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Mechanism of electron bunching by an ultrarelativistically intense laser pulse when crossing a nonuniform-plasma boundary

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Abstract. **Numerical simulations were used to investigate the electron bunching induced by a laser pulse in its penetration into a plasma that has a nonuniform density profile at the vacuum–plasma boundary, which reaches a plateau. A mechanism was found to exist for this plasma which fosters a denser electron grouping in the generated bunch in comparison with the plasma with a sharp boundary. This mechanism is related to the nonstationarity of the wake wave produced by the laser pulse at the beginning of the plasma density plateau.**

Keywords: *laser pulse, electron bunch, generation, injection.*

1. Introduction

In a laser-plasma accelerator, a high-intensity laser pulse produces a wake wave, whose field is several orders of magnitude higher than the accelerating fields attainable with traditional radio-frequency technology, making it possible to accelerate electron bunches to an energy of ~8 GeV over a length of 20 cm [1]. In the interaction with a nonuniform plasma, a laser pulse of ultrarelativistic intensity may also produce ultrashort electron bunches [2], which are of special interest in the generation of ultrashort X-ray pulses employed in the physics of ultrafast processes [3]. The length of the electron bunch injected into the laser-plasma accelerator has a strong effect on its characteristics after acceleration, in particular on its monoenergeticity. This is of paramount importance in the development of free-electron lasers, which require a relative energy spread of less than 0.1% [4]. That is why the investigation of the physical processes leading to the generation of shortest electron bunches is of major practical importance.

One of these processes was discovered by Li et al. [5], who numerically simulated the interaction of a laser pulse with a semi-infinite plasma, which had a diffuse interface with the vacuum in the form of a transition layer with a linearly varying density. They demonstrated that a laser pulse of ultrarelativistic intensity may generate attosecond electron bunches when passing through this plasma boundary. Moreover, this physical process is quasi-one-dimensional and the electron bunch is made up of the electrons that are in the vicinity of the point where the linear density profile becomes a plateau.

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The one-dimensional theoretical analysis of electron bunch production by a laser pulse interacting with semi-infinite plasmas suggested [6–8] that this process has a threshold and is realised when the energy $E_{\rm os}$ of longitudinal electron oscillations after the interaction with the laser pulse exceeds the threshold value $E_{\rm os\,th} = mc^2\gamma_{\rm ph} = mc^2/\sqrt{1 - V_{\rm gr}^2/c^2}$, where $V_{\rm gr}$ is the group velocity of the laser pulse in the density plateau and m is the electron mass. It was found [8] that the type of the plasma–vacuum interface (be it sharp or diffuse) has no effect on the threshold energy but may result in impairment of the characteristics of the generated bunch. The present work is concerned with the numerical simulations of attosecond electron bunch production by laser pulses interacting with the diffuse boundary of a nonuniform plasma so as to elucidate the features of the electron bunch generation mechanism under these conditions.

2. Formulation of the problem

Consider the problem in which a laser pulse in incident on the plasma boundary normally to its surface along the z axis, the carrier frequency being much higher than the plasma frequency. The zero point of the z axis coincides with the beginning of the plasma density plateau, at the point corresponding to $n(z) = n_0$. In the transition layer ($z < 0$) with a characteristic thickness σ, the plasma density varies according to the dependence $n(z) = n_0\exp[-z^2/(2\sigma^2)]$, which is more realistic than the linear one.

In the one-dimensional geometry, for a circularly polarised laser pulse the longitudinal electron motion along the z axis is described by the equations

$$\frac{\mathrm{d}p}{\mathrm{d}t} = |e|\frac{\partial\varphi}{\partial z} - mc^2\frac{\frac{\partial}{\partial z}\left(\frac{eA}{mc^2}\right)^2}{2\sqrt{1+\frac{p^2}{m^2c^2}+\left(\frac{eA}{mc^2}\right)^2}},$$
$$\frac{\mathrm{d}z}{\mathrm{d}t} = u = \frac{p/m}{\sqrt{1+\frac{p^2}{m^2c^2}+\left(\frac{eA}{mc^2}\right)^2}}, \qquad (1)$$

where $A(z, t)$ is the amplitude of the envelope of the vector potential of the laser pulse; $\varphi(z, t)$ is the scalar potential of the charge separation field; and p and u are the electron momentum and velocity. For our numerical simulations, the electron plasma component will be represented in the form of a set of electron macroparticles, i.e. plane layers of thickness $k_{\rm p}\Delta z = 0.0005$ ($k_{\rm p}$ is the wavenumber of the plasma wave), whose motion is defined by Eqns (1).

The amplitude and shape of the laser pulse in Eqns (1) is assumed to be invariable, because the electron bunch, according to the data of Refs [6–9], is generated in a time interval during which the variation of laser pulse parameters may be neglected. For simplicity we also assume that its group velocity depends only on the plasma density and is defined, for a pulse whose length is much shorter than the characteristic size of the transition layer, by the current position $z_{\rm L}$ of its centre on the z axis by the relation

$$V_{\rm gr}(z_{\rm L}) = V_{\rm gr}(n(z_{\rm L})) = c^2 k_0 / \sqrt{c^2 k_0^2 + \omega_{\rm p}^2(z_{\rm L})}\,,$$

where $\omega_{\rm p}^2(z_{\rm L}) = 4\pi e^2 n(z_{\rm L})/m$, and k_0 is the wavenumber of the high-frequency filling of the laser pulse.

The $\varphi(z,t)$ potential and the charge separation field $E_z = -{\rm d}\varphi/{\rm d}z$, which appear in Eqns (1), are found from the Poisson equation

$$\frac{{\rm d}^2\varphi}{{\rm d}z^2} = 4\pi|e|(n_{\rm e} - n_{\rm i}(z)), \qquad (2)$$

where $n_{\rm e}$ and $n_{\rm i}$ are the densities of electrons and immobile ions ($n_{\rm i} = n(z)$). In the integration of Eqn (2) for calculating the charge separation field, we assume that the electron density coincides with the ion density in the plasma interior where the laser pulse has not yet penetrated, which corresponds to setting the boundary conditions $\varphi(\infty) = 0$, ${\rm d}\varphi(\infty)/{\rm d}z = 0$. In this case, the density of plasma electron component along the z axis is defined by the location of electron macroparticles at a given point in time.

3. Numerical simulations

Numerical simulations of electron bunch generation was performed for a laser pulse which passes through the point corresponding to the beginning of plasma density plateau ($z = 0$) at the point in time t_0 and has the envelope

$$a = a_0 \cos^2[(t - t_0)/\tau_{\rm L}]\,{\rm sgn}(\pi\tau_{\rm L}/2 - |t - t_0|),$$

where $a_0 = |e|A_0/(mc^2) = 6.2$ is the dimensionless amplitude of the vector potential; $\tau_{\rm L}$ is the duration of the laser pulse, which corresponds to its half-amplitude duration $\tau_{\rm FWHM} = 1.143\tau_{\rm L} = 12$ fs. It is assumed that the group velocity $V_{\rm gr}$ of laser pulse propagation on the plasma density plateau corresponds to the gamma factor $\gamma_{\rm ph} = 1/\sqrt{1 - V_{\rm gr}^2(n_0)/c^2} = 7$. The plasma density is found from the relation $k_0/k_{\rm p} = \gamma_{\rm ph}$, where $k_{\rm p} = \sqrt{4\pi e^2 n_0/(mc^2)}$, $k_0 = 2\pi/\lambda_0$, $\lambda_0 = 1$ μm. In the interaction with such plasma, the laser pulse excites longitudinal electron oscillations in the plateau with an energy $E_{\rm os} = 7.8617mc^2$. Therefore, we investigate the production of electron bunches in the weak suprathreshold mode, since $E_{\rm os}/(mc^2) - \gamma_{\rm ph} \ll \gamma_{\rm ph}$.

To elucidate the effect of the plasma boundary type on the electron bunch generation by the laser pulse which traverses through this boundary, we performed simulations of this process for the plasma with the transition layer of thickness $\sigma = 40$ μm (Fig. 1) and for the plasma with a sharp boundary (Fig. 2). Figures 1 and 2 show the electron macroparticle distributions in the (z, p) phase plane at the instant of the onset of the crossing of electron trajectories and of wake wave breaking, which results in the self-injection of electrons into the wake wave produced by the laser pulse and in the production of electron bunch.

In Figs 1 and 2, the large black point corresponds to an electron located at the time instant $t_{\rm crf}$ on the phase plane at a point with the coordinates $z_{\rm crf}$, $p_{\rm crf}$, from which the process of intersection of the electron trajectories and their self-injection into the wake wave begins. For a plasma with a diffuse boundary, the first injected electron is a two-sided point of concentration of the electron distribution, and electrons will be collected in the generated bunch simultaneously from two distribution branches. This means that in this case the process of electron injection and bunch generation occurs more intensively than in a plasma with a sharp boundary.

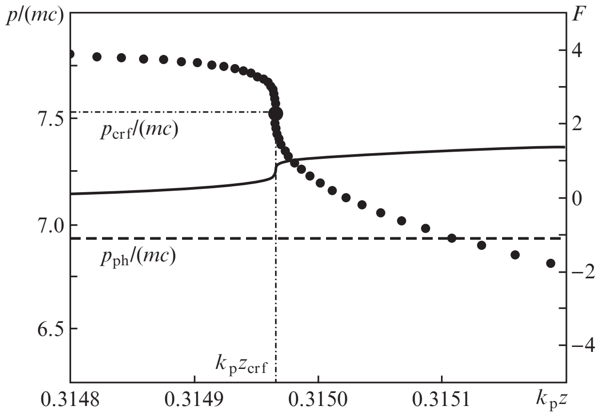

Figure 1. Electron distribution (points) in the (z, p) phase plane at the instant of the onset of bunch generation in the plasma with the transition layer of thickness $\sigma = 40$ μm. The dashed line corresponds to momentum $p_{\rm ph} = mc\sqrt{\gamma_{\rm ph}^2 - 1}$, the solid line stands for the force $F = -|e|E_z/(mc\omega_{\rm p})$ acting on the electrons in the wake wave.

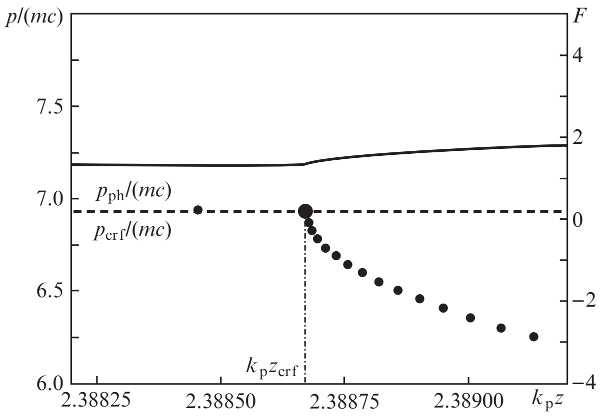

Figure 2. Same as in Fig. 1, for the plasma with a sharp boundary.

Another difference between Figs 1 and 2 is that the energies of the first electrons injected into the wake wave are much different in the plasmas with the sharp and diffuse boundaries. As is known from Refs [6–8], at the instant the plasma electron trajectories begin to intersect, which results in the generation of an electron bunch, the self-injected electron velocity coincides with the phase velocity of the wake wave at the same point. In the case of a sharp boundary [6, 7], the energy of the first injected electron coincides with the gamma factor $\gamma_{\rm ph}$ of the wake wave in the plasma interior, i.e. $p_{\rm crf} = mc\sqrt{\gamma_{\rm ph}^2 - 1}$. The oscillation centre of this electron is at point $z_{0\rm f}$, which is at a distance from the sharp plasma boundary, i.e. from the beginning of the plateau, equal to the amplitude $A_m = \sqrt{(E_{os} - mc^2)/(2\pi e^2 n_0)}$ of its longitudinal oscillations after interaction with the laser pulse.

For the plasma with the diffuse boundary, from Fig. 1 it follows that $p_{crf} > mc\sqrt{\gamma_{ph}^2 - 1}$. This signifies that the oscillation centre of the electron under consideration is at a point which satisfies relation $z_{0f} < A_m$, because it is only under this condition that the electron self-injection into the wake wave may occur in the domain where the gamma factor of the wave does not coincide with γ_{ph} on the plateau.

Concerning the phase velocity of the wake wave generated by the laser pulse, it is well to bear in mind the following. At any point of the z axis and at any point in time, the velocity of the wake wave is defined by two factors: by the group velocity of the laser pulse, which lowers in its transit through the transition layer from the luminal velocity to the constant value defined by the plateau plasma density n_0, as well as by the motion prehistory of the electrons which make up the wake wave at the given point in space. In the present investigation, the parameters of the laser pulse are so selected that the variation of the group velocity in its passage from the boundary plasma layer to the density plateau has no effect on the motion of those electrons which subsequently inject into the wake wave. These and subsequent electrons in the plasma depth are affected by the laser pulse propagating at a constant group velocity $V_{gr}(n_0)$. These electrons form a regular wake wave with a potential $\varphi(z - V_{ph}t)$ and a constant phase velocity $V_{ph} = V_{gr}(n_0)$ until they enter the transition layer, because in the course of oscillations of each electron about its oscillation centre their phase difference persists. However, when an electron oscillates about a point with coordinate $z_0 < A_m$, over time, it enters the transition layer. After this electron and the adjacent ones return to the plateau, their oscillation energy remains as before and, as shown in Ref. [8], satisfies the relation

$$E_{os} = mc^2\left[1/\sqrt{1 - u^2/c^2} + k_p^2(z - z_0)^2/2\right], \quad (3)$$

where z and u are the current coordinate and velocity of the electron with the oscillation centre at the point z_0. But the phase difference of the electron oscillations relative to each other changes, and the wake wave therefore becomes nonstationary in the plasma layer for $z < A_m$. In this case, according to the data of simulations of Ref. [5], the phase velocity of the wake wave, which is formed by these electrons, turns out to be higher than on the plateau, where $V_{ph} = V_{gr}(n_0)$ for $z_0 \geqslant A_m$. However, as the oscillation centres z_0 of the electrons that make up the wake wave approach the A_m value, it gradually lowers to the phase velocity of the wake wave in the plateau depth equal to $V_{gr}(n_0)$.

Early in the electron trajectory intersection, the bunch charge is small and has a minor effect on the motion of other electrons, which, following the first self-injecting electron, begin participating in the process of their trajectory intersection with the trajectories of neighbour electrons. Then, in view of the coincidence of electron velocity u with the phase velocity $V_{ph}(z_{cr})$ of the wake wave, from relation (3) it follows that the phase of electron oscillations about the oscillation centre at which it enters the trajectory intersection process should vary. In this case, it should vary in such a way that for each next electron the instant its trajectory crosses the neighbour electron trajectory occurs at an earlier stage of its own oscillations about the oscillation centre, since the phase velocity of the wake wave becomes lower.

The oscillation phase at which there occurs trajectory intersection for each electron is conveniently expressed in the form of the difference $z_{cr} - z_0$, which corresponds to the deviation z_0 of a given electron from its oscillation centre. Figure 3 shows the dependences of $z_{cr} - z_0$ on the position z_0 of electron oscillation centre for the plasma with a transition layer ($\sigma = 40$ μm) and the plasma with a sharp boundary, which were obtained by numerical simulations. For the ease of comparing these dependences, plotted on the abscissa is the quantity $z_0 - z_{0f}$, where z_{0f} is the position of the electron oscillation centre which the process of trajectory intersection begins. One can see from Fig. 3 that the phase $z_{cr} - z_0$ is practically invariable for the plasma with the sharp boundary, i.e. that the trajectory intersection for each next electron occurs in about the same way as for the previous one. In the plasma with the diffuse boundary ($\sigma = 40$ μm), each next electron enters the trajectory intersection at an increasingly early stage of its oscillations and is therefore injected into the wake wave earlier. This enhances the intensity of electron accumulation in the laser-driven pulsed bunch.

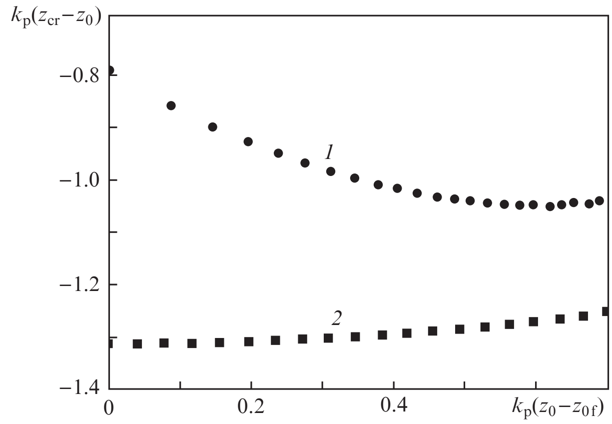

Figure 3. Dependence of the oscillation phase $z_{cr} - z_0$ for the electron entering the trajectory intersection process on the position z_0 of its oscillation centre for the plasmas with diffuse [(*1*); $\sigma = 40$ μm] and sharp (*2*) boundaries.

The simulation data are demonstrated in Fig. 4, which shows the growth of charge density per unit cross section of the generated bunch for the plasmas with diffuse and sharp boundaries. These data suggest that the bunch charge accumulation intensity is much higher for the plasma with the diffuse boundary ($\sigma = 40$ μm) than for the plasma with a sharp boundary. One might therefore expect in this case that electrons would be grouped in the head part of the bunch more densely in space, i.e. over the bunch length. For the same reason the charge accumulation in the bunch reaches saturation earlier and approaches some asymptotic value earlier for the plasma with the diffuse boundary than for the sharp-boundary plasma.

We emphasise that the above description of charge accumulation in the bunch generated by a laser pulse is qualitative and applies primarily to the early stage of the process, while the bunch charge is small. In reality this process is quite complicated, especially at the late bunch generation stage: due to mixing and mutual repulsion, the effect of bunch electrons on the

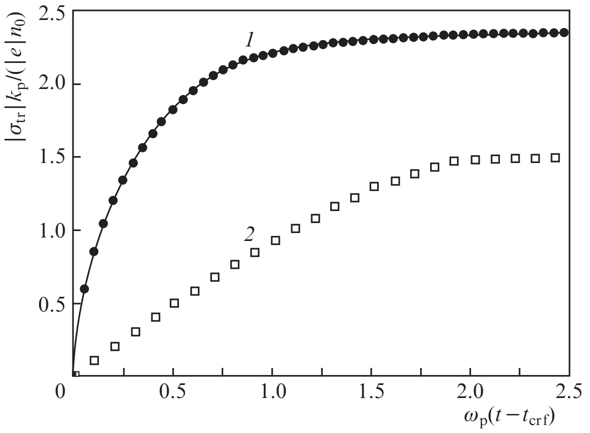

Figure 4. Time dependences of the charge density in the generated bunch per its unit cross section for the plasmas with diffuse [(*1*); σ = 40 μm] and sharp (*2*) boundaries.

capture of the subsequent electrons becomes so significant that so simple an approach to the analysis of electron motion may not be applied. Furthermore, in the time interval comprising the end of bunch generation stage and the onset of the acceleration stage, there sets in the mechanism of electron exchange between the bunch tail and the background plasma, which is not shown in Fig, 4. As a result, a part of the electrons located in the tail part of the bunch are subsequently lost or make up a long plume of low-energy electrons. This affects the eventual length of the electron bunch when we include the entire set of the electrons that escape from the background plasma.

We note, however, that the electrons captured in the bunch at the initial stage of its production are arranged over the bunch length in the order close to the sequence of their injection. The above qualitative reasoning about the injection mechanism and electron grouping in the injected bunch suggests it is possible to separate out, of the entire set of electrons in the bunch, the head part with sufficiently good characteristics by separating electrons in energy. To this end, we performed the simulations of electron bunch production by a laser pulse passing through the plasma boundaries with transition layers of different thickness. Since the effect of charge repulsion affects the bunch length, there is good reason to separate electrons in energy even at the stage of acceleration, when the leading electron of the bunch acquires a sufficiently high energy $E_{ld} \gg mc^2$. In this case, the velocity of the majority of bunch particles becomes so close to the luminal velocity that subsequently the bunch length is hardly changed.

For the bunches obtained when a laser pulse transited the plasma boundary with transition layers of different thickness, Fig. 5 shows the dependences of bunch characteristics on the cutoff energy $E_{cut} < E_{ld}$ for a leading bunch electron energy $E_{ld} = 40mc^2$, which is acquired over an acceleration path of ~10 μm after electron self-injection into the wake wave. By selecting the cutoff energy E_{cut} it is possible to control the characteristics of the selected part of bunch electrons. One can see from Figs 5a and 5c that the separation of electrons in energy permits improving the characteristics of the selected part of the bunch: its length and electron energy spread. However, excessive reduction of the difference $E_{ld} - E_{cut}$ also lowers the charge density per unit cross section in this part of the bunch. Its total value without truncation of the electron distribution in energy may be estimated by the formula

$$\sigma_{tr} \approx -|e|n_0 k_p^{-1} \sqrt{2[E_{os}/(mc^2) - \gamma_{ph}]}\,, \tag{4}$$

which was derived earlier for the plasma with a sharp boundary [6, 7]. Note that, according to Ref. [8], formula (4) provides a reasonably accurate estimate of the bunch charge density per unit cross section also in the case of plasma with a transition layer with a linear density profile. This is indication that the mechanism of the termination of electron accumulation in the generated bunch is universal in nature and depends only slightly on the type of plasma boundary.

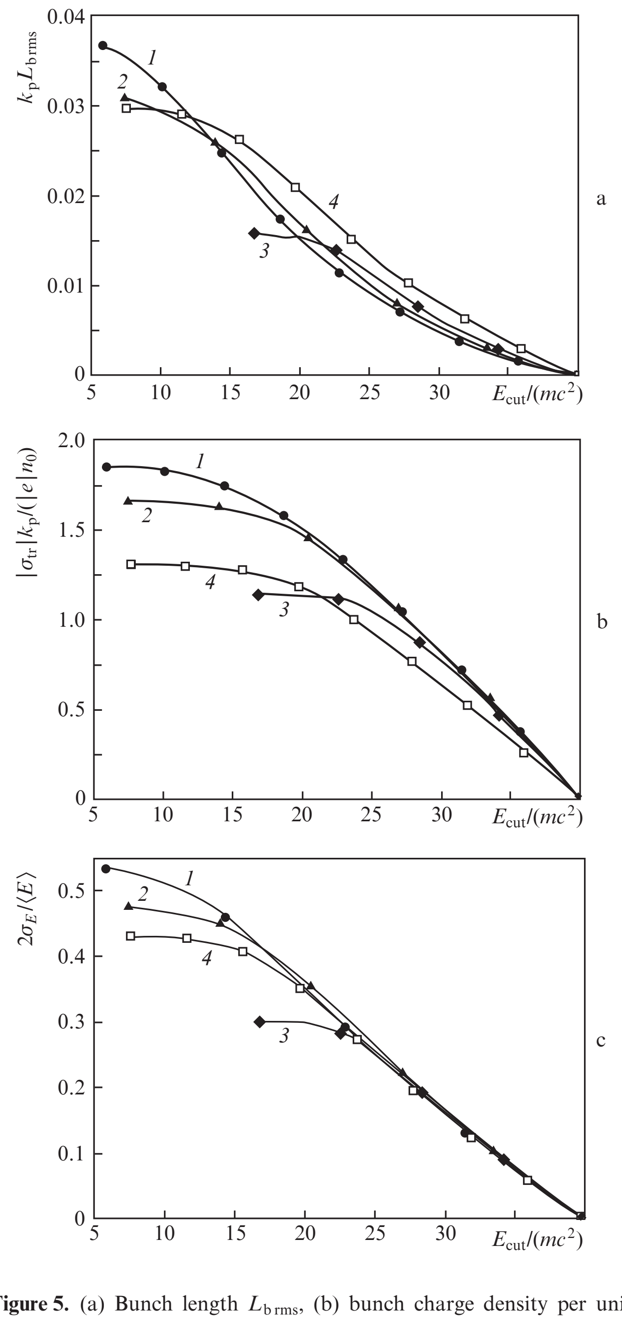

Figure 5. (a) Bunch length $L_{b\,rms}$, (b) bunch charge density per unit cross section, and (c) relative spread in electron energy E in the bunch as functions of the cutoff energy for the plasmas with the transition layer of thickness σ = (*1*) 40, (*2*) 100, and (*3*) 150 μm as well as (*4*) for the plasma with a sharp boundary.

One can also see from Fig. 5 that the thickness of transition layer affects only slightly on the electron energy spread and the charge density in the separated part of the bunch, but its length depends noticeably on this parameter. Hence it follows that the efficiency of electron grouping mechanism in the bunch generated by a laser pulse passing through the plasma boundary with a transition layer depends significantly on the layer thickness. Exact determination of the optimal transition layer thickness calls for the treatment of this problem by analytical methods, which will be done in the future. The data of numerical simulations given in Fig. 5a suggest that shorter electron bunches are generated in the plasma with the transition layer of thickness $\sigma = 40$ μm. In this case, for a cutoff energy $E_{\text{cut}} = 25mc^2$ it is possible to obtain a $\sim$30-as long bunch. The relative electron energy spread in this separated bunch is $\sim$25% and the bunch charge density per unit cross section $|\sigma_{\text{tr}}| \approx 1.2|\text{e}|n_0 k_{\text{p}}^{-1}$.

In the case of two-dimensional geometry of the problem, the bunch charge depends also on the characteristic lateral pulse width σ_{L}. However, it should be rather large, $k_{\text{p}}\sigma_{\text{L}} \gg \sqrt{a_0}$ (see, for instance, Refs [5, 6, 10]) to consider the electron motion to be sufficiently close to one-dimensional, as is assumed in the present investigation. The parameters of the laser pulse used in the present simulations satisfy this relation if it is assumed that the characteristic transverse size of the laser pulse $\sigma_{\text{L}} \approx 20\lambda_0$. In this case, it is valid to say that the above results obtained in the one-dimensional geometry of electron bunch generation by the laser pulse, which penetrates the plasma through a diffuse boundary, provide a qualitatively correct picture of the operation features of the mechanism underlying this process. Then, using the approach outlined in Refs [5, 6] and taking into account formula (4), the bunch charge may be estimated at $q_{\text{tr}} \approx 100$ pC.

4. Conclusions

The one-dimensional numerical simulation of electron bunch production in the passage of an ultrarelativistically intense laser pulse through a plasma boundary with a transition layer permits elucidating the main qualitative characteristics of this physical phenomenon. As shown in our work, owing to the phase velocity variation of the wake wave produced by the laser pulse near the onset of plasma density plateau, there is a mechanism which favours a more intense bunch generation for the plasma with a diffuse boundary than for the plasma with a sharp boundary. As a consequence, the electrons that are accumulated in the bunch group more densely along the bunch length to form bunches with a denser head part. By separating out these electrons from the total set of bunch electrons, for the diffuse-boundary plasma it is possible to obtain bunches with characteristics which are superior to those obtained from the sharp-boundary plasma. By varying the plasma density and transition layer length as well as by selecting the cutoff energy, it is possible to control the bunch parameters and select the optimal values for the bunch injected into a laser-plasma accelerator.

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