

Quantum gates on asynchronous atomic excitations

Yu.I. Ozhigov

Abstract. A method for realising a universal system of quantum gates based on asynchronous excitations of two-level atoms in optical cavities is proposed. The entangling operator of the CSign type is implemented without beam splitters, approximately, using the incommensurability of the Rabi oscillation periods in a cavity with single and double excitations.

Keywords: quantum gate, two-level atom, optical cavity, asynchronous excitation.

1. Introduction

Quantum computing is based on the invasion of quantum theory into the field of complex processes, where the operation of its fundamental laws is not yet studied. Therefore, the design of the simplest schemes of such computations, in which quantum laws would manifest themselves as clearly as possible, is an urgent problem. The dark area here is decoherence, which arises from the interaction of charges and the field, the quanta of which strongly couple the quantum computer to the environment. This makes it necessary to account for photons and control them, or even explicitly use photons in quantum protocols.

Photons as information carriers make it possible to use linear optical devices to realise one-qubit gates, but the design of entangling operations is difficult to implement, since photons do not directly interact with each other. There is a popular KLM scheme [1], where measurements are used as an ersatz interaction and its improved version [2, 3] with teleportation, which significantly increases efficiency, as well as a number of options of this scheme for atoms (see, e.g., [4]). However, the use of classical probabilistic schemes in experiment imposes increased requirements on the efficiency, at least theoretically possible, of quantum gates on single particles. The use of classical probability obscures the main question for a quantum computer: how does coherence manifest itself in complex systems of different particles?

The most *ab initio* methods are more suitable here, the main of which implies considering an optical cavity with several atoms, whose interaction with a single-mode field is

clearly described *ab initio* (for the capabilities of this type of devices, see, e.g., [5]). For example, the CNOT gate was implemented using the external (vibrational) degrees of freedom of an atom [6]. However, the essence of quantum computing is not in the coherent behaviour of an individual qubit, but in scaling a Feynman quantum processor that implements the theoretical capabilities of unitary dynamics in the entire Hilbert state space and gives, e.g., Grover's algorithm [7] on the same hardware as Shor's algorithm [8]. Exploiting external factors to demonstrate the dynamics of the interaction of individual atoms and the field is useful exactly for individual atoms, but the inevitably introduced noises will certainly affect scaling.

In this regard, schemes for implementing gates using minimal means, which are well described *ab initio*, are of value. One of such schemes was proposed by H. Azuma in Ref. [9], where a qubit is constructed from a pair of optical paths where a single photon is running. In this precisely reproducible scheme, the interaction of photons with atoms is used only for the entangling transformation CSign: $|x, y\rangle \rightarrow (-1)^{xy}|x, y\rangle$, which requires two optical cavities; two beam splitters and phase shifters are also needed.

In this paper, we propose a simplified version of the Azuma scheme, in which only one cavity is used, and the beam splitters are replaced by a time shift for the photons entering it. We will have asynchronous states of atoms with Rabi oscillations as logical qubits. This scheme can be modified for purely photonic carriers with a time shift that determines the value of the qubit. However, atoms as carriers of information have the advantage of being much easier to control, as well as the photons emitted by atoms. The advantage of the proposed scheme is its simplicity. The drawback is similar to that of [9], namely, the influence of the triggering time error of the Pockels cell or its analogue, which must be made significantly less than the period of the atom Rabi oscillation in the cavity.

For technical reasons, we will implement the coCSign gate: $|x, y\rangle \rightarrow (-1)^{(x \oplus 1)y}|x, y\rangle$, which changes sign for a single state $|0, 1\rangle$ and is kindred to the CSign gate proposed in [9]. There is no difference, since $\text{CSign} = \sigma_x(x) \text{coCSign} \sigma_x(x)$, where σ_x is the Pauli matrix, and single-qubit gates are realised by linear optical devices

2. Calculation of phase shifts

The key element of the considered scheme is an optical cavity with one two-level atom having an energy gap $\hbar\omega$ between the ground, $|0\rangle$, and excited, $|1\rangle$, energy levels, where ω coincides with the frequency of a photon of a certain mode, confined in the cavity. The atom-field coupling constant g is assumed to

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be small, $g/(\hbar\omega) \ll 1$ (in practice, this ratio should not exceed 10^{-3}), for the possibility of using RWA approximation, in which the Jaynes–Cummings Hamiltonian of the atom-field system has the form [10]

$$H_{JC} = H_0 + H_{\text{int}},$$

$$H_0 = \hbar\omega a^\dagger a + \hbar\omega\sigma^\dagger\sigma, \quad H_{\text{int}} = g(a^\dagger\sigma + a\sigma^\dagger), \quad (1)$$

where a and a^\dagger are the operators of annihilation and creation of a photon; and σ and σ^\dagger are the operators of deexcitation and excitation of the atom. Let us write the basic states of the field and atom in the form $|n\rangle_{\text{ph}}|m\rangle_{\text{at}}$, where $n = 0, 1, 2, \dots$ is the number of photons in the cavity, and $m = 0, 1$ is the number of atomic excitations. We will consider $n = 0, 1, 2$. During the execution of the coCSign gate, the Hamiltonian changes, namely, H_{int} is complemented with the term $H_{\text{jump}} = v(a_i a_j^\dagger + a_j a_i^\dagger)$ (v being the parameter describing the photon transition intensity), which determines the transfer of a photon from the i th cavity to the j th one and back. However, the energy H_0 of independent atoms and the field will not change (the Jaynes–Cummings–Hubbard model, JCH). Therefore, the phase gain associated with H_0 is common to all states and can be disregarded. We will calculate the phase gain relative to either the identity operator I or σ_x , since all the operations considered below are reduced to either the first case, or to the second one with a phase change, so that the phase gain when using, e.g., the operator $-i\sigma_x$ is $-\pi/2$.

Let $\tau_1 = \pi\hbar/g$, $\tau_2 = \pi\hbar/g\sqrt{2}$ be periods of Rabi oscillations for the total energies $\hbar\omega$ and $2\hbar\omega$, respectively. Operators $U_t = \exp[-(i/\hbar)Ht]$, induced by evolution at times of importance, will depend on the total energy of the cavity. If it is equal to $\hbar\omega$, then in the basis $|\phi_0\rangle = |1\rangle_{\text{ph}}|0\rangle_{\text{at}}$, $|\phi_1\rangle = |0\rangle_{\text{ph}}|1\rangle_{\text{at}}$ we have

$$U_{\tau_1/2} = -i\sigma_x, \quad U_{\tau_1} = -I, \quad U_{2\tau_1} = I. \quad (2)$$

For the total energy of the cavity $2\hbar\omega$, we obtain similar relations with τ_1 replaced with τ_2 .

When a photon moves from the j th cavity to the i th one and back, which is achieved by the simultaneous switching on of Pockels cells or similar devices in these cavities, the H_{jump} addition to the H_{int} interaction is implemented, which, in the absence of atoms in the cavities, leads to exactly the same dynamics as Rabi oscillations, but with a period $\tau_{\text{jump}} = \pi\hbar/v$. Let us assume that $v \gg g$, then it is possible to move a photon from cavity to cavity so that the atom does not affect this process at all, and the phase gain can be calculated using formulas analogous to Eqn (2). As noted in Ref. [9], this condition is difficult to fulfill in the experiment; however, there are reasons to consider it as a technical obstacle. If the above condition is fulfilled, the phase gain with the operator σ_x applied to the photons in two cavities will be $-\pi/2$, as for a half of the Rabi oscillation.

Due to the incommensurability of the Rabi oscillation periods τ_1 and τ_2 we can choose such natural numbers n_1 and n_2 , for which the approximate equality

$$2n_2\tau_2 \approx 2n_1\tau_1 + \frac{\tau_1}{2} \quad (3)$$

is valid with high precision. This equality will be the basis for the nonlinear phase shift required for the coCSing implementation.

3. Implementation of coCSign

The qubit state $|0\rangle$ is realised in our model as the state of the optical cavity $|0\rangle_{\text{ph}}|1\rangle_{\text{at}}$, and the qubit state $|1\rangle$ as $|1\rangle_{\text{ph}}|0\rangle_{\text{at}}$. Thus, the state $|01\rangle$, whose phase is to be inverted, has the form $|01\rangle_{\text{ph}}|10\rangle_{\text{at}}$, where the first photon qubit belongs to the x cavity and the second one to the y cavity. Note that after the time $\tau_1/2$, the states 0 and 1 change places as the phase gain becomes equal to $-\pi/2$.

The sequence of operations implementing coCSign is shown in Fig. 1, and the cavities involved are shown in Fig. 2. First, we organise a short exchange of photons between the auxiliary cavity and cavity x , then, with a delay of $\tau_1/2$, a similar exchange with the cavity y . Then, after the time $2n_2\tau_2$, we again organise a short exchange of photons between the auxiliary cavity and cavity x , then, after time $\tau_1/2$, a similar exchange with the cavity y . From our choice of the photon exchange times, it follows that at these moments of time in the cavities participating in the operations there will be either one photon or none; therefore, the switching on of Pockels cells during a small time interval $\delta\tau = \pi\hbar/(2v) \ll \tau_1$ will lead precisely to the transfer of photons.

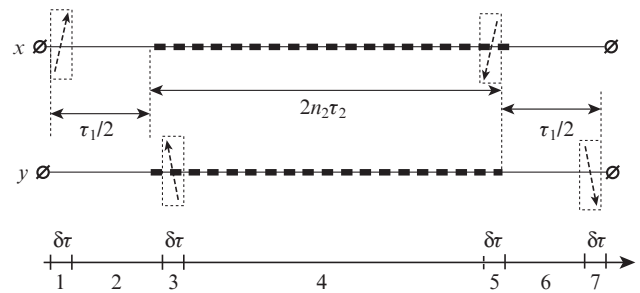


Figure 1. The sequence of operations for the implementation of the coCSign gate on asynchronous atomic excitations in optical cavities, divided into seven sections in time. Photon transition time $\delta\tau = \tau_{\text{jump}}/2 \ll \tau_{1(2)}$. After the triggering of the shown circuit, it is necessary to wait for a time $\tau_1/2$.

Figure 1 shows that the evolution of the state incorporates three long time intervals (interaction of atoms and photons in cavities) and four short intervals (photon jumps from cavity to cavity). Let us consider in detail the phase change for the state $|00\rangle$.

Interval 1 (short). Nothing happens because there is no photon in the cavity x and in the auxiliary cavity.

Interval 2 (long). Joint evolution of two cavities, each of which contains an atom and a photon; phase gain is $-\pi/2 - \pi/2$.

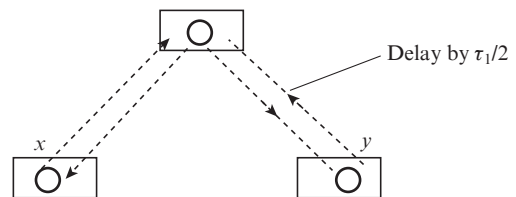


Figure 2. Three cavities, x , y and an auxiliary one, in the implementation of the coCSign gate.

Interval 3 (short). The photon in the cavity y is transferred into the auxiliary cavity; phase gain is $-\pi/2$.

Interval 4 (long). Evolution of two cavities with energy $\hbar\omega$ in each of them; phase gain is $-\pi/2 - \pi/2$.

Interval 5 (short). Nothing happens because there is no photon in the involved cavities.

Interval 6 (long). Joint evolution of two cavities, each containing an atom and a photon; phase gain is $-\pi/2 - \pi/2$.

Interval 7 (short). The photon from the auxiliary cavity is transferred to the cavity y ; phase gain $-\pi/2$.

As a result, the total phase gain is $-8\pi/2$, which is equivalent to zero. In this case, the state $|00\rangle$ turns into $|11\rangle$; after waiting for time $\tau_1/2$ it returns into $|00\rangle$ with a phase gain π , but this gain is common for all initial states and, therefore, can be ignored.

Further, following the same scheme, we consider transition $|01\rangle \rightarrow |10\rangle$ with phase gain $-\pi$, which is due to the fulfillment of Eqn (3), transition $|11\rangle \rightarrow |00\rangle$ with phase gain 0, and transition $|10\rangle \rightarrow |01\rangle$ with phase gain 0. After waiting for a time $\tau_1/2$, all states take their initial form with a common phase factor π .

4. Physical limitations of the coCSign gate quality

The advantage of the considered coCSign gate scheme is that, unlike, e.g., the KLM scheme [1], it is fully implemented using the standard JCH model and, in the ideal case, does not require any operations beyond the limits of the model validity. From calculations presented in Ref. [11] it follows that to achieve a satisfactory error in such entangling gates based on nonlinearity in cavities, it is sufficient to take the number of incommensurate periods n_1 and n_2 , not exceeding a few tens, which corresponds to the number of observed Rabi oscillations in optical cavities.

However, the JCH scheme itself has limitations imposed on its parameters, so that an arbitrary choice of their values can go beyond the limits of the scheme applicability. In [9], one of such limitations, which follows from experiments, is noted, namely, the limitation of the Pockels cell response rate. However, this limitation is not the only one.

The coefficient of the atom–field coupling in the cavity has the form

$$g = \sqrt{\hbar\omega/V} dE(x), \quad (4)$$

where V is the effective volume of the cavity; d is the dipole moment of the transition between the ground and excited states; $E(x) = \sin(\pi x/L)$ is the factor depending on the spatial location of the atom in the cavity; and L is the length of the cavity. For reliable confinement of a photon in the cavity, the cavity length should be $L = l\lambda/2$, where λ is the photon wavelength; in experiments, l is taken to be 1 in order to reduce the effective volume of the cavity, which makes it possible to obtain a few tens of Rabi oscillations (see, e.g., [5]).

We cannot choose the parameter ν of the photon transition intensity too large because of the energy–time uncertainty relation for photons, since a very short time interval during which a photon passes from cavity to cavity automatically means a large uncertainty of its energy. This leads to a rapid loss of the photon, whose wavelength begins to differ greatly from the doubled cavity length.

Taking into account that the photon frequency in experiments with the Rb atom is approximately 10^{10} s^{-1} , and assum-

ing the upper boundary value of the possible frequency uncertainty to be 10^9 s^{-1} (the real value is much less), from the uncertainty relation $\delta\omega\delta t \approx 1$ we find the lower estimate for the photon transition time window $\delta\tau \approx 10^{-9} \text{ s}$. For the period of the Rabi oscillation $\tau \approx 10^{-6} \text{ s}$, we obtain the inequality $10^{-9} \text{ s} \leq \delta\tau \ll 10^{-6} \text{ s}$ for the time window, which means the possibility of a single gate triggering with an error exceeding 10^{-3} . Such error, unfortunately, does not allow building a Feynman quantum computer based on this processor, if we consider its only option.

Note that the above inequalities are only a matter of discussion and by no means can serve to estimate the real error of the gate under consideration, since the uncertainty relation operates in conjunction with other factors of decoherence including the inaccuracy of Eqn (3) and the limited photon lifetime in the cavity. For example, we could increase the possible range of $\delta\tau$ variation by decreasing the period of the Rabi oscillation via the reduction of the spatial location parameter $E(x)$, but this will lead to a decrease in the photon lifetime in the cavity, which is no less fatal for the gate.

A similar difficulty occurs in all photon gate designs. The way out is to use many processors, i.e., to combine the quantum effect with the effect of classical parallelisation. A similar technique is used in other known schemes of photonic computers, e.g., in the already mentioned KLM scheme.

In the literature, there are no higher-accuracy estimates of the combined action of three factors: errors in determining the time $n_2\tau_2$, errors due to the limitation of the photon transfer time window, and limitation of the number of oscillations. The proposed scheme is the simplest known one, and all its disadvantages are inherent in other similar schemes. Its suitability for quantum computing can be evaluated only in experiment, and the considered scheme is one of the best candidates for implementation.

5. Conclusions

We have proposed a coCSign entanglement gate scheme based on asynchronous atomic excitations in optical cavities, which is fully described by the JCH model and is much simpler than the known schemes of this type. It is shown that beam splitters, as in the closest known analogue, the Azuma gate, can be replaced with time shifts using only one auxiliary cavity instead of two. To implement the proposed gate, an additional optical cavity and organisation of the transfer of photons from cavity to cavity in a time window are required, on the possible values of which two types of restrictions are imposed. The first type is associated with the technical response rate of the Pockels cell, and the second one with the fundamental energy-time uncertainty relation. Similar limitations exist in other photonic computer designs.

The simplicity of the considered scheme in comparison with the known schemes makes it a probable candidate for experimental implementation. The quantum computing effect of such a scheme can be achieved by using classical parallelism as an addition to the traditional Feynman gate scheme on individual processors.

The main advantage of the proposed scheme for implementing gates is its simplicity and the possibility of accurately following the theoretical JCH model, which, despite the difficulties mentioned above, inspires optimism in relation to scalability and comparison of the theory of a quantum computer with experiments on a large number of qubits.

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