was resolved in its time and is no longer of practical importance because the manufacture of CDs is currently very limited, if any. Also, the issue hardly has something new for theory, and possible results can hardly be useful because of their inherently limited applicability.

Acknowledgements. This work was supported by the RF Ministry of Science and Higher Education (state research task No. 0023-2019-0002).

References

1. Bogatov A.P., Drakin A.E. *Quantum Electron.*, **49** (8), 717 (2019) [*Kvantovaya Elektron.*, **49** (8), 717 (2019)].
2. Ivanov A.V., Kurnosov V.D., Kurnosov K.V., Romantsevich V.I., Ryaboshtan Yu.A., Chernov R.V. *Quantum Electron.*, **36** (10), 918 (2006) [*Kvantovaya Elektron.*, **36** (10), 918 (2006)].
3. Kurnosov V.D., Kurnosov K.V. *Quantum Electron.*, **48** (9), 807 (2018) [*Kvantovaya Elektron.*, **48** (9), 807 (2018)].
4. Basov N.G., Nikitin V.V., Semenov A.S. *Usp. Fiz. Nauk*, **97** (4), 561 (1969).
5. Suhara T. *Semiconductor Laser Fundamentals* (New York–Basel: Marcel Dekker Inc., 2004) Ch. 6.6.3.
6. Lamb W.E., Jr. *Phys. Rev.*, **134** (6a), A1429 (1964).
7. *Quantum Optics and Electronics* (New York: Gordon and Breach, 1965; Moscow: Mir, 1966).
8. Bogatov A.P., Eliseev P.G., Sverdlov B.N. *IEEE J. Quantum Electron.*, **QE-11** (7), 510 (1975).
9. Pippard A.B. *The Physics of Vibration* (Cambridge: Cambridge Univ. Press, 1983; Moscow: Vysshaya Shkola, 1989).
10. Schiff L.I. *Quantum Mechanics* (New York: McGraw-Hill, 1949; Moscow: Inostrannaya Literatura, 1957).
11. Batrak D.V., Bogatova S.A., Borodaenko A.V., Drakin A.E., Bogatov A.P. *Quantum Electron.*, **35** (4), 316 (2005) [*Kvantovaya Elektron.*, **35** (4), 316 (2005)].
12. Bogatov A.P., Boltaseva A.E., Drakin A.E., Belkin M.A., Konyaev V.P. *Quantum Electron.*, **30** (4), 315 (2000) [*Kvantovaya Elektron.*, **30** (4), 315 (2000)].
13. Gorlachuk P.V., Ivanov A.V., Kurnosov V.D., Kurnosov K.V., Marmalyuk A.A., Romantsevich V.I., Simakov V.A., Chernov R.V. *Quantum Electron.*, **48** (6), 495 (2018) [*Kvantovaya Elektron.*, **48** (6), 495 (2018)].
14. Basov N.G., Morozov V.N., Nikitin V.V., Semenov A.S. *Fiz. Tekh. Poluprovodn.*, **1**, 1570 (1967).
15. Bogatov A.P., Eliseev P.G., Ivanov L.P., Logginov A.S., Manko M.A., Senatorov K.Ya. *IEEE J. Quantum Electron.*, **QE-9** (2), 392 (1973).
16. Miftakhutdinov D.R., Batrak D.V., Bogatov A.P., Drakin A.E., Plisyuk S.A. *Quantum Electron.*, **36** (8), 751 (2006) [*Kvantovaya Elektron.*, **36** (8), 751 (2006)].
17. Batrak D.V., Bogatov A.P., Kamenets F.F. *Quantum Electron.*, **33** (11), 941 (2003) [*Kvantovaya Elektron*, **33** (11), 941 (2003)].

18. Batrak D.V., Bogatov A.P. *Quantum Electron.*, **37** (8), 745 (2007) [*Kvantovaya Elektron*, **37** (8), 745 (2007)].
19. Meller A.S., Khandokhin P.A., Khanin Ya.I. *Sov. J. Quantum Electron.*, **16** (11) 1502 (1986) [*Kvantovaya Elektron*, **13** (11), 2278 (1986)].
20. Letokhov V.S. *Zh. Eksp. Teor. Fiz.*, **55** (3), 1077 (1968).
21. Popov Yu.M., Shuikin N.N. *Zh. Eksp. Teor. Fiz.*, **58** (5), 1727 (1970).