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# Laser cooling limits in fields with a polarisation gradient of atoms with different recoil energies

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***Abstract.*** **Based on the numerical solution of the quantum kinetic equation for the atomic density matrix, which makes it possible to accurately take into account the recoil effects in the interaction of atoms with field photons, we have studied the limits of laser cooling of atoms using closed optical transitions characterised by different recoil parameters (the ratio of the recoil energy to the natural linewidth). It is shown that for optical transitions with an insufficiently small recoil parameter, the polarisation effects, which lead to the possibility of sub-Doppler laser cooling, lose their efficiency and do not ensure an attainment of the temperature below the Doppler limit. The analysis performed allows one to outline the boundaries of the sub-Doppler theory of laser cooling of atoms.**

***Keywords:*** *laser cooling of atoms, field polarisation, optical transitions, density matrix, recoil parameter.*

## 1. Introduction

In the late 1970s a new direction of atomic and laser physics emerged, i.e. laser cooling of neutral atoms and ions, which is now widely developed. Laser-cooled atoms are used in precision spectroscopy and in quantum frequency standards [1–3], for the implementation of Bose–Einstein condensation [4, 5], in the modelling of quantum effects in condensed media, in the investigation of interatomic collisions, and in other studies [6, 7].

For a theoretical description of the mechanical action of the light on atoms, it is necessary to take into account the processes of changes not only of the internal degrees of freedom of atoms in an external laser field, but also of the translational degrees as a result of recoil effects (transfer of momentum and kinetic energy) during the interaction of atoms with single field photons, as well as their correlations, which greatly complicates the task. At the initial stage of research, quasi-classical approaches became widespread, which made it possible, within the framework of certain approximations, to separate the processes of the rapid evolution of the internal degrees of freedom of atoms and processes associated with the slower evolution of the translational degrees of freedom, and to describe the kinetics of atoms in terms of the forces acting on atoms from the resonant electromagnetic field, and diffusion as a result of abrupt absorption/emission of field photons [8–15]. The main limitation for the application of semi-classical approaches is the smallness of the momentum transferred to atoms interacting with field photons with respect to the momentum distribution of atoms: $\hbar k/\Delta p \ll 1$, as well as the smallness of the recoil parameter: $\varepsilon_{\rm R} = \omega_{\rm R}/\gamma \ll 1$ (the ratio of the recoil energy $\hbar\omega_{\rm R} = \hbar^2k^2/2M$ obtained by a motionless atom of mass $M$ as a result of absorption/emission of field photons to the natural linewidth $\gamma$, $k = \omega/c$ is the wave vector).

An undoubted advantage of the semi-classical approaches is that in a number of cases they allow one to obtain analytical expressions for the forces acting on atoms in light fields and for the diffusion coefficients. This made it possible to describe the main mechanisms of laser cooling in optical molasses – Doppler [8–10, 16] and sub-Doppler cooling, arising for atoms with a ground state degenerate in the angular momentum projection in fields with a polarisation gradient [10, 17–19 ], as well as to present the principles of operation of a magneto-optical trap.

Simultaneously, quantum approaches were developed to solve the problem of laser cooling of atoms. Methods based on the wave function formalism became widespread, in which the processes of decoherence of the wave function as a result of spontaneous emission have modelled in terms of quantum jumps (the so-called quantum Monte Carlo methods) [20, 21]. Due to the need to take into account a large number of degrees of freedom of a quantum system, the direct solution of the problem of atomic kinetics within the framework of the density matrix formalism was not possible and required additional approximations and simplifications. For example, the low-intensity field approximation was used to describe the limits of laser cooling in the lin⊥lin configuration [22, 23], which made it possible to reduce the equation for the density matrix to a simplified one describing the evolution of only sublevels of the ground state. In these works, the additional approximation $\sqrt{U_0/(\hbar\omega_{\rm R})} \ll |\delta|/\gamma$ was adopted, i.e., it was assumed that the distance between the vibrational levels in the

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potential of the optical lattice of the field is much greater than their width. Here $U_0$ is the optical shift, which determines the depth of the optical grating, and $\delta = \omega - \omega_0$ is the detuning of the laser field with a frequency $\omega$ from the frequency of the resonant atomic transition $\omega_0$. For fields that do not produce an optical potential, for example, in the $\sigma_+ - \sigma_-$ configuration, this approach is not applicable, i.e.- for such fields the problem is greatly simplified in the momentum representation [24–26].

Note that the approximation used in [22, 23] describes well the atoms localised in the optical potential. It is violated for atoms at higher vibrational levels, where the distance between the levels decreases, and becomes inapplicable for atoms performing above-barrier motion. In some cases, the fraction of 'hot' atoms can be large, which leads to discrepancies with more accurate approaches [27–29]. In these works, we proposed a universal quantum approach, which allows a stationary solution of the quantum kinetic equation to be found for the atomic density matrix in fields of an arbitrary one-dimensional configuration with full allowance for quantum recoil effects, while it does not have the restrictions mentioned above. In a number of cases, it was shown that for atoms with insufficiently small parameters $\varepsilon_R$, the recoil effects become more significant, which leads to significant discrepancies in the final temperature of cold atoms with this approach and with the use of semi-classical models and approximate quantum theories [25, 30–33]. For atoms with narrow optical lines, $\varepsilon_R > 1$, the recoil effects become most critical, leading to the exit from the resonance contour of the interaction of atoms with light in single events of absorption and emission of field photons. In this case, laser cooling of atoms in a monochromatic field is possible under conditions of field broadening [31]. At large recoil parameters $\varepsilon_R$, the use of light waves with a polarisation gradient for atoms with levels degenerate in the angular momentum projection does not allow obtaining temperatures lower than in the case of atoms with a nondegenerate ground state [34], when the kinetics of atoms can be described within a simple two-level model.

In this work, we analyse in detail the limits of laser cooling of atoms in fields with a polarisation gradient using optical transitions characterised by small recoil parameters, $\varepsilon_R < 1$, when, according to the semi-classical theory, sub-Doppler laser cooling is achieved. On the basis of the developed quantum approach, we investigate the limits of laser cooling in a wide range of recoil parameters. It is shown in the work that the efficiency of sub-Doppler friction mechanisms in fields with lin$\perp$lin and $\sigma_+ - \sigma_-$ configurations is different and depends significantly on $\varepsilon_R$. The analysis performed allows us to outline the limits of applicability of the sub-Doppler theory of laser cooling at various recoil parameters for fields with a polarisation gradient in the $\sigma_+ - \sigma_-$ and lin$\perp$lin configurations, which are most often used in problems of deep laser cooling.

## 2. Statement of the problem

Let us consider laser cooling of atoms with a closed optical transition $J_g \to J_e$, where $J_g$ and $J_e$ are the total angular momenta of the ground (g) and excited (e) states. An atom resonantly interacts with a monochromatic field, which is a combination of two counterpropagating waves propagating along the $z$ axis:

$$\boldsymbol{E}(z,t) = E_0(\boldsymbol{e}_1 e^{ikz} + \boldsymbol{e}_2 e^{-ikz})e^{-i\omega t} + \text{c.c.} \quad (1)$$

Here $E_0$ is the complex amplitude of the counterpropagating light waves. The polarisations of the counterpropagating waves, $\boldsymbol{e}_1$ and $\boldsymbol{e}_2$, can be expressed in terms of the components of the vectors in the cyclic basis:

$$\boldsymbol{e}_n = \sum_{\sigma=0,\pm1} e_n^{\sigma}\boldsymbol{e}_{\sigma},\ n = 1, 2, \quad (2)$$

where $\boldsymbol{e}_{\pm1} = \mp(\boldsymbol{e}_x \pm i\boldsymbol{e}_y)/\sqrt{2}$ and $\boldsymbol{e}_0 = \boldsymbol{e}_z$ are the unit vectors in the cyclic basis. Note that the components $e_n^0 = 0$ due to the orthogonality of the vectors $\boldsymbol{e}_n$ and $\boldsymbol{k}$. Below we restrict ourselves to the analysis of two field configurations most frequently used in sub-Doppler laser cooling:

– the lin$\perp$lin configuration of the light field with $\boldsymbol{e}_1 = \boldsymbol{e}_x$ and $\boldsymbol{e}_2 = \boldsymbol{e}_y$ is formed by counterpropagating waves with orthogonal linear polarisations; and

– the $\sigma_+ - \sigma_-$ configuration of the light field with $\boldsymbol{e}_1 = \boldsymbol{e}_+$ and $\boldsymbol{e}_2 = \boldsymbol{e}_-$ is formed by counterpropagating waves with orthogonal circular polarisations.

These configurations represent fields where only one field parameter changes in space, and the other parameters remain unchanged along $z$. For example, for the lin$\perp$lin configuration, the variable parameter is the ellipticity of the light field, which cyclically changes from right circular to left circular along the $z$ coordinate. For the $\sigma_+ - \sigma_-$ configuration, the field polariwation at each point is linear, but the orientation angle depends on the $z$ coordinate (see, e.g., [17, 35]). The presented field configurations are a special case of the $\varepsilon_1 - \theta - \varepsilon_2$ configuration, which is a combination of counterpropagating waves with elliptical polarisations and a relative angle $\theta$ in their mutual orientation, at which additional sub-Doppler laser cooling mechanisms are possible, which do not arise in fields with orthogonal polarisations [13, 19, 36, 37].

The evolution of a low-density atomic ensemble, when the interatomic interaction can be neglected, is determined by the quantum kinetic equation for the atomic density matrix $\hat{\rho}$

$$\frac{\partial}{\partial t}\hat{\rho} = -\frac{i}{\hbar}[\hat{H},\hat{\rho}] + \hat{\Gamma}\{\hat{\rho}\}, \quad (3)$$

where $\hat{H}$ is the Hamiltonian, and $\hat{\Gamma}\{\hat{\rho}\}$ describes the relaxation of atomic levels in the process of spontaneous decay. The Hamiltonian of the atom, $\hat{H}$, is split into the sum of contributions:

$$\hat{H} = \frac{\hat{p}^2}{2M} + \hat{H}_0 + \hat{V}, \quad (4)$$

where the first term is the kinetic energy operator; the second term, $\hat{H}_0 = -\hbar\delta\hat{P}_e$, is the Hamiltonian of a free atom in the basis of a rotating wave; and

$$\hat{P}_e = \sum_{\mu} |J_e,\mu\rangle\langle J_e,\mu| \quad (5)$$

is the projection operator on the levels of the excited state $|J_e,\mu\rangle$, characterised by the total angular momentum $J_e$ and the projection of the angular momentum $\mu$ onto the quantisation axis ($-J_e \leqslant \mu \leqslant J_e$). The last term $\hat{V}$ describes the interaction of an atom with field (1), which in the electric dipole approximation has the form

$$\hat{V} = \hat{V}_1 e^{ikz} + \hat{V}_2 e^{-ikz} + \text{h.c.},$$

$$\hat{V}_n = \hbar\frac{\Omega}{2}(\hat{\boldsymbol{D}}\cdot\boldsymbol{e}_n) = \hbar\frac{\Omega}{2}\sum \hat{D}_\sigma e_n^\sigma,\ n = 1, 2, \tag{6}$$

where $\Omega$ is the Rabi frequency. The matrix components of the operator $\hat{D}_\sigma$ in the circular basis are expressed in terms of the Clebsch–Gordan coefficients:

$$\hat{D}_\sigma = \sum_{\mu,m} C^{J_e,\mu}_{J_g,m;1,\sigma}\,|J_e,\mu\rangle\langle J_g,m|. \tag{7}$$

Here, the subscripts $m$ describe the projection of the angular momentum $J_g$ of the ground state onto the quantisation axis. The last term $\hat{\Gamma}\{\hat{\rho}\}$ of kinetic equation (3), which describes the relaxation of the atomic density matrix with allowance for recoil effects, has the form (see, e.g., [29]):

$$\hat{\Gamma}\{\hat{\rho}\} = -\frac{\gamma}{2}(\hat{P}_e\hat{\rho} + \hat{\rho}\hat{P}_e)$$

$$+\gamma\frac{3}{2}\left\langle \sum_{\xi=1,2}(\hat{\boldsymbol{D}}\cdot\boldsymbol{e}_\xi(\boldsymbol{k}))^\dagger e^{-i\boldsymbol{k}\hat{\boldsymbol{r}}}\hat{\rho}e^{i\boldsymbol{k}\hat{\boldsymbol{r}}}(\hat{\boldsymbol{D}}\cdot\boldsymbol{e}_\xi(\boldsymbol{k}))\right\rangle_{\Omega_k}, \tag{8}$$

where $\langle\ldots\rangle_{\Omega_k}$ denotes averaging over the directions of emission of a spontaneous photon with momentum $\hbar\boldsymbol{k}$ with two orthogonal polarisations $\boldsymbol{e}_\xi(\boldsymbol{k})$.

To solve the quantum kinetic equation (3), it is convenient to use the coordinate representation for the atomic density matrix $\hat{\rho}(z_1, z_2)$, where the spontaneous relaxation operator takes the simplest form:

$$\hat{\Gamma}\{\hat{\rho}(z_1,z_2)\} = -\frac{\gamma}{2}\left[\hat{P}_e\hat{\rho}(z_1,z_2) + \hat{\rho}(z_1,z_2)\hat{P}_e\right]$$

$$+ \sum_{\sigma=0,\pm1}\kappa_\sigma(q)\hat{D}^\dagger_\sigma\hat{\rho}(z_1,z_2)\hat{D}_\sigma. \tag{9}$$

Here $q = kz_1 - kz_2$, and the functions $\kappa_\sigma(q)$ are as follows:

$$\kappa_0(q) = 3\left(\frac{\sin(q)}{q^3} - \frac{\cos(q)}{q^2}\right),$$

$$\kappa_{\pm1}(q) = \frac{3}{2}\left(\frac{\cos(q)}{q^2} + \frac{\sin(q)}{q} - \frac{\sin(q)}{q^3}\right). \tag{10}$$

To search for a stationary solution to Eqn (3) and analyse the limits of sub-Doppler laser cooling, we use the previously proposed approach and implemented both for the Wigner representation of the atomic density matrix [27, 28] and for the coordinate one [29]. In the coordinate representation, the stationary solution (3) is a periodic function of $z$ and can be expressed as a series of harmonics:

$$\hat{\rho}(z,q) = \sum_n \hat{\rho}^{(n)}(q)e^{inkz}. \tag{11}$$

Thus, the problem is reduced to finding the matrices of amplitudes $\hat{\rho}^{(n)}(q)$, for which an equation can be written in the form of recursion:

$$n\frac{\hbar k^2}{M}\frac{\partial}{\partial q}\hat{\rho}^{(n)} = \hat{\mathcal{L}}_0\{\hat{\rho}^{(n)}\}$$

$$+\hat{\mathcal{L}}_+\{\hat{\rho}^{(n-1)}\} + \hat{\mathcal{L}}_-\{\hat{\rho}^{(n+1)}\}; \tag{12}$$

the latter can be solved using the generalised continued fraction method for operators $\hat{\mathcal{L}}$:

$$\hat{\mathcal{L}}_+\{\hat{\rho}\} = -\frac{i}{\hbar}\left(\hat{W}_1\hat{\rho}e^{\frac{iq}{2}} - \hat{\rho}\hat{W}_1e^{-\frac{iq}{2}}\right),$$

$$\hat{\mathcal{L}}_-\{\hat{\rho}\} = -\frac{i}{\hbar}\left(\hat{W}_2\hat{\rho}e^{-\frac{iq}{2}} - \hat{\rho}\hat{W}_2e^{\frac{iq}{2}}\right),$$

$$\hat{\mathcal{L}}_0\{\hat{\rho}\} = -\frac{i}{\hbar}\left(\hat{H}_0\hat{\rho} - \hat{\rho}\hat{H}_0\right) + \hat{\Gamma}\{\hat{\rho}\}. \tag{13}$$

Here $\hat{W}_1$ and $\hat{W}_2$ are the linear combinations of operators of interaction with the field: $\hat{W}_1 = \hat{V}_1 + \hat{V}_2^\dagger$, $\hat{W}_2 = \hat{V}_2 + \hat{V}_1^\dagger$.

The amplitudes $\hat{\rho}^{(n)}$ are the functions of the variable $q$ and contain information about the quantum coherence of atomic states between two spatial points$z_1 = z + q/2k$ и $z_2 = z - q/2k$. Since the spatial coherence decays with distance as $q/k$, then for numerical calculations it is sufficient to restrict the problem to the search for $\hat{\rho}^{(n)}(q)$ in some interval from $-q_{max}$ to $q_{max}$. In the Wigner representation, this constraint determines the refinement of the density matrix in the momentum space $\Delta p \approx \hbar k\pi/q_{max}$. In most calculations, we used $q_{max} \leqslant 20$; however, in some cases (to obtain more detailed results in the momentum space) we increased $q_{max}$ up to values of $\sim$100.

## 3. Stationary solution of the problem of laser cooling of atoms in fields with a polarisation gradient

First, let us define the main parameters of the problem. The kinetics of atoms, described by Eqn (3), is determined by both the light field parameters and the atomic parameters. The first parameters include the intensity $I$, frequency $\omega$, and spatial configuration determined by the polarisations of counterpropagating waves that form the field. Atomic parameters include the dipole moment of the optical transition $d$, angular momenta of the ground $J_g$ and excited $J_e$ states, frequency $\omega_0$, the natural width $\gamma$ of the optical transition, and mass $M$ of the atom. Among this extensive list, three dimensionless parameters can be distinguished that determine the stationary solution of the quantum kinetic equation (3):

$\delta/\gamma$ is the dimensionless detuning;

$\dfrac{\Omega}{\gamma} = \sqrt{\dfrac{I}{2I_{sat}}}$ is the dimensionless Rabi frequency

($I_{sat} = 2\pi^2\gamma\hbar c/\lambda^3$ is determined by the dipole moment of the optical transition, see, e.g., [35]); and

$\varepsilon_R = \omega_R/\gamma$ is the recoil parameter.

The polarisation configuration of the light field and the type of optical transition $J_g \to J_e$ remain additional parameters of the problem.

The recoil parameter varies within a fairly wide range (see Table 1). For most optical transitions used for laser cooling, it varies from extremely small values of $\varepsilon_R < 10^{-3}$ for $D_2$ lines of alkali elements (Cs, Rb) to values of the order of $10^{-1}$, when the effects associated with the release of atoms as a result of recoil from resonant contour of interaction with the field become more significant. For narrow optical transitions, such as the $^1S_0 \to {}^3P_1$ intercombination transitions of the alkaline earth elements Sr, Ca, and Mg, the recoil parameter $\varepsilon_R \geqslant 1$.

**Table 1.** Recoil parameter and optical transitions for a number of neutral atoms.

| Atom | Optical transition | $\gamma\times(2\pi)^{-1}$/MHz | $\lambda$/nm | $\varepsilon_R$ |
|---|---|---|---|---|
| $^{174}$Yb | $6^1S_0 \to 6^1P_1$ | 28 | 399 | $3\times10^{-4}$ |
| | $6^1S_0 \to 6^3P_1$ | 0.18 | 556 | $2\times10^{-2}$ |
| $^{87}$Sr | $5^1S_0 \to 5^1P_1$ | 32 | 461 | $3\times10^{-4}$ |
| | $5^1S_0 \to 5^3P_1$ | $7\times10^{-3}$ | 689 | 0.6 |
| | $5^3P_2 \to 4^3D_3$ | $2.7\times10^{-3}$ | 2900 | 0.1 |
| $^{40}$Ca | $4^1S_0 \to 4^1P_1$ | 34.2 | 423 | $0.8\times10^{-3}$ |
| | $3^1S_0 \to 3^3P_1$ | $4\times10^{-4}$ | 657 | 28 |
| $^{24}$Mg | $3^1S_0 \to 3^1P_1$ | 78 | 285 | $1.3\times10^{-3}$ |
| | $3^3P_2 \to 3^3D_3$ | 26.7 | 384 | $2.1\times10^{-3}$ |
| | $3^1S_0 \to 3^3P_1$ | $31.2\times10^{-6}$ | 457 | $1.2\times10^{3}$ |
| $^{133}$Cs | $6^2S_{1/2} \to 6^2P_{3/2}$ | 5 | 852 | $4\times10^{-4}$ |
| $^{85}$Rb | $5^2S_{1/2} \to 5^2P_{3/2}$ | 5.9 | 780 | $6\times10^{-4}$ |
| $^{39}$K | $4^2S_{1/2} \to 4^2P_{3/2}$ | 6.2 | 767 | $1.4\times10^{-3}$ |
| $^{23}$Na | $3^2S_{1/2} \to 3^2P_{3/2}$ | 9.9 | 589 | $2.5\times10^{-3}$ |
| $^7$Li | $2^2S_{1/2} \to 2^2P_{3/2}$ | 5.9 | 671 | $10^{-2}$ |
| $^1$H | $1^2S_{1/2} \to 2^2P_{3/2}$ | 99.58 | 122 | 0.13 |
| $^{27}$Al | $3^2P_{3/2} \to 3^2D_{5/2}$ | 11.6 | 309 | $6.6\times10^{-3}$ |
| $^{52}$Cr | $a^7S_3 \to z^7P_4$ | 5 | 426 | $4\times10^{-3}$ |
| $^{56}$Fe | $a^5D_4 \to z^5F_5^o$ | 2.58 | 372 | $10^{-2}$ |
| $^{169}$Tm | $^2F_{7/2}^o \to (5, 3/2)_{9/2}$ | 10 | 411 | $0.7\times10^{-3}$ |
| | $^2F_{7/2}^o \to (6, 5/2)_{9/2}$ | 0.35 | 531 | $10^{-2}$ |
| $^{69,71}$Ga | $4^2P_{3/2} \to 4^2D_{5/2}$ | 25 | 294 | $1.3\times10^{-3}$ |
| $^{107}$Ag | $5^2S_{1/2} \to 5^2P_{3/2}$ | 2.2 | 328 | $0.8\times10^{-3}$ |
| $^{115}$In | $5^2P_{3/2} \to 5^2D_{5/2}$ | 20.7 | 326 | $0.8\times10^{-3}$ |
| $^{199}$Hg | $6^1S_0 \to 6^3P_1$ | 1.32 | 254 | $1.3\times10^{-2}$ |

Below we will consider a rather large range of variation of the recoil parameters ($10^{-4} \leqslant \varepsilon_R \leqslant 10^{-1}$) for atoms with optical transitions used for laser cooling. For such atoms, we investigate the limits of sub-Doppler laser cooling in fields with a polarisation gradient.

We note separately that at an extremely low intensity of light waves within the approximations [22, 23] describing Sisyphean laser cooling in a lin$\perp$lin configuration field, there remains only one parameter characterising the stationary solution. This parameter,

$$U_0 = \frac{|\delta|}{3\omega_R}\frac{|\Omega|^2}{(\delta^2+\gamma^2/4)}, \tag{14}$$

determines the optical shift of the ground state sublevels. In accordance with the definitions of these works, instead of the dimensionless parameter $\Omega/\gamma$, which determines the light field intensity, we will use the combined parameter $U_0$ that will allow us to compare our results with those presented in [22, 23].

### 3.1. Limits of sub-Doppler cooling of atoms with extremely small values of the recoil parameter, $\varepsilon_R \leqslant 10^{-3}$

We present the results obtained for the average kinetic energy of atoms with optical transitions characterised by extremely small recoil parameters $\varepsilon_R \leqslant 10^{-3}$ in the fields of lin$\perp$lin- and $\sigma_+ - \sigma_-$ configurations. The considered values of the recoil parameter correspond to the conditions of the semi-classical description of the kinetics of atoms in light fields [8–15], and for the lin$\perp$lin configuration at low intensities of the light field, the results should correspond to the results of [22, 23]. As an example, consider the optical transition $J_g = 1 \to J_e = 2$, for which sub-Doppler laser cooling is possible both in the lin$\perp$lin and $\sigma_+ - \sigma_-$ configurations [17] This will make it possible to compare the limits of the laser cooling in these two most commonly used light field configurations.

Figure 1a shows the average kinetic energy of cold atoms as a function of $U_0$ in the lin$\perp$lin-configuration field at $\varepsilon_R = 4\times10^{-4}$, which corresponds to laser cooling of Cs atoms with the use of the $D_2$ line. For comparison, a similar dependence for atoms with $\varepsilon_R = 10^{-3}$ is shown in Fig. 1e. Note that the dependence of the average kinetic energy on the parameter $U_0$ tends to a universal dependence at extremely large detunings, which corresponds to the results obtained in the secular approximation $\sqrt{U_0/(\hbar\omega_R)} \ll |\delta|/\gamma$ [22, 23]. The minimum achievable energies $E_{min} \approx 22\hbar\omega_R$ (for $\varepsilon_R = 4\times10^{-4}$) and $E_{min} \approx 20\hbar\omega_R$ (for $\varepsilon_R = 10^{-3}$) established by us are somewhat less than those obtained in [22, 23]. This difference is explained by the difference in angular momenta for the considered optical transitions: $J_g = 1 \to J_e = 2$ in our calculations and $J_g = 1/2 \to J_e = 3/2$ in [22, 23]. For the optical transition $J_g = 1/2 \to J_e = 3/2$, the results exactly correspond to the results of [22, 23] in the limit of large detunings, as was shown in [29].

The momentum distribution of cold atoms for the lin$\perp$lin and $\sigma_+ - \sigma_-$ configurations (Figs 1b, 1d, 1f, and 1h) is not equilibrium and cannot be described in terms of temperature. At low intensities of light fields (for small $U_0$), a momentum distribution is observed, consisting of a narrow central part due to the sub-Doppler contribution to the friction force and a wide substrate due to Doppler friction mechanisms. In this case, despite a decrease in the width of the central part of the distribution, the average kinetic energy of cold atoms increases due to a decrease in the fraction of atoms in it.

The value of the average kinetic energy of atoms, $E_{kin}/(\hbar\omega_R) \approx 1/(4\varepsilon_R)$, corresponding to the equilibrium distribution described by the Gaussian function, with the Doppler temperature $k_B T_D \approx \hbar\gamma/2$, is approximately 625 (for atoms with $\varepsilon_R = 4\times10^{-4}$, Fig. 1a) and 250 ($\varepsilon_R = 10^{-3}$, Fig. 1e). As can be seen from the results presented, the attained values of the average kinetic energy of atoms are much lower than the energy of the Doppler limit, which corresponds to the well-known concepts of sub-Doppler laser cooling [10–14, 17].

Similar results were obtained for laser cooling in the field of the $\sigma_+ - \sigma_-$ configuration (Figs 1c and 1g). As in the case of the lin$\perp$lin configuration, for atoms with extremely small recoil parameters, the dependence of the average kinetic energy of atoms tends to a certain universal dependence on the parameter $U_0$ at sufficiently large detunings of the light field, $|\delta|/\gamma \geqslant 5$. Dashed horizontal line in the figures corresponds to the average kinetic energy of atoms having an equi-

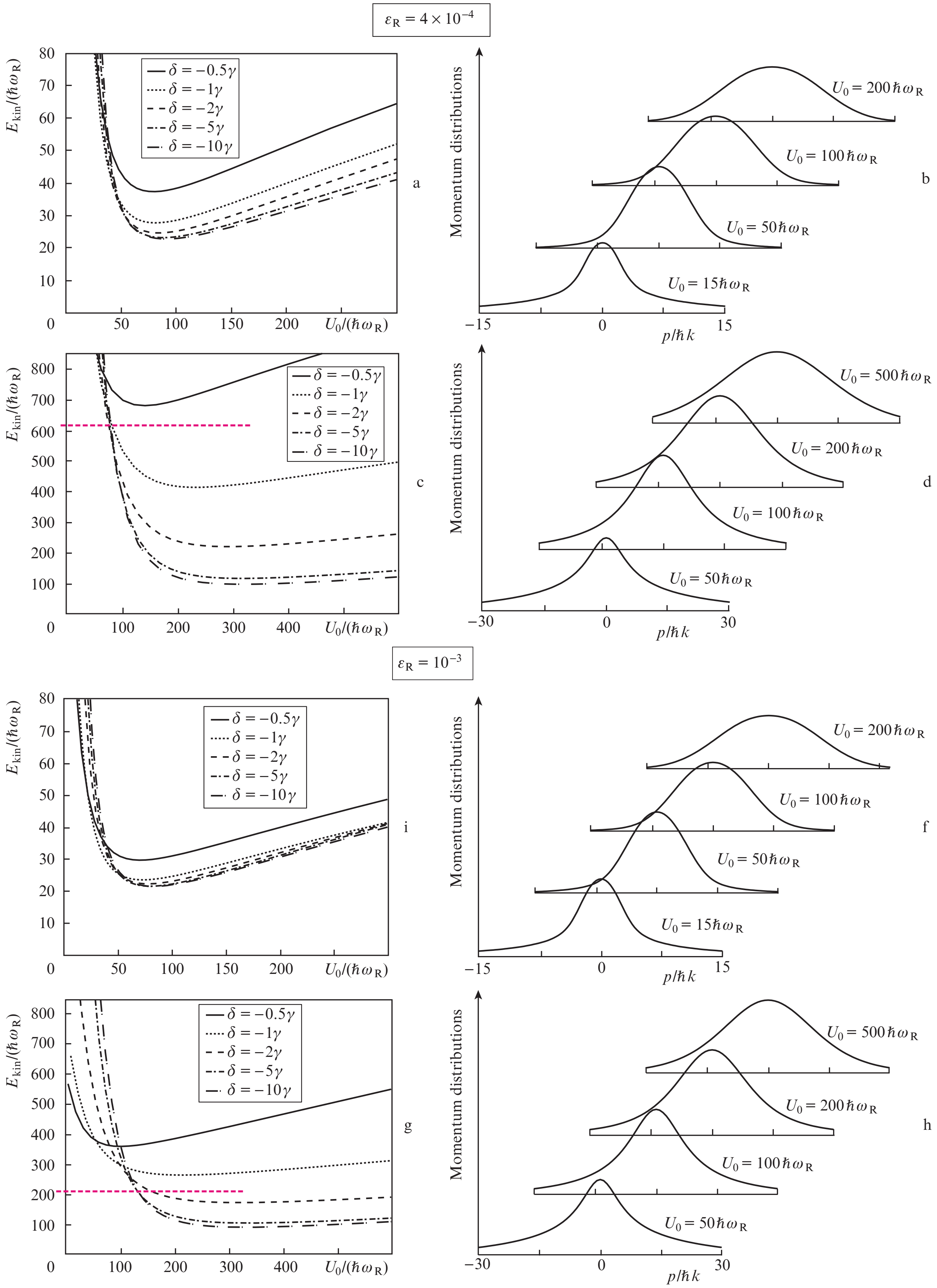


**Figure 1.** Dependences of the average kinetic energy of cold atoms on the field shift $U_0$ for various detunings $\delta$ in the fields of (a, e) lin ⊥ lin and (c, g) $\sigma_+ - \sigma_-$ configurations, as well as momentum distributions of atoms at different $U_0$ for detuning $\delta = -2\gamma$ in the (b, f) lin ⊥ lin and (d, h) $\sigma_+ - \sigma_-$ fields with values of the recoil parameter $\varepsilon_R$ = (a–d) $4\times10^{-4}$ and (e–h) $10^{-3}$. The dashed horizontal line denotes the kinetic energy of atoms corresponding to the Doppler limit of laser cooling, $k_B T_D = \hbar\gamma/2$.

librium distribution with the Doppler temperature $k_B T_D = \hbar\gamma/2$. Sub-Doppler mechanisms of laser friction in the field of the $\sigma_+ - \sigma_-$ configuration ensure the achievement of an energy below the Doppler limit; however, the lin⊥lin configuration of the light field looks more attractive for the implementation of the deepest laser cooling of atoms under conditions of optical molasses.

### 3.2. Limits of sub-Doppler cooling of atoms with an insufficiently small recoil parameter ($\varepsilon_R \leqslant 10^{-2}$)

More significant differences in the sub-Doppler cooling of atoms appear for larger values of the recoil parameter $\varepsilon_R$. The results of calculating the average kinetic energy of cold atoms for $\varepsilon_R = 10^{-2}$ are shown in Figs 2a and 2c. There is a limited range of $U_0$ parameters and detunings at which energies below the Doppler limit are reached in the lin⊥lin configuration field. However, in the field of the $\sigma_+ - \sigma_-$ configuration, the sub-Doppler mechanisms of laser cooling lose efficiency and do not lead to energies below the Doppler limit (Fig. 2c), despite the fact that all the necessary conditions for this are fulfilled within the framework of semi-classical models [10–14], including the smallness of the parameter $\varepsilon_R \ll 1$.

As can be seen from the dependences shown in Fig. 2c, laser cooling of atoms in the field of the $\sigma_+ - \sigma_-$ configuration does not at all ensure the attainment of an energy below the Doppler limit. This makes it possible to explain the results of laser cooling of Mg atoms using laser fields resonant to the $3s3p^3P_2 \to 3s3d^3D_3$ optical transition [38], as well as the results of laser cooling of Sr atoms using the $5s5p^3P_2 \to 5s4d^3D_3$ optical transition [39] (see below the results obtained for the recoil parameter $\varepsilon_R = 10^{-1}$) when the laser cooling temperature did not reach values below the Doppler limit.

A comparison of the presented quantum approach with the semi-classical one, which takes into account the nonlinear dependence of the forces and diffusion coefficients, for atoms with a similar recoil parameter was carried out in [33], where it was shown that the distribution of atoms in the momentum space is substantially nonequilibrium (see also Figs 2b and 2d). Approximating the distribution function of atoms in the momentum space by two Gaussian functions, we can distinguish two fractions of atoms: 'cold', having a sub-Doppler temperature, and 'hot', with a temperature of the order of and above the Doppler limit. An analysis of the results of laser cooling of Tm atoms using the $4f^{13}(^2F^o)6s^2\,^2F^o_{7/2} \to 4f^{12}(^3H_6)5d_{5/2}6s^2$ $(6, 5/2)_{9/2}$ optical transition ($\varepsilon_R \approx 10^{-2}$) showed that the semi-classical approach overestimates the fraction of cold atoms, and this leads to an estimate of the average energy of atoms below the Doppler limit. The results obtained on the basis of the quantum approach are in good agreement with the experimental results both in the fraction of cold atoms and in their temperature [33].

### 3.3. Limits of laser cooling of atoms with narrow lines

For atoms with narrow optical lines ($\varepsilon_R \approx 10^{-1}$), the recoil effects in the interaction with single field photons become more significant, which leads to a low efficiency of sub-Doppler laser cooling mechanisms also in a lin⊥lin-configuration field, i.e., under conditions action of sub-Doppler Sisyphean mechanisms of laser cooling. The action of the latter in the case under study leads to the appearance of an inconspicuous narrow structure near zero momenta against the background of a wider momentum distribution determined by Doppler friction (Figs 2f and 2h), and they do not make any noticeable contribution to the kinetic energy of atoms. In contrast to the cases of small $\varepsilon_R$ considered above, the minimum values of the kinetic energy of cold atoms are achieved at lower modulus values of the detuning of the field $\delta \approx -2\gamma$ in the lin⊥lin-configuration field and at $\delta \approx -\gamma$ in the $\sigma_+ - \sigma_-$-configuration field (Figs2e–h).

Note that for even larger recoil parameters ($\varepsilon_R > 1$), sub-Doppler laser cooling mechanisms can have a negative effect, leading to atomic energies higher than in the case of the two-level model [31]. In this case, to achieve the deepest laser cooling, it may be preferable to use fields with uniform polarisation, i.e., those generated by counterpropagating waves with the same polarisations [34].

## 4. Conclusions

The study of the limits of sub-Doppler laser cooling of atoms in fields formed by counterpropagating waves with orthogonal circular ($\sigma_+ - \sigma_-$) and linear (lin⊥lin) polarisations is based on the numerical solution of the quantum kinetic equation for the atomic density matrix with full allowance for quantum recoil effects in the interactiion with field photons. The characteristic dependences of the atomic energy attainable upon laser cooling on the parameters of the light field for $\varepsilon_R < 1$ are obtained.

It is shown that the well-known theory of sub-Doppler cooling of atoms is valid only for extremely small values of the recoil parameter $\varepsilon_R \leqslant 10^{-3}$. In this case, for sufficiently large red frequency detunings ($|\delta|/\gamma \geqslant 5$), the kinetic energy of cold atoms is a certain universal function of only one parameter $U_0$ (optical shift), which depends on the choice of the light field configuration and the angular momenta of the ground $J_g$ and excited $J_e$ states, both in the field of lin⊥lin and $\sigma_+ - \sigma_-$ configurations.

For atoms with optical transitions characterised by larger values of the recoil parameter ($\varepsilon_R \approx 10^{-2}$–$10^{-1}$), sub-Doppler laser cooling mechanisms become less efficient, especially in the field of the $\sigma_+ - \sigma_-$ configuration used in magneto-optical traps. In this case, the minimum values of the kinetic energy of atoms are achieved at lower values of the red frequency detuning ($|\delta|/\gamma \approx 1$) than in the case of $\varepsilon_R \leqslant 10^{-3}$. For atoms with optical transitions characterised by the recoil parameter $\varepsilon_R \approx 10^{-2}$, the optimal detuning at low intensities of the light field is $\delta/\gamma \approx -1/2$, which corresponds to the optimal conditions for achieving the Doppler limit of laser cooling within the framework of the two-level model [8–10]. For recoil parameters $\varepsilon_R > 10^{-1}$, sub-Doppler mechanisms do not work at all, as a result of which the limiting energy during laser cooling becomes comparable to the Doppler limit.

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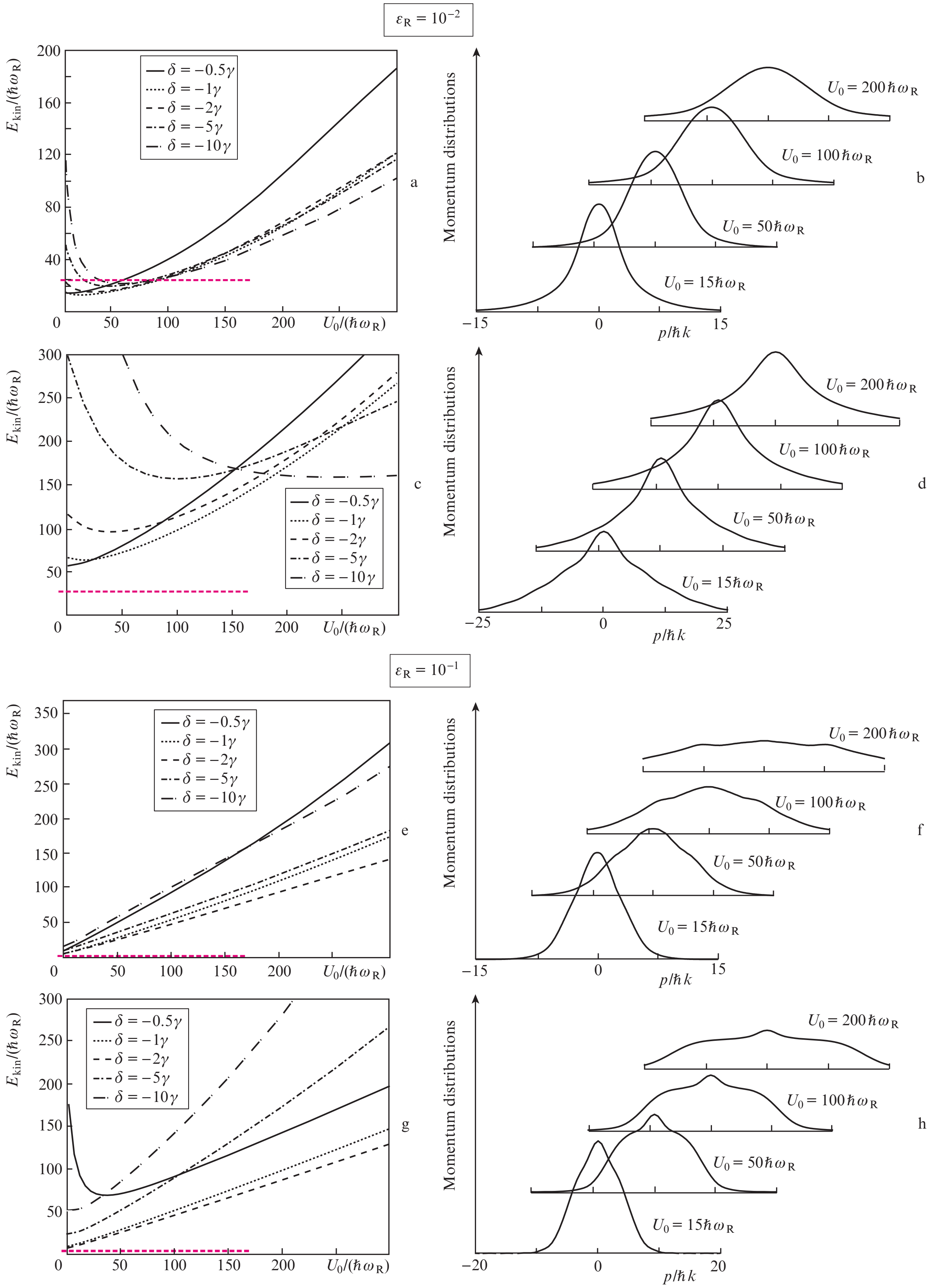


**Figure 2.** Dependences of the average kinetic energy of cold atoms on the field shift $U_0$ for various detunings $\delta$ in the fields of (a, e) lin ⊥ lin and (c, g) $\sigma_+ - \sigma_-$ configurations, as well as momentum distributions of atoms at different $U_0$ for detuning $\delta = -2\gamma$ in the (b, f) lin ⊥ lin and (d, h) $\sigma_+ - \sigma_-$ fields with values of the recoil parameter $\varepsilon_R$ = (a–d) $10^{-2}$ and (e–h) $10^{-1}$. The dashed horizontal line denotes the kinetic energy of atoms corresponding to the Doppler limit of laser cooling, $k_B T_D = \hbar\gamma/2$.

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