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Effect of defocusing on the quality of quantum ghost images

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Abstract. **The effect of defocusing due to the finite thickness of the crystal, in which parametric scattering occurs, on the spatial resolution of ghost images is considered. The necessary relationships are presented, and methods of levelling this distorting factor and improving the quality of ghost images are proposed.**

Keywords: *ghost images, image quality, entangled photons, point spread function.*

Recently, considerable attention has been paid to the quality of quantum ghost images [1–5], which is associated not only with significant gaps in the theory, but also with the obviously unsatisfactory spatial resolution achieved in experiments. As to the effect of defocusing due to the finite thickness of the crystal in which parametric scattering occurs, it is not mentioned at all. Although this factor is insignificant for the used small-aperture optical systems, in the future, with an increase in the relative aperture and the nonlinear crystal thickness, it will have to be taken into account.

Ghost images [6] are one of the options for solving the problem of studying light-sensitive objects, the direct optical observation of which is difficult. To form ghost images, a source of correlated light beams is required, one of which interacts with the object, and the other does not (Fig. 1). In the object channel, the detector provides information only about the total intensity of the transmitted radiation. The conjugate beam does not interact with the object, but is recorded by the CCD matrix, which allows measurement of the spatial correlation function of the intensity between the two channels.

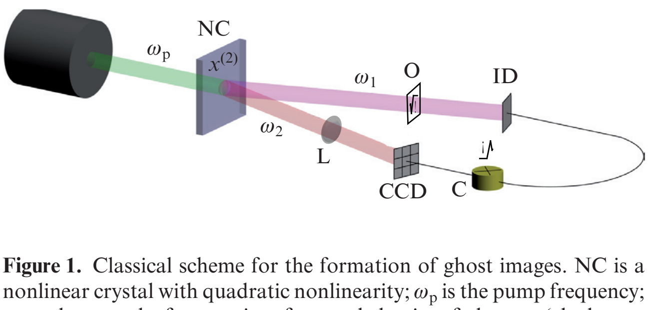

Figure 1. Classical scheme for the formation of ghost images. NC is a nonlinear crystal with quadratic nonlinearity; ω_p is the pump frequency; ω_1 and ω_2 are the frequencies of entangled pairs of photons (the beams diverge due to the use of a noncollinear process of parametric scattering); O is an object; ID is an integrating detector in the object channel; L is an optical lens; CCD is an array of photodetectors in the reconstruction channel; C is an intensity correlator (coincidence circuit).

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One of the important arguments in favour of quantum ghost images is the creation of the most gentle lighting conditions for the object under study, in which the effect of radiation on the object (sometimes irreversible) is minimal [7]. This is especially important when living beings are irradiated, e.g., with X-rays.

In this paper, we not only present the basic relations necessary to take into account the defocusing that arises in ghost images, but also propose design schemes that allow increasing the spatial resolution.

The non-normalised point spread function of an optical system with conventional defocusing Δz is proportional to the pupil function (see, e.g, [8]):

$$a(x,y) \propto f\left(\frac{xL}{\Delta z}, \frac{yL}{\Delta z}\right), \tag{1}$$

where x and y are coordinates in the image plane; $f(\xi,\eta)$ is the dimensionless function of the exit pupil; ξ and η are the pupil coordinates; and L is the distance from the exit pupil to the image plane. In the absence of defocusing ($\Delta z \to 0$), the quantity $a(x,y)$ becomes proportional to the Dirac delta function $\delta(x,y)$.

In our case, defocusing depends on the point with the longitudinal coordinate z in the crystal, at which the biphoton is generated. If the pump radiation is assumed to be independent of the z coordinate, then all these points are equally probable, and the point spread function

$$a(x,y) \propto \int_0^{l/2\cos(\phi/2)} f\left(\frac{xL}{n\Delta z}, \frac{yL}{n\Delta z}\right) \mathrm{d}(\Delta z) \tag{2}$$

for the optimal case, when the optical system is aligned to the centre of a crystal of thickness l, and the defocusing effect is 'halved'. Here ϕ is the angle at which the signal and idler photons diverge in the crystal, and n is its refractive index for phase matching in the frequency-degenerate ($\omega_1 = \omega_2$) noncollinear regime.

Formula (2) is similar to the formula describing dynamic defocusing in photo cameras, when the defocusing changes during the exposure time [9].

The pupil function is determined by the transverse size of the pump beam. It usually has a Gaussian profile; therefore, the integrand in Eqn (2) is proportional to the exponential function

$$f\left(\frac{xL}{n\Delta z}, \frac{yL}{n\Delta z}\right) \propto \exp\left[-\left(\frac{L}{nd\Delta z}\right)^2 (x^2+y^2)\right], \tag{3}$$

where d is the transverse size of the pump beam, if the objective is located in the object channel, or its image obtained using the subsequent optical system, as shown in Fig. 1.

It is clear that the uttermost spatial resolution cannot be better than the reciprocal of a, unless, of course, further mathematical processing is used to achieve superresolution (see, e. g., [10–12]). Anyway, a decrease in the value of a is always desirable, since no mathematical processing can completely restore the distorted image. How can this be done?

One can place a nonlinear crystal between two lenses so that it would work in parallel beams (Fig. 2). In this case, the object and the ghost image must be in the focal planes of these lenses. This is a successful layout option, since it allows not only to eliminate the distortions associated with defocusing, but also to completely compensate for the aberrations introduced by the crystal: From the point of view of geometrical optics, it corresponds to a plane-parallel plate introducing at least spherical aberration into converging or diverging beams. In collimated beams, there are no aberrations.

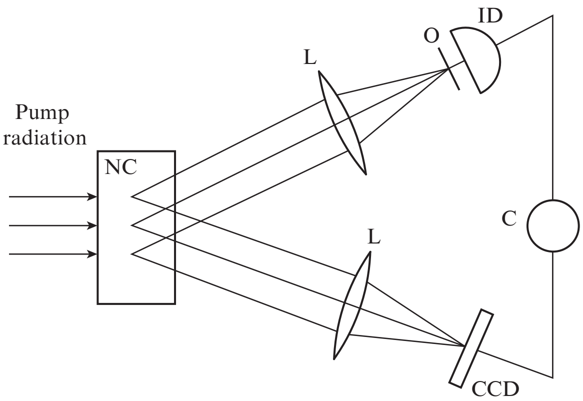

Figure 2. Schematic of the ghost image formation with a parallel path of rays between the lenses L. NC is a nonlinear crystal with quadratic nonlinearity; beams of entangled pairs of photons illuminate the object O and the CCD photodetector array in the reconstructing channel; the object and the array are in the focal planes of the optical lenses L; ID is an integrating detector in the object channel; C is an intensity correlator (coincidence circuit).

Fairly speaking, for the currently used relative apertures of optical systems for forming ghost images, the effect of the considered defocusing is negligible. For example, in the experiment [13], where a ghost image of a slit was actually recorded, the estimated value of a did not exceed a few micrometres. At the same time, a low aperture ratio has a negative effect on the quality of any images, since it causes a diffraction limitation of the spatial resolution, and it is this factor that is crucial for ghost images (see, e. g., [1–3]).

Why, then, in schemes with parametric scattering, is it impossible to achieve a high aperture ratio? First of all, because of the small angle of parametric capture, in which exponential amplification is observed. It is exactly this angle that ultimately determines the relative aperture. How to increase it? The easiest way is to reduce the thickness of the crystal. However, the possibilities for such a solution are not unlimited, because it reduces the efficiency of the parametric process.

What will happen if we use the backward four-photon mixing, as in PC mirrors? The only difference in this case is that the seed should be vacuum fluctuations rather than an external signal. In an isotropic medium with cubic nonlinearity, parametric oscillation can occur in any direction. Consequently, there are no fundamental limitations imposed on the aperture ratio. On the other hand, the spatial correlation of the signal and idler photons propagating in strictly opposite directions, following from the momentum conservation law, provides excellent opportunities for the formation of ghost images: the signal photon will illuminate the object, and the idler one will be recorded by the CCD matrix. In this case, from the point of view of geometric optics, the rays corresponding to both photons must necessarily be on the same straight line. To reduce aberrations, as in the case of a three-photon process, it makes sense to use an optical system layout in which a nonlinear medium would operate in a parallel beam of rays (Fig. 3).

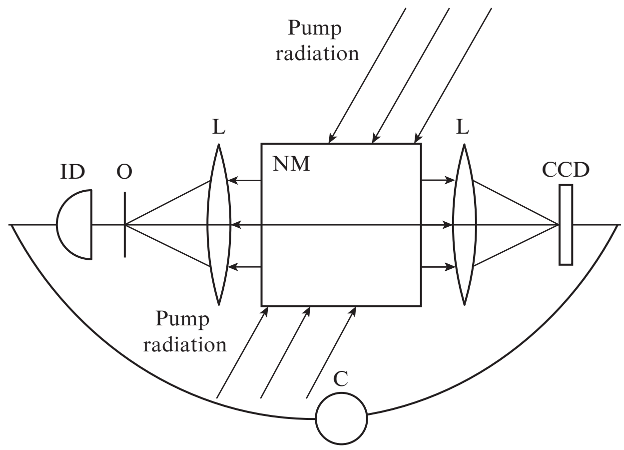

Figure 3. Schematic of the ghost image formation using backward four-photon mixing and parallel path of rays between the lenses L and the nonlinear medium NM with cubic nonlinearity: beams of entangled pairs of photons illuminate the object O and the matrix of photodetectors CCD in the reconstruction channel. The object and the matrix are in the focal planes of the optical lenses L; ID is an integrating detector in the object channel; C is an intensity correlator (coincidence circuit).

Such a design scheme, as we see it, can lead to a drastic increase in the quality of ghost images.

Consideration of issues related to the spatial resolution of ghost images would be incomplete without mentioning their formation using classical thermal light sources (see, e. g., [14]). The main factor that determines the limiting resolution of such systems is the radius of spatial coherence of radiation. Delta-correlated light is ideal for illuminating objects. The speckle size in the object plane, taking into account the linear transverse magnification of the optical system, determines the limiting spatial resolution of ghost images.

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