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Asymptotic theory of ponderomotive dynamics of an electron in the field of a focused relativistically intense electromagnetic envelope

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Abstract. **Based on a formal asymptotic solution of Maxwell's equations for a field propagating in a vacuum and of the relativistic Newton equation for an electron driven by the corresponding Lorentz force, we construct a description of the relativistic ponderomotive dynamics of an electron in the field of an intense focused electromagnetic envelope. A small parameter for the asymptotic expansion is proportional to the ratio of the radiation wavelength to the focal spot radius. Using the obtained averaged model of ponderomotive dynamics, electron beam scattering patterns are plotted with regard to an angle relative to the electromagnetic field propagation axis for laser pulses with a Gaussian transverse intensity distribution, as well as the energy spectra of scattered electrons corresponding to individual ranges of this angle. Scattered particles are absent in some ranges of the polar angle.**

Keywords: *relativistic intensity, ponderomotive force, laser acceleration of electrons.*

1. Introduction

The problems of strongly nonlinear dynamics of electrons in electromagnetic fields have become a topic of particular interest with the advent of laser physics of relativistic intensities. This field of research was formed when the level of attainable laser intensities exceeded a so-called relativistic intensity of approximately 10^{18} W cm^{-2} [1]. At optical field intensities comparable with or exceeding the relativistic one, the dynamics of electrons born due to ionisation of matter at the leading edge of the laser pulse or injected into the focal region turns out to be substantially relativistic, and, as a result of interaction with a focused optical field, electrons can acquire significant energies. In the generalised sense, the motion of an electron under the action of an electromagnetic pulse, considered on a time scale significantly exceeding the optical cycle, represents ponderomotive dynamics, accompanied with an energy gain by an electron. The construction of a mathematically rigorous theory of ponderomotive dynamics of an electron in the field of a focused electromagnetic envelope of relativistic intensity is the goal of this study.

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The general scenario of the relativistic dynamics of an electron under the action of the Lorentz force generated by a high-power pulse of focused laser radiation has been studied in detail at the level of numerical analysis in many papers (see, for example, Refs [2–8]): it comprises electron capture by an optical field in which it oscillates, gaining energy on average, and then an electron ejection from the focal zone with some, in many cases relativistic, residual energy. The model used to describe this process includes the relativistic Newton equation with the corresponding Lorentz force, as well as the expressions for an optical envelope propagating in vacuum. In some papers, these expressions were formulated only with allowance for longitudinal corrections to the electromagnetic field potential [9, 10], while in more detailed studies, corrections to the expressions for the transverse field components [11, 12] caused by the finite pulse duration are additionally taken into account.

The concept of the ponderomotive dynamics of an electron in a high-frequency field was formulated in the classical work [13], in which the ponderomotive force responsible for the average electron motion was shown to be proportional to the radiation intensity gradient (see also [14]). It should be noted that the results of [13] were obtained in the quasi-linear approximation, that is, as applied to a field of moderate intensity. In connection with the growth of achievable laser intensities, generalisations of the theory of ponderomotive dynamics were proposed for the case of relativistic intensities. The approach developed in [13] was reproduced in the majority of works devoted to this problem: After the transformation of the initial equations, *a priori* assumptions were made about the nature of the oscillations of the quantities in them (including the relativistic factor for the electron) and on this basis the equations themselves were averaged over time [10, 15, 16]. A step in the direction of constructing a more general theory of ponderomotive dynamics of an electron in a strong field using the Krylov–Bogoliubov method was made in [17], where, however, in the most practically important case of linear polarisation of electromagnetic radiation, consideration was limited to the quasi-relativistic problem because of the complexity of complete characterisation of the nature of the relativistic factor oscillations.

An important aspect of the problem of ponderomotive dynamics – the fundamental difference between averaging over the period of field oscillations and over the phase of electron oscillations – was noted in [18]. According to the results of this work, averaging over the phase of the oscillations of the particle must be performed under conditions of a significant longitudinal displacement of the electron under the action of a laser pulse, which, in any case, takes place in the case of relativistic intensity. In contrast to [18], in paper [13] averaging was performed over time. Averaging over the proper time of the electron, which is proportional to the phase of its oscillations, was performed in [19], but in this work the authors studied a comparatively less important case of circular polarisation of the electromagnetic field, for the amplitude of which no specific expressions were provided.

In some papers, relativistic ponderomotive dynamics was studied in the framework of a one-dimensional problem [20, 21]. Significant in this case is the fact that in this approximation the corresponding equations have an invariant relating the transverse and longitudinal components of the electron momentum. A hypothesis was expressed about the presence of a similar invariant in the three-dimensional problem [22], but a rigorous theory of such an invariant remained absent.

In this paper, we construct a formal asymptotic solution of the relativistic Newton equation for an electron driven by the Lorentz force produced by a linearly polarised focused electromagnetic envelope of high intensity. The small parameter for the asymptotic series is $\epsilon = \lambda/(2\pi w_0)$, where λ and w_0 are the wavelength and radius of the focal spot of the optical field, respectively, and the solution below is calculated up to corrections of order of ϵ. Within the framework of the asymptotic algorithm used, the phase of the oscillations of the electron, to which its 'fast' and 'slow' times are directly related, plays the role of the variable on which the desired functions depend. The expression for the electromagnetic field presented in Section 2 includes corrections in the parameter ϵ to its longitudinal and transverse components, since they turn out to be necessary below for the correct solution of the equations of electron motion using the Krylov–Bogoliubov method. The lowest approximation for the asymptotic solution includes arbitrary constants depending only on the 'slow' time, which are further determined on the basis of the first approximation from the condition that its secular components are equal to zero. In view of this, it should be noted that the main goal of introducing first-order corrections into expressions for the field is to ensure the correctness of the result in the lower approximation rather than to increase the accuracy of the approximation. It turns out that the conditions for eliminating secularity are in fact the electron motion equations averaged over the phase of its oscillations, that is, the desired model of the ponderomotive dynamics of the electron in the field of a relativistically intense focused optical envelope. Thus, when constructing this model, it is possible to avoid any *a priori* assumptions about the nature of the oscillations of various dynamic quantities and even direct averaging of the motion equations. The completely three-dimensional initial equations, as shown in Section 2, have an adiabatic invariant, which is similar to the exact invariant of the corresponding one-dimensional problem and, in particular, relates the energy and direction of electron motion after the interaction with the field.

The resulting model of ponderomotive dynamics can be significantly simplified when modelling the scattering of a low-density ensemble of electrons by a linearly polarised relativistically intense laser pulse with an axisymmetric amplitude distribution. If at the initial moment of time the velocity distribution of electrons is isotropic, then there is a uniform distribution of scattered electrons over the azimuthal angle in a cylindrical coordinate system with the axis coinciding with the direction of propagation of the laser pulse and the reference origin placed in the focus. In Section 4, for the case of a Gaussian transverse distribution of the optical field intensity, we present typical solutions to the problem of the ponderomotive dynamics of an individual electron in the field of a focused electromagnetic envelope of relativistic intensity, as well as angular (relative to the direction of laser pulse propagation) and electron energy distributions formed when the electron ensemble is scattered by high-power laser radiation.

2. Asymptotics of the solutions to the electron motion equations in a relativistically intense optical field

Nonlinear relativistic dynamics of an electron in an electromagnetic field obeys Newton's equations, which in a normalised form can be expressed as:

$$\gamma\partial_t x = p_x, \quad \gamma\partial_t y = p_y, \quad \gamma\partial_t z = p_z \tag{1}$$

$$\partial_t \boldsymbol{p} = \partial_t \boldsymbol{A} - \gamma^{-1}(\boldsymbol{p}\times(\nabla\times\boldsymbol{A})), \quad \gamma = \sqrt{1+\boldsymbol{p}^2}, \tag{2}$$

where $\nabla = (\partial_x, \partial_y, \partial_z)$; $\partial_{x,y,z,t}$ are the derivatives with respect to the corresponding variables; γ is the relativistic factor; and $\boldsymbol{A}$ is the vector potential of the electromagnetic field propagating in vacuum (hereafter, ∂_t denotes the time derivative of the time-depending coordinates and momentum $\boldsymbol{p}$ and simultaneously the partial derivative of the vector potential). Consider the case when the field is a focused envelope; let the coordinates and time be normalised to the focal spot radius w_0 and to w_0/c, respectively, the vector potential be normalised to mc^2/e, and the electron momentum be normalised to mc. Then Maxwell's equations (under the Coulomb gauge) take the form

$$\Delta\boldsymbol{A} - \partial_t^2\boldsymbol{A} = 0, \quad (\nabla, \boldsymbol{A}) = 0,$$

and in the case of linearly polarised electromagnetic field, they can, in particular, have the asymptotic solution:

$$A_x = \exp(\mathrm{i}\theta)\left[a(\tau,x,y,s) + \sum_{m=1}^{\infty}\epsilon^m a_{xm}(\tau,x,y,s)\right] + \text{c.c.},$$

$$A_y = 0, \quad A_z = \exp(\mathrm{i}\theta)\sum_{m=1}^{\infty}\epsilon^m a_{zm}(\tau,x,y,s) + \text{c.c.}$$

The variables in it, which are interpreted below as fast and slow proper times of the electron, are determined by the relations $\theta = (t-z)/\epsilon$, $s = \epsilon\theta$; $\epsilon = \lambda/(2\pi w_0)$, $\tau = 2\epsilon z$.

As shown in [11, 12], the equations of zero and first approximation for the amplitude of the vector potential have the form

$$-4\mathrm{i}\partial_\tau a + \Delta_\perp a = 0, \quad -4\mathrm{i}\partial_\tau a_{x1} + \Delta_\perp a_{x1} = 4\partial_{\tau s}^2 a,$$

$$-\mathrm{i}a_{z1} + \partial_x a = 0,$$

where $\Delta_\perp = \partial_x^2 + \partial_y^2$, and the structure of their solutions is determined by the expressions

$$a(\tau,x,y,s) = a_0(s)u(x,y,\tau), \tag{3}$$

$$a_{x1}(\tau,x,y,s) = \mathrm{i}\partial_s a_0(s)\partial_\tau[\tau u(x,y,\tau)], \tag{4}$$

$$a_{z1}(\tau,x,y,s) = -\mathrm{i}a_0(s)\partial_x[u(x,y,\tau)], \tag{5}$$

in which the function $u(x,y,\tau)$, in turn, is a solution of the Schrödinger equation

$$-4i\partial_\tau u + \Delta_\perp u = 0.$$

The function $a_0(s)$ defines the temporal envelope of the pulse.

The simplest solution to the above equation corresponds to an axisymmetric distribution of the amplitude of a Gaussian pulse:

$$u(x,y,\tau) = \frac{\Lambda(\tau,r)}{\sqrt{\tau^2+1}}\exp[i\psi(\tau,r)],$$

$$\Lambda(\tau,r) = \exp\left(-\frac{r^2}{\tau^2+1}\right),\ \ \psi(\tau,r) = -\frac{\tau r^2}{\tau^2+1} + \arctan\tau,$$

where $r = \sqrt{x^2+y^2}$. A more general solution of the same equation is a mode defined through the Laguerre polynomial and has the form:

$$u_{l,\delta}(x,y,\tau) = u(x,y,\tau)2^{l/2}\left(\frac{r}{\sqrt{\tau^2+1}}\right)^l L_\delta^l\left(\frac{2r^2}{\tau^2+1}\right)$$

$$\times\exp[i(l+2\delta)\arctan\tau]\sin(l\varphi+\varphi_0),$$

where $\varphi = \arctan(y/x)$ [11, 12]. In the particular case, when $\delta = 0$, $l = 0$ and $\varphi_0 = \pi/2$, this expression describes a Gaussian pulse, and in the general case, it describes a focused pulse with an axisymmetric amplitude distribution. An even more general solution to this problem is a linear combination of Laguerre modes.

The goal of this work is to construct asymptotic solutions with respect to the small parameter ϵ for equations (1)–(5), that is, to solve the problem of the relativistic dynamics of an electron in an electromagnetic field using the same assumptions under which the problem of the propagation of a focused electromagnetic envelope in vacuum is solved. Below, when constructing such solutions, we have obtained the results that do not imply the presence of axial symmetry of the amplitude distribution of the focused envelope and are fully applicable in the case of any of the Laguerre modes or their superposition, and for the particular case of an axisymmetric optical field, we have obtained and investigated a model significantly simplified in comparison with the general equations.

It should be noted that, in view of the traditional architecture of the asymptotic algorithm used below, which involves the final determination of lower-order solutions using higher-order solutions, small parameter corrections to the solution of Maxwell's equations describing the electromagnetic field are fundamentally necessary when solving the formulated problem of the electron dynamics even in the lowest approximation. On the whole, the implemented asymptotic method for solving the equations of the electron dynamics is adapted to the structure of the above expressions for the electromagnetic field. With a different structure of the electromagnetic field, as, for example, in the case of an electron accelerated by an interference field of optical pulses of a complicated configuration [23, 24], the nature of this dynamics and, accordingly, the solutions describing it can be significantly different.

Consider equations (1) and (2) with a vector potential defined by equations (3)–(5). Since the equations of the electron dynamics obtained after calculating the Lorentz force in accordance with the above solutions for the vector potential are ordinary differential equations in time, they have $\partial_t = (j/\epsilon\gamma)\partial_\theta$, where

$$j = \gamma - p_z, \tag{6}$$

and the problem can be rewritten in terms of functions that depend not on time t, but on θ. In the process of constructing asymptotics with respect to ϵ, the equations will be solved in the variables $s = \epsilon\theta$ and θ. The asymptotic series for the coordinates and components of the electron momentum are expressed as:

$$x(t) = x_0(s,\theta) + \epsilon x_1(s,\theta) + \cdots,$$

$$p_x(t) = p_{x0}(s,\theta) + \epsilon p_{x1}(s,\theta) + \cdots,$$

$$y(t) = y_0(s,\theta) = \epsilon y_1(s,\theta) + \cdots,$$

$$p_y(t) = p_{y0}(s,\theta) + \epsilon p_{y1}(s,\theta) + \cdots,$$

$$\tau(t) = \tau_0(s,\theta) + \epsilon\tau_1(s,\theta) + \cdots,$$

$$p_z(t) = p_{z0}(s,\theta) + \epsilon p_{z1}(s,\theta) + \cdots.$$

In this case, lower-order equations will be ordinary differential equations with respect to the variable θ, and arbitrary functions of the variable s will emerge in their solutions as integration constants. These functions, in turn, will be determined from the condition that the secular growth of solutions of equations of higher approximations is absent. The completely nonlinear zero-order solution presented below is identical to the well-known solutions of the corresponding one-dimensional problem. Below, we will also present general solutions of equations for first-order approximations; moreover, the equality-to-zero conditions for the secular terms, that is, terms proportional to θ, entering into them will turn out to be the averaged solutions to the problem of the dynamics of an electron in the lower approximation. This approach avoids the direct 'averaging' of nonlinear equations, which, as a rule, is based on *a priori* assumptions about the nature of the oscillations of the quantities included in these equations.

Let $m(x,y,\tau) = \mathrm{Re}\,u(x,y,\tau)$ and $n(x,y,\tau) = \mathrm{Im}\,u(x,y,\tau)$. Zero approximations obtained by substituting relations (3)–(5) and asymptotic series for coordinates and electron momentum into equations (1), (2) are as follows:

$$x_0(s,\theta) = x_{0a}(s),\ \ y_0(s,\theta) = y_{0a}(s),\ \ \tau_0(s,\theta) = \tau_{0a}(s), \tag{7}$$

$$p_{x0}(s,\theta) = a_0(s)[m(x_{0a}(s), y_{0a}(s), \tau_{0a}(s))\cos\theta$$
$$-n(x_{0a}(s), y_{0a}(s), \tau_{0a}(s))\sin\theta] + p_{x0a}(s), \tag{8}$$

$$p_{y0}(s,\theta) = p_{y0a}(s). \tag{9}$$

The functions $x_{0a}(s)$, $y_{0a}(s)$, $\tau_{0a}(s)$, $p_{x0a}(s)$, and $p_{y0a}(s)$ in them must be additionally defined. It is easy to show that, as in the one-dimensional problem, for these lower-order solutions the quantity j defined by equation (6) is an invariant, that is, does not depend on θ:

$$\gamma_0(s,\theta) - p_{z0}(s,\theta) = j_0(s), \quad (10)$$

where

$$\gamma_0(s,\theta) = \sqrt{1 + p_{x0}^2(s,\theta) + p_{y0}^2(s,\theta) + p_{z0}^2(s,\theta)};$$

moreover, the function $j_0(s)$ also has to be determined from higher order equations. As follows from (10),

$$p_{z0}(s,\theta) = \frac{p_{x0}^2(s,\theta) + p_{y0}^2(s,\theta) - j_0^2(s) + 1}{2j_0(s)}.$$

Thus, the relativistic factor for an electron and its energy are expressed as

$$\gamma_0(s,\theta) = \frac{p_{x0}^2(s,\theta) + p_{y0}^2(s,\theta) + j_0^2(s) + 1}{2j_0(s)}, \quad E = \gamma_0(s,\theta) - 1. \quad (11)$$

In the higher order, the last of equations (1) has a solution

$$\tau_1(s,\theta) = \tau_{1a}(s) - \theta\tau'_{0a}(s).$$

Since the secular growth in the variable θ, generated by the last term of this solution, leads to the violation of the assumptions in the used asymptotic series, it should be assumed that the function $\tau_{0a}(s)$, which is part of (7) and is not defined in the lower approximation, is a constant, so that

$$\tau_{0a}(s) = \tau_{0a}, \quad \tau_1(s,\theta) = \tau_{1a}(s).$$

Similarly, terms proportional to θ are found in the following approximation to other solutions, and the corresponding conditions for the absence of secular growth must be imposed on these solutions.

Denote for brevity:

$$m_0(s) = m(x_{0a}(s), y_{0a}(s), \tau_{0a}), \quad n_0(s) = n(x_{0a}(s), y_{0a}(s), \tau_{0a}),$$

$$m_1(s) = \frac{\partial m(x_{0a}(s), y_{0a}(s), \tau_{0a})}{\partial x}, \quad n_1(s) = \frac{\partial n(x_{0a}(s), y_{0a}(s), \tau_{0a})}{\partial x},$$

$$m_2(s) = \frac{\partial m(x_{0a}(s), y_{0a}(s), \tau_{0a})}{\partial y}, \quad n_2(s) = \frac{\partial n(x_{0a}(s), y_{0a}(s), \tau_{0a})}{\partial y},$$

$$m_3(s) = \frac{\partial m(x_{0a}(s), y_{0a}(s), \tau_{0a})}{\partial \tau}, \quad n_3(s) = \frac{\partial n(x_{0a}(s), y_{0a}(s), \tau_{0a})}{\partial x}.$$

First-order solutions for the transverse coordinates of the electron and the transverse components of the momentum are expressed as:

$$x_1(s,\theta) = \frac{a_0(s)[m_0(s)\sin\theta + n_0(s)\cos\theta]}{j_0(s)} + x_{1a}(s) + \left(\frac{p_{x0a}(s)}{j_0(s)} - x'_{0a}(s)\right)\theta, \quad (12)$$

$$y_1(s,\theta) = y_{1a}(s) + \left(\frac{p_{y0a}(s)}{j_0(s)} - y'_{0a}(s)\right)\theta, \quad (13)$$

$$p_{x1}(s,\theta) = \sum_{k=1,2}[\alpha_{x,k}(s)\cos(k\theta) + \beta_{x,k}(s)\sin(k\theta)] + p_{x1a}(s) - \left(p'_{x0a}(s) + \frac{a_0^2(s)[m_0(s)m_1(s) + n_0(s)n_1(s)]}{2j_0(s)}\right)\theta, \quad (14)$$

$$p_{y1}(s,\theta) = \sum_{k=1,2}[\alpha_{y,k}(s)\cos(k\theta) + \beta_{y,k}(s)\sin(k\theta)] + p_{y1a}(s) - \left(p'_{y0a}(s) + \frac{a_0^2(s)[m_0(s)m_2(s) + n_0(s)n_2(s)]}{2j_0(s)}\right)\theta, \quad (15)$$

where the coefficients included in the terms oscillating in θ are given by the equations

$$\alpha_{x,1}(s) = a_0(s) \times\left(-\frac{n_1(s)p_{x0a}(s)}{j_0(s)} + m_1(s)x_{1a}(s) + m_2(s)y_{1a}(s) + m_3(s)\tau_{1a}(s)\right) - a'_0(s)\,[n_0(s) + n_3(s)\tau_{0a}],$$

$$\alpha_{x,2}(s) = a_0^2(s)\frac{m_0(s)n_1(s) + n_0(s)m_1(s)}{4j_0(s)},$$

$$\beta_{x,1}(s) = -a_0(s) \times\left(\frac{m_1(s)p_{x0a}(s)}{j_0(s)} + n_1(s)x_{1a}(s) + n_2(s)y_{1a}(s) + n_3(s)\tau_{1a}(s)\right) - a'_0(s)\,[m_0(s) + m_3(s)\tau_{0a}],$$

$$\beta_{x,2}(s) = a_0^2(s)\frac{m_0(s)m_1(s) - n_0(s)n_1(s)}{4j_0(s)},$$

$$\alpha_{y,1}(s) = -a_0(s)\frac{n_2(s)p_{x1a}(s)}{j_0(s)},$$

$$\alpha_{y,2}(s) = -a_0^2(s)\frac{n_0(s)m_2(s) + m_0(s)n_2(s)}{4j_0(s)},$$

$$\beta_{y,1}(s) = -a_0(s)\frac{m_2(s)p_{x1a}(s)}{j_0(s)},$$

$$\beta_{y,2}(s) = a_0^2(s)\frac{n_0(s)n_2(s) - m_0(s)m_2(s)}{4j_0(s)}.$$

The longitudinal coordinate of the electron in the second order has the form

$$\tau_2(s,\theta) = \sum_{1,2}[\sigma_k(s)\cos(k\theta) + \delta_k(s)\sin(k\theta)] + \tau_{2a}(s) + \left\{\frac{2[p_{x0a}^2(s) + p_{y0a}^2(s) + 1] + a_0^2(s)[m_0^2(s) + n_0^2(s)]}{2j_0^2(s)} - \tau'_{1a}(s) - 1\right\}\theta, \quad (16)$$

where the coefficients included in the terms oscillating in θ are determined by the relations

$$\sigma_1(s) = \frac{2a_0(s)n_0(s)p_{x0a}(s)}{j_0^2(s)}.$$

$$\sigma_2(s) = \frac{a_0^2(s)n_0(s)m_0(s)}{2j_0^2(s)},$$

$$\delta_1(s) = \frac{2a_0(s)m_0(s)p_{x0a}(s)}{j_0^2(s)},$$

$$\delta_2(s) = \frac{a_0^2(s)\left[m_0^2(s) - n_0^2(s)\right]}{2j_0^2(s)}.$$

The longitudinal momentum of an electron is expressed as:

$$p_{z1}(s,\theta) = \frac{p_{x0}(s,\theta)p_{x1}(s,\theta) + p_{y0}(s,\theta)p_{y1}(s,\theta)}{j_0(s)} \tag{17}$$

$$+\frac{\gamma_0\left\{\Pi(s)j_0(s) + a_0^2(s)\left[m_1(s)\sin\theta + n_1(s)\cos\theta\right]\right\} + \theta j_0'(s)}{j_0(s)}.$$

The functions $x_{1a}(s)$, $y_{1a}(s)$, $p_{x1a}(s)$, $p_{y1a}(s)$, $\tau_{2a}(s)$, and $\Pi(s)$ needed to determine first-order corrections in ϵ can be calculated on the basis of the following approximations, but they are not needed to determine the conditions for the absence of secularity in the first order. These conditions, as follows from equations (12)–(17), consist in the fact that

$$p_{x0a}(s) = j_0(s)x'_{0a}(s), \qquad p_{y0a}(s) = j_0(s)y'_{0a}(s), \tag{18}$$

$$p'_{x0a}(s) = -\frac{a_0^2(s)\left[m_0(s)m_1(s) + n_0(s)n_1(s)\right]}{2j_0(s)}, \tag{19}$$

$$p'_{y0a}(s) = -\frac{a_0^2(s)\left[m_0(s)m_2(s) + n_0(s)n_2(s)\right]}{2j_0(s)}, \tag{20}$$

$$j_0(s) = \mathrm{const}, \tag{21}$$

$$\tau'_{1a}(s) = \frac{a_0^2(s)\left[m_0^2(s) + n_0^2(s)\right]}{2j_0^2(s)} + \frac{p_{x0a}^2(s) + p_{y0a}^2(s) + 1 - j_0^2(s)}{j_0^2(s)}. \tag{22}$$

Equations (19) and (20) can be written in the form

$$p'_{x0a}(s) + \frac{a_0^2(s)}{2j_0}\partial_x W(x_{0a}(s), y_{0a}(s), \tau_{0a}) = 0, \tag{23}$$

$$p'_{y0a}(s) + \frac{a_0^2(s)}{2j_0}\partial_y W(x_{0a}(s), y_{0a}(s), \tau_{0a}) = 0, \tag{24}$$

$$W(x,y,\tau) = \frac{m^2(x,y,\tau) + n^2(x,y,\tau)}{2} = \frac{|u(x,y,\tau)|^2}{2}, \tag{25}$$

and, therefore, the function $W_{\mathrm{p}} = (x,y,\tau,s) = a_0^2(s)\,W(x,y,\tau)$ represents the ponderomotive potential of the electron in the electromagnetic field under study. The longitudinal component of the momentum is calculated from equation (10). With accuracy of the order of ϵ, equations (18)–(25) form the problem of the relativistic ponderomotive dynamics of an electron in the field of a focused linearly polarised electromagnetic envelope (and a formally strict relativistic generalisation of the result [13]).

Obviously, from the point of view of the initial asymptotic series, the quantities $x_{0a}(s)$, $y_{0a}(s)$, $\tau_{1a}(s)$, $p_{x0a}(s)$, and $p_{y0a}(s)$ are the coordinates and components of the electron momentum averaged over the phase of its oscillations in the electromagnetic field. Thus, the equations for these average values arise without additional assumptions about the nature of the oscillations themselves and even without the averaging procedure *per se*, representing the conditions for the absence of secularly growing terms in the first order of the asymptotic behaviour of the solution of the original problem with respect to the small parameter ϵ.

The condition for the applicability of the developed approximation is the smallness of the parameter ϵ, which in terms of the focal spot radius w_0 and laser radiation wavelength λ means $w_0 \gg \lambda/2\pi$. In addition, the ponderomotive approximation is not applicable directly to the axis of the laser pulse propagation (on this axis, the Jacobian used below for conversion into cylindrical coordinates vanishes, and the relation determining the relationship between the electron energy and its angle of motion with respect to the axis of the pulse propagation gives a divergence). However, already at distances from the propagation axis of the optical field, which are of the order of the small parameter, the ponderomotive approximation yields acceptable results.

Equations (23)–(25) are derived in the framework of a fairly general formulation of the problem of the motion of an electron under the action of a linearly polarised relativistically intense electromagnetic field propagating in vacuum. A number of models of ponderomotive dynamics from previous works can be obtained from the above presented model as special cases when we introduce additional initial assumptions or they reveal structural similarities with it. Since $\gamma = 1$ in the nonrelativistic case, the variable $s = \epsilon\theta$ introduced above in the description of nonrelativistic dynamics can, following the logic of paper [18], be identified with time provided that there is no essential longitudinal component in the dynamics of the electron. Gaponov and Miller [13] assumed that the field does not have a spatiotemporal envelope and experiences rapid oscillations in time, but not in a spatial variable, which justified in this case the assumption that the longitudinal displacement of the electron is small. Assuming $s \approx t$ and $a_0(s) \equiv 1$, it is not difficult to obtain from the equations (23)–(25) the classical problem formulated in [13]. In work [19], the problem of the ponderomotive dynamics of an electron in an oscillating field, structurally close to equations (23)–(25), was obtained in the case of circular polarisation of electromagnetic radiation, which was mathematically simpler, but relatively difficult to implement in the relativistic range of intensities; nevertheless, the issue of the presence of an adiabatic invariant in this problem and its role in determining the longitudinal component of the electron momentum was not considered in this study. In paper [17], the result for linear polarisation was obtained by averaging over laboratory time without elucidating the limits of applicability of this approach and only for an electromagnetic field of moderate intensity.

3. Equations of ponderomotive dynamics in cylindrical coordinates

We consider equations (18)–(21) in cylindrical coordinates by setting $x_{0a}(s) = r(s)\cos[\varphi(s)]$, $y_{0a}(s) = r(s)\sin[\varphi(s)]$, $p_{x0a}(s) = \rho(s)\cos[\psi(s)]$, and $p_{y0a}(s) = \rho(s)\sin[\psi(s)]$. Here, the functions $\varphi(s)$ and $\psi(s)$ play the role of azimuthal angles in the spaces of coordinates and momenta in the framework of this description of electron motion. Then, $\delta\psi(s) = \varphi(s) - \psi(s)$ and $W(x,y,\tau) = c(r^2,\varphi,\tau_{0a})$.

Equations (18)–(21) transform into the system:

$$r'(s) = \frac{\rho(s)\cos[\delta\psi(s)]}{j_0}, \tag{26}$$

$$\rho'(s) = -\frac{a_0^2(s)}{4j_0 r(s)}\sin[\delta\psi(s)]\partial_\varphi c\left(r^2(s),\varphi(s),\tau_{0a}\right) - 2\cos[\delta\psi(s)]r^2(s)\frac{\partial c\left(r^2(s),\varphi(s),\tau_{0a}\right)}{\partial r^2}, \tag{27}$$

$$\varphi'(s) = -\frac{\sin[\delta\psi(s)]\rho(s)}{j_0 r(s)}, \tag{28}$$

$$\psi'(s) = -\frac{a_0^2(s)}{4j_0 r(s)\rho(s)}\left\{\cos[\delta\psi(s)]\frac{\partial c\left(r^2(s),\varphi(s),\tau_{0a}\right)}{\partial\varphi} + 2\sin[\delta\psi(s)]r^2(s)\frac{\partial c\left(r^2(s),\varphi(s),\tau_{0a}\right)}{\partial\tau^2}\right\}. \tag{29}$$

Taking into account equation (10), relating the longitudinal and transverse components of the momentum, the corresponding angle between the directions of electron motion and field propagation, as well as the electron energy after the interaction, are expressed in terms of ρ after the interaction, which is denoted below as $\rho_{\rm f}$, using the following relations:

$$\Xi = \arctan\frac{2j_0\rho_{\rm f}}{1+\rho_{\rm f}^2 - j_0^2},$$

$$E = \frac{\rho_{\rm f}^2 + (j_0-1)^2}{2j_0}.$$

In the case of an axisymmetric distribution of the field amplitude, these equations can be reduced to a problem for three unknowns. Let $c(r^2,\varphi,\tau_{0a}) = c_0(r^2,\tau_{0a})$. Then, after subtracting equation (28) from equation (29), the problem takes on a more compact form and equations

$$\rho'(s) = -\frac{a_0^2(s)r(s)\cos[\delta\psi(s)]\dfrac{\partial c_0\left(r^2(s),\tau_{0a}\right)}{\partial r^2}}{2j_0}, \tag{30}$$

$$\delta\psi'(s) = -\frac{\sin[\delta\psi(s)]\left[a_0^2(s)r^2(s)\dfrac{\partial c_0\left(r^2(s),\tau_{0a}\right)}{\partial r^2} - 2\rho^2(s)\right]}{2j_0 r(s)\rho(s)} \tag{31}$$

are added to equation (26) instead of (27)–(29). In particular, for a Gaussian pulse

$$c_0\left(r^2,\tau_{0a}\right) = \frac{\exp\left[-2r^2/\left(\tau_{0a}^2+1\right)\right]}{\tau_{0a}^2+1}.$$

The calculations below are performed for the time profile of the electromagnetic envelope, defined as $a_0(s) = q\exp[-(s-d)^2/\sigma^2]$, where q is the maximum field amplitude, σ is the pulse duration, and d is the distance from the point at which the maximum field is reached, up to the electron at the initial moment of time.

Most calculations presented in this paper were performed at $q = 33$ and $\sigma = 4$. At a wavelength of $\lambda = 800$ nm and, in particular, at $\epsilon = 0.1$, this approximately corresponds to a laser pulse with a focal spot 1.3 μm in diameter, a duration of 17 fs and an intensity of 2.3×10^{21} W cm^{-2}.

In this paper, a more compact model, represented by equations (26), (30), and (31), is used to study the individual ponderomotive dynamics of an electron and the ponderomotive expansion of an ensemble of electrons under the action of intense linearly polarised laser radiation, because within the framework of this model, as will be shown below, the averaging over random initial directions of the momenta of charged particles occurs naturally. At the same time, as an initial illustration, Fig. 1 shows two examples of solving the unreduced problem (26)–(29) of an energy increase by an electron in interaction with a relativistically intense envelope, clarifying its relationship with problem (26), (30), and (31). In these calculations, the initial conditions for the azimuthal angles φ and ψ are different; nevertheless, they are selected so that the initial value of $\delta\psi$ is the same in both cases. The initial conditions for ρ are small random quantities (an almost immobile electron). Obviously, the solutions for φ and ψ quickly converge, and for the initial conditions under consideration, the quantity φ changes little and ψ significantly, which is explained by the presence of a small denominator in the right-hand side of equation (29) at the initial stage of the process because of the smallness of the initial value of ρ. In this case, as a result,

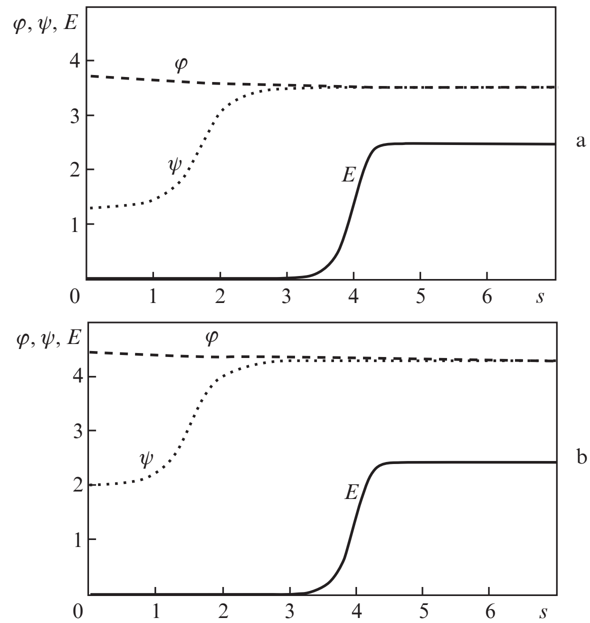

Figure 1. Relativistic ponderomotive dynamics of an electron in cylindrical coordinates: electron energy E, azimuth of coordinate φ, and azimuth of momentum ψ. Optical field parameters: Gaussian pulse, $q = 33$, $\sigma = 4$, $\epsilon = 0.1$, and $d = 10$ (interaction with a remote target). Here and in Figs 2–4, the initial values of the electron momentum components are random values within 0.1 % of the relativistic threshold (the value of the adiabatic invariant in the calculations fluctuates correspondingly). Initial conditions: $\rho(0) = 0.1$; (a) $\varphi(0) = 3.7$, $\psi(0) = 1.3$; (b) $\varphi(0) = 4.4$, $\psi(0) = 2.0$. Calculations are presented for $\tau_{0a} = 0$ (focal plane). The variable s is the proper time of the electron, the difference $\delta\psi(0) = \varphi(0) - \psi(0)$ is the same in both cases and the curves of the energy gain by the electron are identical.

the electron is displaced from the focal spot in the direction characterised by the final value common for both azimuthal angles (in the case shown in Fig. 1, it practically coincides with the azimuth of the initial electron position), and at the above-defined angle Ξ to the optical propagation axis.

As can be seen in Fig. 1, the curves of energy gain by electrons in cases (a) and (b) are identical, which is consistent with the fact that the variables φ and ψ enter the reduced problem derived from equations (26)–(29) not separately, but only through a combination of $\delta\psi$, and this quantity has the same value in both cases. Thus, any solution of equations (26), (30), and (31) allows us to determine the parameters of the ponderomotive dynamics for a continuum of the corresponding solutions to the original problem (26)–(28).

4. Dynamics of electrons in a relativistically intense optical field with an axisymmetric intensity distribution: trajectories, scattering patterns, and energy spectra

In solving the problem of scattering an ensemble of electrons by a relativistically intense laser pulse, it is natural to assume that, prior to the interaction, noise-level momenta of the charged particles are distributed isotropically. From the point of view of the above model, this means that for any value of the azimuthal angle φ on the plane of transverse coordinates, there are particles with all possible initial values of the azimuthal angle ψ in the space of the transverse components of the electron momentum in the range from 0 to 2π. Moreover, the value of $\delta\psi$ will also be uniformly distributed over the $0-2\pi$ interval for each value of φ. Based on this circumstance, we can conclude that the calculations using equations (26), (30), and (31) should be performed for the full range of values of the difference of the angles $\delta\psi$, and the scattering of an ensemble of electrons by a laser pulse with an axisymmetric intensity distribution is uniform in the azimuthal angle.

It should be noted that in the case under consideration the uniform distribution of electrons along the azimuthal angle occurs despite the fact that only the field intensity distribution is axisymmetric in the lower approximation, and electromagnetic radiation is linearly polarised and, therefore, the problem has a preferential azimuthal direction. The explanation of this paradigm consists in the fact that the angle of departure of an individual electron from the focal spot depends on the difference $\delta\psi$, and this quantity, in turn, is random in nature and assumes all possible values for various electrons of the scattered ensemble, thereby actually being responsible for the axial symmetry of the scattering pattern of the ensemble of electrons.

It can be expected that a similar symmetry will also be observed in the case of scattering of an ensemble of electrons by a circularly polarised field for which there is no preferential azimuthal direction. A complete study of this issue, however, would require consistent implementation of the above-described asymptotic algorithm for other conditions, which is an independent task and does not seem significant, since there are no methods for generating circularly polarised radiation of relativistic intensity.

A number of numerical solutions of equations (26), (30), and (31) for the case of a Gaussian pulse were obtained in the present work using the Runge–Kutta method implemented in a standard package of computer algebra with an adaptive step and step-by-step accuracy control. The initial conditions for electrons in the presented calculations are set such that the particles initially possess random momenta of small magnitude (within 0.1% of the relativistic threshold, understood as $\rho = 1$, which in terms of non-normalised quantities corresponds to the electron momentum equal to mc). This, in turn, leads for each electron to small deviations of the adiabatic invariant j_0 from unity. Two representative calculations of the trajectories of relativistic ponderomotive dynamics based on equations (26)–(28) are presented in Figs 2 and 3 (the laser pulse parameters are listed in the captions to the figures, the calculations are given for electrons originally located in the focal plane, and the initial distances from electrons to the symmetry axis of the optical field intensity distribution are indicated directly in the figures). In the case illustrated by Fig. 2, the ensemble of electrons is scattered by a laser pulse incident on it from a large distance. Thus, the electron expansion

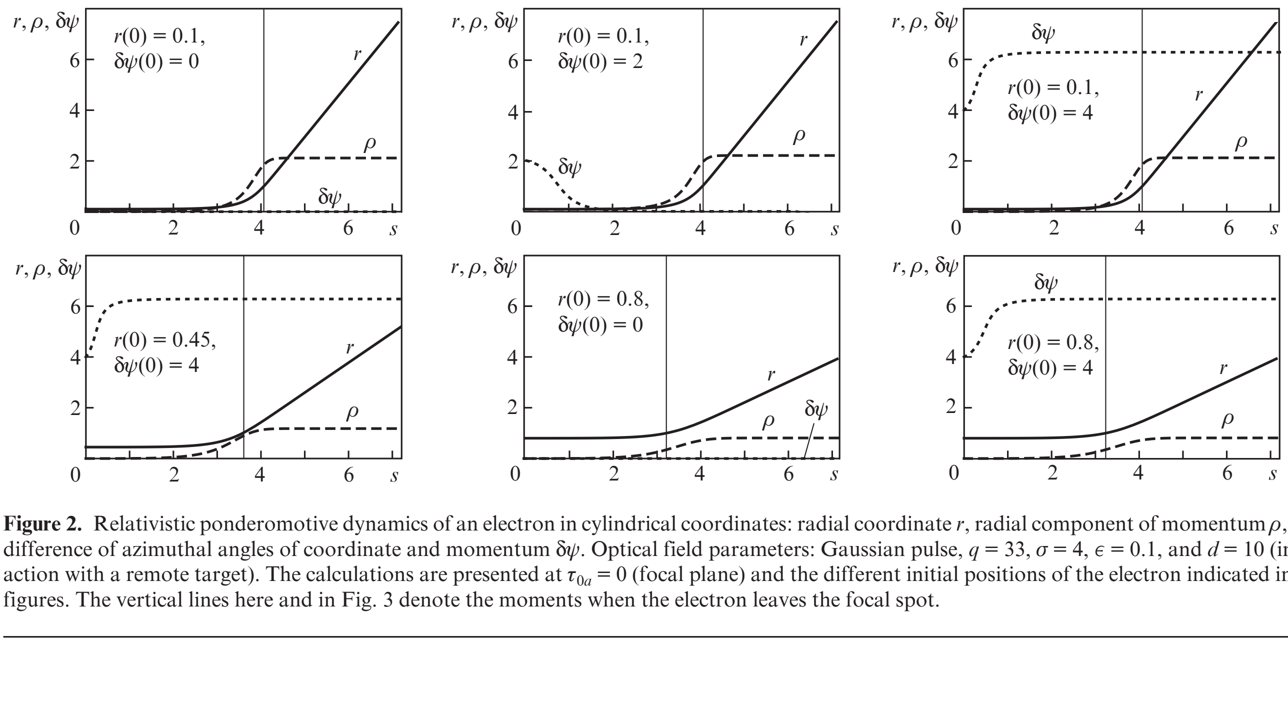

Figure 2. Relativistic ponderomotive dynamics of an electron in cylindrical coordinates: radial coordinate r, radial component of momentum ρ, and difference of azimuthal angles of coordinate and momentum $\delta\psi$. Optical field parameters: Gaussian pulse, $q = 33$, $\sigma = 4$, $\epsilon = 0.1$, and $d = 10$ (interaction with a remote target). The calculations are presented at $\tau_{0a} = 0$ (focal plane) and the different initial positions of the electron indicated in the figures. The vertical lines here and in Fig. 3 denote the moments when the electron leaves the focal spot.

takes place under the action of the pulse front, where the intensity takes moderate values, which also explains the moderate, despite a substantially relativistic laser radiation intensity, energy range of the electron scatter from the focal spot (this effect was discovered both during direct numerical simulation of electron scattering by laser pulses and in experiments [12, 25]). In the case shown in Fig. 3, the electrons were initially placed inside the region of an intense laser field, as would be the case in the situation of ionisation self-injection [26, 27] (in this paper, we do not attempt to self-consistently include ionisation in the model of the electron ensemble scattering). Under the conditions that correspond to Fig. 3, the electrons are scattered by the central part of the time sweep of the laser pulse and, accelerated by a high-intensity field, are ejected with substantially relativistic energies. As can be seen from Figs 2 and 3, the angle difference $\delta\psi$ quickly becomes equal to zero or a multiple of 2π as the electron gains kinetic energy. Finally, the capture of the electron by the field ceases when it reaches the boundary of the focal spot, which, as is also seen from Figs 2 and 3, is almost exactly determined by the relation $r(s)/\sqrt{1+\tau^2} = 1$.

For two cases shown in Fig. 2, Fig. 4 displays the averaged longitudinal component of the momentum and the longitudinal displacement of the electron for two cases of ponderomotive dynamics in a relativistically intense electromagnetic field. As follows from the figure, the ponderomotive dynamics of an electron in a relativistic intense field is accompanied by a significant increase in the longitudinal component of its momentum and a significant shift along the field propagation axis. Moreover, the dependences for the averaged longitudi-

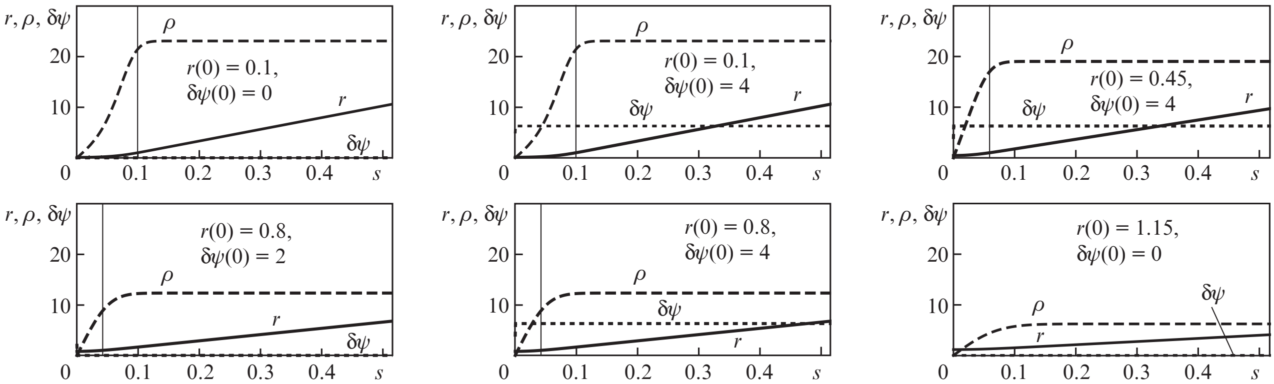

Figure 3. Relativistic ponderomotive dynamics of an electron in cylindrical coordinates: radial coordinate r, radial component of momentum ρ, and difference of azimuthal angles of coordinate and momentum $\delta\psi$. Optical field parameters: Gaussian pulse, $q = 33$, $\sigma = 4$, $\epsilon = 0.1$, and $d = 0$ (interaction under the conditions of self-injection).

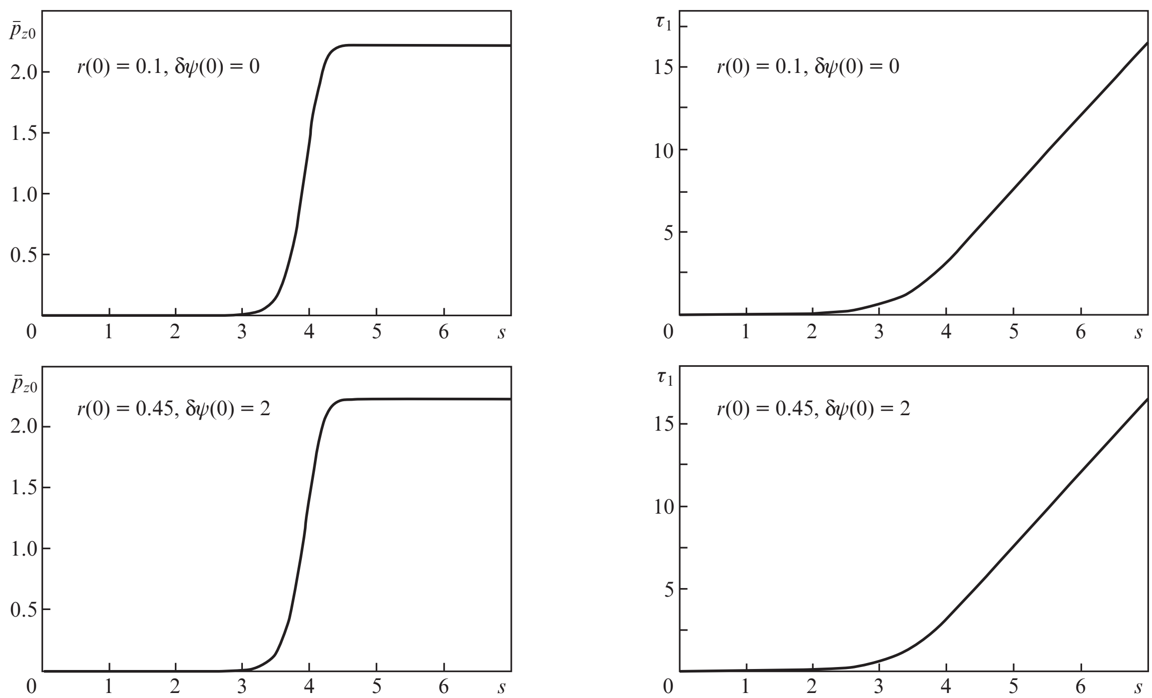

Figure 4. Averaged longitudinal momentum component and longitudinal electron displacement for two cases of ponderomotive dynamics in a relativistically intense electromagnetic field. Optical field parameters: Gaussian pulse, $q = 33$, $\sigma = 4$, $\epsilon = 0.1$, and $d = 10$ (interaction with a remote target). The initial conditions are shown in the figures.

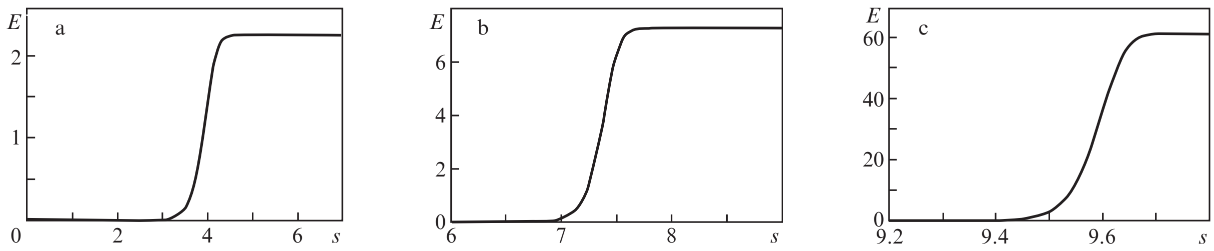

Figure 5. Energy gain by an electron in the process of ponderomotive dynamics during the interaction with laser pulses of various durations: $\sigma =$ (a) 4, (b) 2, and (c) 0.5. Other calculation parameters: $q = 33$, $\epsilon = 0.03$, $d = 10$, $\tau_{0a} = 0$, $r(0) = 0.1$, and $\delta\psi(0) = 2$.

nal component of the momentum are almost identical to those for the amplitude of the transverse momentum ρ presented for the corresponding cases in Fig. 2, which directly follows from the equation for the longitudinal component of the momentum at $j_0 \approx 1$, that is, for an initially almost stationary electron .

Figure 5 shows the results of calculations of the energy gain by an electron under the action of optical envelopes of various durations, in particular, for an envelope with a longitudinal size less than the focal spot. The character of the dynamics as a whole remains the same as in Fig. 1, although shorter pulses correspond to significantly higher electron output energies. It should be noted, however, that the asymptotic algorithm proposed in this work is not applicable for the description of pulses with a duration of the order of or less than that of the optical cycle and the dynamics of charged particles induced by them, and this topic requires independent investigation.

Let Ξ be the ejection angle of the released electron with respect to the axis of the laser pulse propagation. As follows from equations (6) and (11), this angle and the residual electron energy E obey the relationship

$$\Xi = \arctan\frac{\rho}{p_{z0}} = \arctan\frac{\sqrt{2j_0E-(j_0-1)^2}}{E-(j_0-1)}, \qquad (32)$$

which allows us to determine Ξ by the corresponding solutions of equations (26)–(28) or problems (26), (30), and (31) using equation (11). Moreover, as noted above, for each value of Ξ, the electrons scattered by the field with a concentration corresponding to this angle are uniformly distributed over the azimuthal angle.

The constructed theory of ponderomotive dynamics was applied to simulate the scattering of an ensemble of electrons, whose density is low enough to neglect Coulomb interactions between charges, by a relativistically intense laser pulse. According to estimates [28], a description of this situation on the basis of separate Newton equations for electrons is acceptable at electron concentrations of the order of 10^{16} cm^{-3} or less (already at concentrations of the order of 10^{17} cm^{-3}, collective effects due to the Coulomb interaction come into play; however, this requires a fluid dynamic or kinetic description of the field–plasma system, where the electrons are accelerated by wakefields [29], rather than taking into account Coulomb corrections). Under the condition of a sufficiently low electron concentration, it is acceptable to assume that the actual number of particles is proportional to the target density in model calculations. In the framework of this work, 5000 particles were involved in each calculation. Figure 6 shows the pattern of the electron scattering over the angle with respect to the field propagation direction for the same laser radiation parameters for which the trajectories in Fig. 2 are plotted. In the depicted polar coordinate system, the marked polar angles correspond to the boundaries of the ranges of angles of electron escape from the focal spot with respect to the direction of propagation of the laser pulse, and the distance from the reference point is proportional to the number of particles in this angular range. The angles are expressed in units of energy into which they are converted according to the above expression (32) for Ξ. In azimuth, the scattering is uniform. Ranges of angles are selected so that to reveal to the greatest extent the structure of the distribution of electrons. The subponderomotive noise was filtered out of the numerical data, that is, the scatter with the energy at the level of accuracy of the asymptotic method implemented in Section 2. Initially located close to the propagation axis of the field, the electrons are scattered with relatively high energies and, therefore, according to relation (32), into small angles. Electrons located relatively far from the axis are scattered in large quantities into large angles. Figure 7, for the same case, shows the energy spectra of electron scattering into angles $0.59 < \Xi < 0.67$ and $0.67 < \Xi < 0.79$ covering the highly relativistic energy range (for these parameters of the laser pulse, very small electron scattering angles are unattainable under conditions of interaction with a remote target). Similarly, for the parameters of laser radiation corresponding to the trajectories in Fig. 3, Figs 8 and 9 present the scattering pattern of the expansion of an ensemble of electrons with a scale in

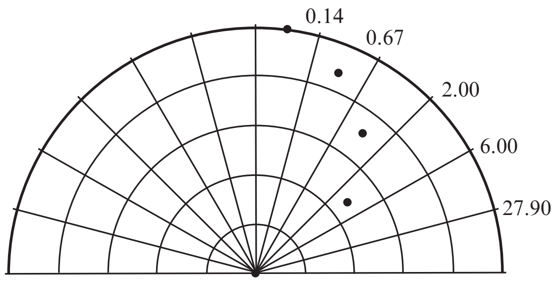

Figure 6. Pattern of electron scattering along the angle Ξ relative to the propagation axis of a relativistically intense laser pulse with the same parameters as in Fig. 2 (interaction with a remote target). The radial coordinates of the given points correspond to normalised numbers of electrons, the angular sectors correspond to ranges of Ξ, and the energy equivalents of the angles in units of mc^2 calculated according to equation (32) for $j_0 = 1$ are given on the angle scale. For each value of Ξ, the electrons scattered by the field are uniformly distributed over the azimuthal angle ψ, i.e., around the axis of propagation. At $\lambda = 800$ nm, the calculation parameters correspond, in particular, to a laser pulse with a focal spot with a diameter of 1.3 μm, a duration of 17 fs, and an intensity of 2.3×10^{21} W cm^{-2}.

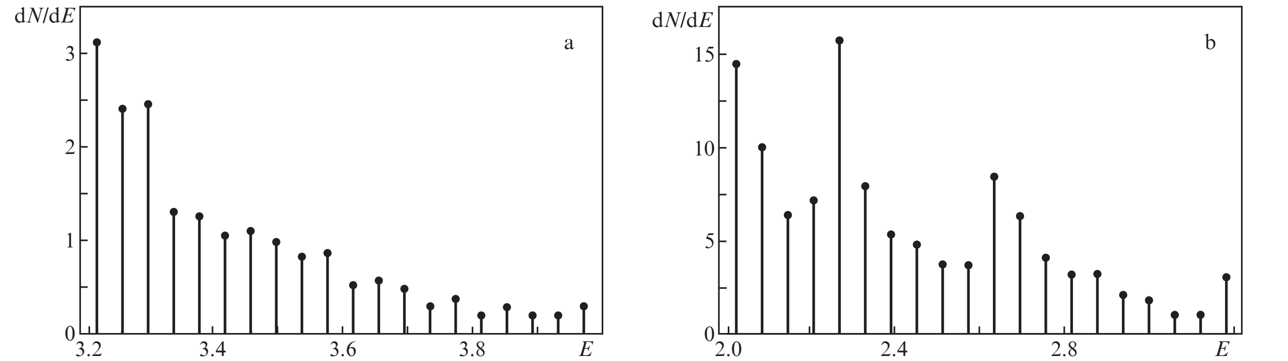

Figure 7. Energy spectra of electrons scattered in separate angular ranges relative to the propagation axis of a relativistically intense laser pulse with parameters corresponding to Fig. 2 (interaction with a remote target): (a) $0.59 < \Xi < 0.67$ and (b) $0.67 < \Xi < 0.79$. For each value of Ξ, the pattern of scattering spectra is the same for all values of the azimuthal angle ψ. At $\lambda = 800$ nm, the calculation parameters correspond, in particular, to a laser pulse with a focal spot with a diameter of 1.3 μm, a duration of 17 fs, and an intensity of 2.3×10^{21} W cm^{-2}.

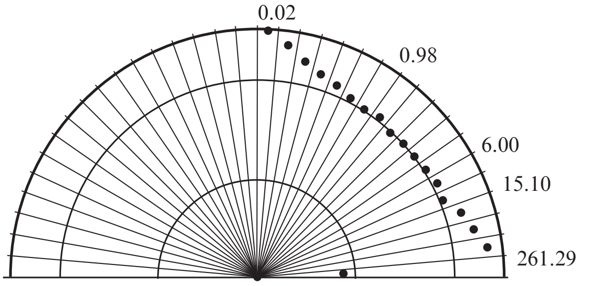

Figure 8. Pattern of electron scattering along the angle Ξ relative to the propagation axis of a relativistically intense laser pulse with the same parameters as in Fig. 3 (interaction under self-injection conditions). The radial coordinates of the given points correspond to normalised numbers of electrons, the angular sectors correspond to ranges of Ξ, and the energy equivalents of the angles relative to the propagation axis of the laser pulse (in units of mc^2) calculated according to equation (32) for $j_0 = 1$ are given on the angle scale. For each value of Ξ, the electrons scattered by the field are uniformly distributed over the azimuthal angle ψ, i.e., around the axis of propagation. At $\lambda = 800$ nm, the calculation parameters correspond, in particular, to a laser pulse with a focal spot diameter of 1.3 μm, a duration of 17 fs, and an intensity of 2.3×10^{21} W cm^{-2}.

energy units and energy spectra corresponding to angles $0.08 < \Xi < 0.1$ and $0.45 < \Xi < 0.63$. The energy intervals in Figs 7 and 9 correspond to the spread of the energies of electrons falling in the considered angular ranges.

5. Conclusions

Based on a strict asymptotic solution of the equations for electromagnetic radiation in vacuum and the equation of motion of an electron in a propagating field, we have constructed a description of the ponderomotive dynamics of an electron under the action of a focused relativistically intense laser pulse. The parameter for asymptotic expansion is proportional to the ratio of the radiation wavelength to the radius of its focal spot.

The electromagnetic field in vacuum is represented by expressions that include the ground state, which corresponds to a focused envelope, as well as small-parameter corrections to the transverse field component caused by the finiteness of the pulse duration and corrections in the form of a longitudinal field component. Approximate solutions of the Newton relativistic equation are obtained as functions of the proper time of the electron using the Krylov–Bogoliubov method, which leads to the equations of motion averaged over the phase of the electron oscillations and makes it possible to reveal the adiabatic invariant corresponding to the equations of the original problem. This invariant defines, in particular, the relationship between the electron energy acquired as a result of interaction with the field and the angle of its exit from the focal region with respect to the axis of the laser pulse propagation.

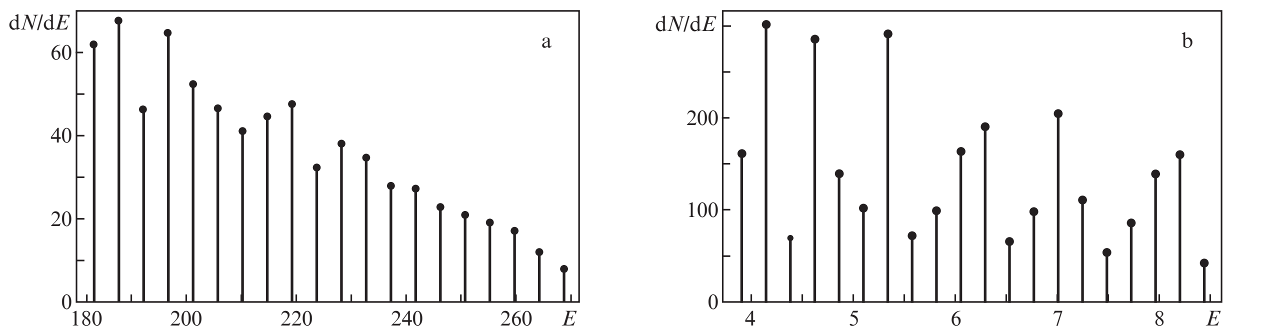

Figure 9. Energy spectra of electrons scattered in separate angular ranges relative to the propagation axis of a relativistically intense laser pulse with parameters corresponding to Fig. 3 (interaction under self-injection conditions): (a) $0.089 < \Xi < 0.1$ and (b) $0.45 < \Xi < 0.63$. For each value of Ξ, the pattern of scattering spectra is the same for all values of the azimuthal angle ψ. At $\lambda = 800$ nm, the calculation parameters correspond, in particular, to a laser pulse with a focal spot with a diameter of 1.3 μm, a duration of 17 fs, and an intensity of 2.3×10^{21} W cm^{-2}.

In the case of scattering of an electron ensemble by an optical field with an axisymmetric distribution of the amplitude, the equations of the relativistic ponderomotive dynamics are simplified as a result of their averaging over the initial directions of the electron momenta.

The theory presented is used to construct averaged individual electron trajectories in a relativistically intense optical field with a Gaussian transverse intensity distribution, as well as the corresponding patterns and electron energy spectra for an electron ensemble scattered by laser radiation.

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