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Electromagnetic waves in an optical photonic lattice

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Abstract. **Laser light propagation in a photonic lattice consisting of two parallel waveguide arrays is theoretically studied using the coupled mode method, with the interaction of each waveguide with the nearest neighbours and between the waveguides of the arrays being taken into account. Analytical expressions are obtained that make it possible to accurately predict the presence of localisation of light depending on the coupling constants. Particular solutions of a system of coupled waves are found, which describe strongly localised light propagating without transverse diffraction along the entire lattice at certain values of the coupling constants. The emergence of spatially limited transverse diffraction of light is predicted.**

Keywords: *photonic lattice, waveguide array, localised light, coupled mode method.*

1. Introduction

Much attention is presently paid to the study of linear states of light in photonic lattices with nonstandard geometry and different periods. The main goal of these investigations is to analyse the possibility of localisation of light in such structures without the use of additional impurities and without any limitation of the light power. As shown in [1], flat-band (FB) lattices have a special linear spectrum, in which at least one of the linear bands is completely flat. This means that the modes of the flat band do not diffract and remain localised in space along the entire length of the system. The authors of Refs [2 – 5] considered Lieb photonic lattices, i.e. square lattices, which consist of three combined unit cells and are characterised by a dispersion law with three tight-binding bands, with one being perfectly flat (linear). Mukherjee et al. [4] studied experimentally and numerically the features of dispersionless localisation of light in a Lieb lattice formed by an array of optical waveguides.

Xia et al. [5] presented the results of an experimental study of distortion-free image transmission through optically induced photorefractive photonic Lieb lattices with different periodicities [6, 7]. These works generated great interest because they prove that waves can remain localised in the continuum even in the absence of any defect [8], disorder [9] or nonlinearity [10]. Theoretical and experimental studies of FB and related phenomena were carried out for one-dimensional and two-dimensional sawtooth gratings [11, 12], as well as for Kagome [13 – 15] and Fano [16] lattices. It is shown that macroscopically degenerate flat bands in periodic lattices contain compact localised states due to the presence of a 'random' flat band and can be observed as a result of the fine tuning of the inter-site coupling coefficients [11], which forms destructive interference and local symmetry depending on the lattice geometry.

Vicencio et al. [17] considered the possibility of propagation of localised discrete fundamental solitons in a model of a two-dimensional Kagome lattice with defocusing nonlinearity. Mukherjee and Thomson [18] experimentally demonstrated the photonic realisation of a dispersionless flat band in a quasi-one-dimensional photonic rhombic lattice in the tight-binding approximation. Goblot et al. [19] studied the nonlinear response of cavity polaritons in the flat band of a one-dimensional Lieb lattice with the formation of gap solitons with quantized size. They showed that in such a system it is possible to support two dispersive and one nondispersive (flat) band.

Theoretical studies of sawtooth lattices, in which quantum topological excitations are observed, were carried out in [20], and the possibility of observing Bose – Einstein condensation in the flat band for a one-dimensional sawtooth lattice and a two-dimensional Kagome lattice in the Bose – Hubbard model was considered in [21]. Baboux et al. [22] demonstrated that bosonic condensation of exciton polaritons is possible with the occurrence of a nondispersive energy band in a geometrically frustrated lattice of optical cavities. Rojas-Rojas et al. [23] reported the experimental results of studying the localisation of light in FB lattices of various geometries. Quasi-one-dimensional and two-dimensional FB lattices were investigated in the quantum regime. Quantum states were constructed that remain localised along the entire length of the lattice, do not experience diffraction, and do not depend on external parameters. The properties of separability of flat-band quantum states during information transmission through multicore fibres were also investigated. The properties of tunable one-dimensional sawtooth and zigzag optical lattices with ultracold atoms and polar molecules in an optical lattice are described in [24, 25].

Recently, studies of the properties of zigzag waveguide arrays [26 – 31], where the second-order coupling plays an important role in diffraction effects, have been of great interest. The authors of these works obtained an analytical expression for the trajectory of an oscillating beam. The solutions found for the beam trajectory make it possible to determine the periods of beam oscillations and the position of the trajectory turning points.

In addition to studies of fundamental flat bands observed in the above experiments, there are many theoretical dis-

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cussions about the existence of a flat band and the possibility of conservation and stability of localised states in space, as well as about the lifetime of flat states [32] and their stability in relation to the lattice geometry and coupling parameters [33]. Special attention is paid to artificial FB lattices created by structured potentials. Currently, significant experimental advances have been made in the realisation and study of flat bands for electrons, cold atoms, photons, and polaritons, as well as in the development of methods for controlling the lattice settings to obtain the desired lattice geometry and the study of compact localised states. The main attention is focused on the observation of new quantum and nonlinear effects of interaction in lattices [34], on the prospects for further studies of flat bands in photonic lattices under resonant optical pumping with the formation of polaritons and exciton-polaritons in metamaterials, coupled microcavities, superconducting microwave circuits [35], as well as on the study of compact discrete breather-periodic and spatially compact localised solutions for translationally invariant flat networks and for dispersive networks with fine-tuned nonlinear dispersion [36].

Real et al. [37] experimentally studied the main properties of a Stub photonic lattice and demonstrated for the first time the possibility of exciting a linear localised FB mode propagating without diffraction in a Stub. In addition, the feasibility of combining this mode with neighbouring modes to generate arbitrary linear combinations was shown, with the help of which the possibility of performing three all-optical logical operations was demonstrated. The possibility of using the properties of the photonic lattice Stub with FB for the development of new all-optical logic gates was predicted. The presented brief review of the literature indicates that the study of the features of localised light propagation in photonic lattices of various geometries is of interest both from a theoretical and an applied point of view and, as a result, is an urgent problem. Complex geometries of arrays, the use of new materials such as metamaterials, photonic crystals, photonic lattices, and the variety of their properties, as well as the prospect of implementing all-optical logic elements based on these materials, provide unique possibilities for controlling light propagation. The results of these studies are important for the applied field of modern optics, i.e. transformation optics. In this paper, we present the main results of a theoretical study of the effects of light propagation in one of such systems, namely, in a photonic optical lattice of waveguides, which consists of two parallel arrays of waveguides, taking into account their coupling with the nearest neighbours. The influence of the coupling constants and their ratios on the possibility of localising and maintaining a localised state in space is also considered.

2. Statement of the problem. Basic equations

The starting point of our consideration is a system of equations for the amplitudes of coupled modes of an optical photonic lattice consisting of two infinite waveguide arrays: an array of waveguides A_n (A) and an array of alternating waveguides B_n (B) and C_n (C) parallel to it (Fig. 1). Nodes A, B, and C make up a cell that periodically repeats in space and forms an optical photonic lattice. One can see from Fig. 1 that each waveguide A_n interacts only with waveguides B_n of the parallel array, and each waveguide B_n, in addition, interacts with the nearest neighbouring waveguides C_n and C_{n-1} of the same array. The system of equations in the geometry of Fig. 1 will have the form

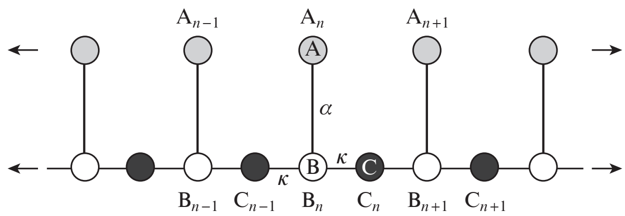

Figure 1. Layout of the waveguides of two coupled parallel infinite arrays in a photonic lattice.

$$
\begin{aligned}
&\mathrm{i}\frac{\mathrm{d}b_n}{\mathrm{d}x} + \kappa(c_n + c_{n-1}) + \alpha a_n = 0,\\
&\mathrm{i}\frac{\mathrm{d}c_n}{\mathrm{d}x} + \kappa(b_n + b_{n+1}) = 0,\\
&\mathrm{i}\frac{\mathrm{d}a_n}{\mathrm{d}x} + \alpha b_n = 0,
\end{aligned}
\tag{1}
$$

where α is the coupling constant of between arrays; x is the coordinate along the waveguide; κ is the coupling constant of the waveguide with the nearest neighbours in the arrays B and C; and a_n, b_n, and c_n are the normalised field amplitudes of the propagating modes in the nth waveguide (Fig. 1). We consider a homogeneous array of waveguides; therefore, all waveguides in the array are characterised by the same propagation constants. The evolution of light in such an array occurs along the waveguides in a direction orthogonal to the transverse periodic structure. The system of equations (1) is described in [16, 37, 38]. We use the Fourier transform for the functions $a_n(x)$, $b_n(x)$, and $c_n(x)$:

$$
\begin{aligned}
&a_n(x) = \frac{1}{2\pi}\int_{-\pi}^{\pi} a(\theta,x)\exp(-\mathrm{i}n\theta)\,\mathrm{d}\theta,\\
&b_n(x) = \frac{1}{2\pi}\int_{-\pi}^{\pi} b(\theta,x)\exp(-\mathrm{i}n\theta)\,\mathrm{d}\theta,\\
&c_n(x) = \frac{1}{2\pi}\int_{-\pi}^{\pi} c(\theta,x)\exp(-\mathrm{i}n\theta)\,\mathrm{d}\theta.
\end{aligned}
\tag{2}
$$

Substituting (2) into (1), for the Fourier transforms $a(\theta,x)$, $b(\theta,x)$, and $c(\theta,x)$ we obtain a system of first-order coupled linear differential equations:

$$
\begin{aligned}
&\mathrm{i}\frac{\mathrm{d}b}{\mathrm{d}x} + \kappa[1 + \exp(\mathrm{i}\theta)]c + \alpha a = 0,\\
&\mathrm{i}\frac{\mathrm{d}c}{\mathrm{d}x} + \kappa[1 + \exp(-\mathrm{i}\theta)]b = 0,\\
&\mathrm{i}\frac{\mathrm{d}a}{\mathrm{d}x} + \alpha b = 0.
\end{aligned}
\tag{3}
$$

We will seek solutions for the field amplitudes a_n, b_n, and c_n in the system of equations (1) in the form of a plane wave:

$$a_n(x) = a\exp(\mathrm{i}px),\; b_n(x) = b\exp(\mathrm{i}px),$$

$$c_n(x) = c\exp(\mathrm{i}px), \tag{4}$$

where p plays the role of the Bloch wave vector of the lattice [39]. Then system (1) takes the form

$$\alpha a - pb + \kappa[1 + \exp(\mathrm{i}\theta)]c = 0,$$

$$\kappa[1 + \exp(-\mathrm{i}\theta)]b - pc = 0, \tag{5}$$

$$pa - \alpha b = 0$$

and will represent a system of linear homogeneous algebraic equations for the field amplitudes a, b, and c, a nontrivial solution of which is possible if

$$\begin{vmatrix} \alpha & -p & \kappa[1+\exp(\mathrm{i}\theta)] \\ 0 & \kappa[1+\exp(-\mathrm{i}\theta)] & -p \\ p & -\alpha & 0 \end{vmatrix} = 0. \tag{6}$$

Expression (6) makes it possible to determine the dispersion law for linear modes propagating in the transverse direction in an array of waveguides when they propagate in the longitudinal direction x along the array. Expanding the determinant, we obtain a cubic equation for finding the eigenvalues of the wave vectors of the array, which characterise the excited linear state propagating without diffraction along the direction of the array [37, 40]:

$$p^3 - p\kappa^2[1 + \exp(-\mathrm{i}\theta)][1 + \exp(\mathrm{i}\theta)] - p\alpha^2 = 0, \tag{7}$$

whose solution will have the form

$$p_1 = 0,\; p_{2,3} = \pm\sqrt{2\kappa^2(1+\cos\theta)+\alpha^2} = \pm\beta. \tag{8}$$

Solutions (8) represent a spectrum of linear states. One of them is the straight line $p_1 = 0$, which characterises a linear flat band of the spectrum, and the other two are linear bands of opposite signs (Fig. 2). One can see that there is a 'gap' between the linear bands at $\alpha \neq 0$, the width of which is 2α; at $\alpha = 0$ and the condition $\theta = \pm\pi$, the gap is absent in the spectrum (Fig. 2b). The behaviour of the bands $p_{2,3}$ is determined by the values of the coupling constants between waveguide arrays and between waveguides in the array, while the flat band p_1 does not depend on the coupling constants in the lattice. The presence of a spectral linear flat band indicates that the amplitudes of the linear modes b_n belonging to this band are equal to zero, which agrees with the results of Refs [37, 38].

The solutions to the system of equations (3) depend on arbitrary constants determined by the initial conditions. This system can be reduced to one second-order differential equation:

$$\frac{\mathrm{d}^2 b}{\mathrm{d}x^2} + \beta^2 b = 0.$$

Using expressions (8), we will seek a solution to the equation for the field amplitude $b(\theta, x)$ in the form

$$b(\theta, x) = B_1 + B_2\exp(\mathrm{i}px) + B_3\exp(-\mathrm{i}px), \tag{9}$$

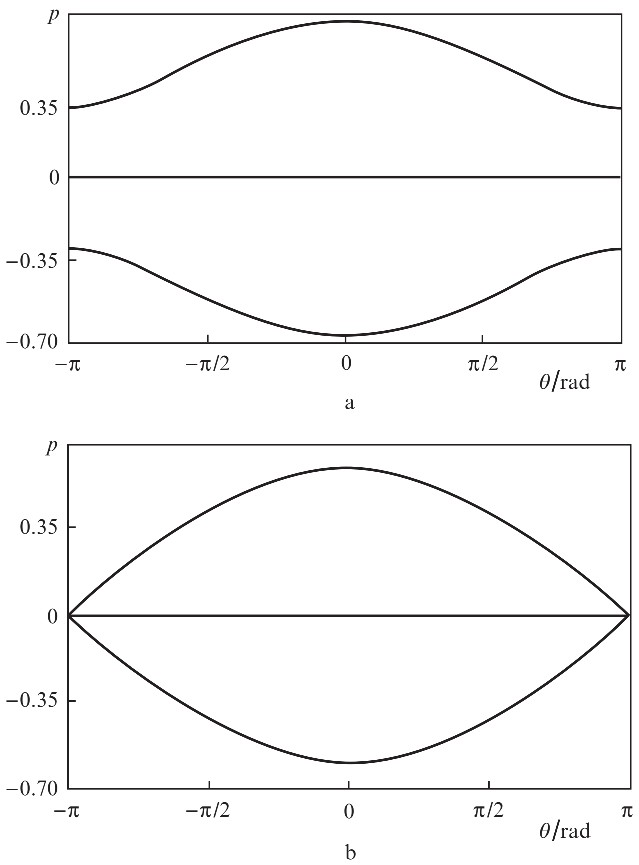

Figure 2. Spectrum (dispersion relation) of the photonic lattice at $\alpha =$ (a) 0.3 and (b) 0; $\kappa = 0.3$.

where arbitrary constants B_1, B_2 and B_3 can be found from the initial conditions

$$b_n|_{x=0} = \delta_{n0},\; a_n|_{x=0} = c_n|_{x=0} = 0$$

(δ_{n0} is the Kronecker symbol), which at $n = 0$ (the end face of the zero waveguide B_0 is pumped) take the form

$$b_0|_{x=0} = 1,\; a_0|_{x=0} = c_0|_{x=0} = 0. \tag{10}$$

Substituting (10) into (9) and using the first equation of system (3) with allowance for the initial condition

$$\mathrm{i}\frac{\mathrm{d}b(\theta,x)}{\mathrm{d}x}\bigg|_{x=0} = 0,$$

we obtain the system of equations

$$B_1 + B_2 + B_3 = 1,$$

$$B_2 - B_3 = 0.$$

to determine arbitrary constants.
As a result, we find

$$B_1 = 0, \tag{11}$$

$$B_2 = B_3 = \frac{1}{2}, \tag{12}$$

and the solutions to the system of equations (3) for the Fourier transforms can be written in the form:

$$b(\theta, x) = B_2\exp(\mathrm{i}px) + B_3\exp(-\mathrm{i}px),$$
$$a(\theta, x) = \mathrm{i}\frac{\alpha}{p}\sin(px) + A_1, \quad (13)$$
$$c(\theta, x) = \mathrm{i}\frac{\kappa}{p}[1 + \exp(-\mathrm{i}\theta)]\sin(px) + C_1,$$

where A_1 and C_1 are the integration constants. Taking into account the initial conditions (10) and (11), (12), we obtain $A_1 = 0$ and $C_1 = 0$. Then the final solutions to the system of equations (3) take the form

$$b(\theta, x) = \cos(px),$$
$$a(\theta, x) = \mathrm{i}\frac{\alpha}{p}\sin(px), \quad (14)$$
$$c(\theta, x) = \mathrm{i}\frac{\kappa}{p}[1 + \exp(-\mathrm{i}\theta)]\sin(px).$$

Substituting (14) into (2), we find the solutions for the functions $a_n(x)$, $b_n(x)$, and $c_n(x)$:

$$b_n(x) = \frac{1}{\pi}\int_0^{\pi} \mathrm{d}\theta\cos(n\theta)\cos(\beta x),$$
$$a_n(x) = \frac{\mathrm{i}\alpha}{\pi}\int_0^{\pi} \mathrm{d}\theta\cos(n\theta)\frac{\sin(\beta x)}{\beta}, \quad (15)$$
$$c_n(x) = \frac{\mathrm{i}\kappa}{\pi}\int_0^{\pi} \mathrm{d}\theta\,\{\cos(n\theta) + \cos[(n+1)\theta]\}\frac{\sin(\beta x)}{\beta}.$$

Setting $4\kappa^2/\alpha^2 \ll 1$, expanding β in series, and using the known relation

$$\exp(\mathrm{i}\alpha x\cos\theta) = \sum_{k=-\infty}^{+\infty}\mathrm{i}^k J_k(x)\exp(\mathrm{i}\kappa\theta),$$

where $J_k(x)$ is the kth-order Bessel function, we obtain a solution for $b_n(x)$:

$$b_n(x) = \mathrm{i}^n J_n\left(\frac{\kappa^2}{\alpha}x\right)\cos\left[\alpha\left(1 + \frac{\kappa^2}{\alpha^2}\right)x\right]. \quad (16)$$

Solution (16) is the nth-order Bessel function modulated by a cosine and describes the propagation of modes in waveguides B_n along the second array.

Let us consider other initial conditions when pumping is performed at the end face of the zero waveguide A_0 of the first array:

$$a_n|_{x=0} = \delta_{n0},\ b_n|_{x=0} = c_n|_{x=0} = 0. \quad (17)$$

Then expressions (11) and (12) for the initial conditions (17) take the form

$$B_1 = 0, \quad (18)$$

$$B_2 - B_3 = \frac{\alpha}{2p}, \quad (19)$$

and the solution to the first equation of system (3)

$$\mathrm{i}\left.\frac{\mathrm{d}b(\theta,x)}{\mathrm{d}x}\right|_{x=0} + \alpha = 0$$

taking into account (18) and (19) will be as follows:

$$b(\theta,x) = \frac{\alpha}{2p}\exp(\mathrm{i}px) - \frac{\alpha}{2p}\exp(-\mathrm{i}px) = \mathrm{i}\frac{\alpha}{p}\sin(px).$$

Let us find a solution for the amplitudes $a(\theta, x)$ and $c(\theta, x)$ from the second and third equations of system (3) taking into account $b(\theta, x)$:

$$a(\theta, x) = \frac{\alpha^2}{p^2}\cos(px) + A_1,$$
$$c(\theta, x) = \frac{\alpha\kappa}{p^2}[1 + \exp(-\mathrm{i}\theta)]\cos(px) + C_1.$$

Then, taking into account the initial conditions (17), we determine the integration constants A_1 and C_1 at $x = 0$:

$$A_1 = 1 - \frac{\alpha^2}{p^2},\ C_1 = -\frac{\alpha\kappa}{p^2}[1 + \exp(-\mathrm{i}\theta)]. \quad (20)$$

Then

$$b_n(x) = \frac{\mathrm{i}\alpha}{\pi}\int_0^{\pi}\mathrm{d}\theta\cos(n\theta)\frac{\sin(\beta x)}{\beta},$$
$$a_n(x) = \delta_{n0} - \frac{2\alpha^2}{\pi}\int_0^{\pi}\mathrm{d}\theta\cos(n\theta)\frac{\sin^2(\beta x)}{\beta^2}, \quad (21)$$
$$c_n(x) = -\frac{\alpha\kappa}{\pi}\int_0^{\pi}\mathrm{d}\theta\,\{\cos(n\theta) + \cos[(n+1)\theta]\}\frac{\sin^2(\beta x)}{\beta^2}.$$

Let us consider the case of complete excitation of a unit cell of an optical photonic lattice, when pumping is performed at the end face of all zero waveguides of both arrays. The initial conditions take the form

$$a_n|_{x=0} = \delta_{n0},\ b_n|_{x=0} = \delta_{n0},\ c_n|_{x=0} = \delta_{n0}. \quad (22)$$

Taking into account (22), we write the solutions to the system of equations (3) for the Fourier transforms:

$$b(\theta, x) = \cos(px),$$
$$a(\theta, x) = \mathrm{i}\frac{\alpha}{p}\sin(px) + A_1, \quad (23)$$
$$c(\theta, x) = \mathrm{i}\frac{\kappa}{p}[1 + \exp(-\mathrm{i}\theta)]\sin(px) + C_1.$$

Let us define the integration constants at $x = 0$:

$$A_1 = 1 - \mathrm{i}\frac{\alpha}{p},\ C_1 = 1 - \mathrm{i}\frac{\kappa}{p}[1 + \exp(-\mathrm{i}\theta)],$$

and write the solution to system (1):

$$b_n(x) = \frac{1}{\pi}\int_0^\pi \mathrm{d}\theta\cos(n\theta)\cos(\beta x).$$

$$a_n(x) = \delta_{n0} - \frac{\mathrm{i}\alpha^2}{\pi}\int_0^\pi \mathrm{d}\theta\cos(n\theta)\frac{1-\sin(\beta x)}{\beta}, \qquad (24)$$

$$c_n(x) = \frac{\mathrm{i}\kappa}{\pi}\int_0^\pi \mathrm{d}\theta\,\{\cos(n\theta) + \cos[(n+1)\theta]\}\frac{1-\sin(\beta x)}{\beta}.$$

As for the intensities of propagating waves $|a_n(x)|^2$, $|b_n(x)|^2$ and $|c_n(x)|^2$, we can easily obtain for them the expression

$$\sum_{n=-\infty}^{+\infty}(|a_n|^2+|b_n|^2+|c_n|^2) = 1. \qquad (25)$$

This relationship is the law of conservation of energy in the system: at any point, regardless of the x coordinate, the sum of energies over all waveguides of both arrays is conserved and is equal to the energy pumped from the end face ($x = 0$) of the zero waveguide.

3. Results and discussion

Let us first consider the case when pumping is performed only at the end face of the waveguide B_0 of the second array. We investigate the spatial distribution of the light intensity in the waveguides of the arrays for different values of the coupling constants κ and α.

If $\alpha \neq 0$ and $\kappa \neq 0$, then both waveguide arrays are excited. The spatial distribution of the light intensity in the arrays depends on the relationship between the constants and in unpumped waveguides is a sequence of intensity maxima separated by field zeros. Under the condition $4\kappa^2/\alpha^2 \ll 1$, the light is localised in the zero waveguides of the arrays, and the spatial distribution of the light intensity in them is periodic. Such localisation at a sufficiently large distance along the array is possible under the condition of strong coupling. In this case, the light is highly localised and propagates without transverse diffusion along the optical lattice. The maximum intensity is concentrated in the zero waveguides A_0 and B_0, where the periodic pump regime is observed, while the waveguides C_0 and C_1 are excited in the waveguides C_n (Fig. 3a). In this case, the intensity of the pumped light in the waveguides C_n is very low, and the pump period coincides with the pump period in the waveguides A_n. The light intensity in the waveguides B_n of the second array is determined by expression (16).

In the case $\alpha > \kappa$, both waveguide arrays are excited and a strong spatial diffusion of light in the transverse direction is observed in all waveguides of both arrays (Fig. 3b). In this case, the intensities of the light propagating in the waveguides A_n and B_n are the same, while the light intensity in the waveguides C_n of the second array increases. The intensity of the light in the pumped waveguide rapidly decreases as it propagates along the array due to the presence of strong transverse diffraction and the light pumping between the arrays; the region of the excited waveguides of both arrays depends on the constant κ.

If $\alpha = \kappa$, then the spatial intensity distribution is a set of minima and maxima of the propagating light located symmetrically with respect to the zero waveguides A_0 and B_0, while the waveguides C_n exhibit symmetry of the intensity distribution relative to the excited waveguides C_0 and C_1 (Fig. 3b). The main part of the light intensity is concentrated in the waveguides B_n of the second array. Figure 3d shows the spatial distributions of the light intensity in the first and second waveguide arrays for $\alpha < \kappa$. In this case, the light intensity distribution is a complex superposition of oscillations in all waveguides of both arrays, and a rapid attenuation of the light is observed in them due to the presence of transverse diffusion of light. One can see that for these values of the constants, the intensity of the propagating light is concentrated in the waveguides B_n and C_n, which is due to the value of the parameter κ.

Let us now consider the case when pumping is performed only at the end face of the zero waveguide A_0 of the first array. We investigate the spatial distribution of the light intensity in the arrays of waveguides of the optical lattice for the same values of the coupling constants κ and α as in the previous case.

If $\alpha \gg \kappa$, then both arrays of the waveguides are excited. The spatial intensity distribution in the waveguides is a periodic sequence of intensity maxima separated by field zeros, most of the light is localised in the zero waveguides of arrays A and B, and the light intensity in the waveguides C_0 and C_1 is very low (Fig. 4a). In the case when $\alpha > \kappa$, the behaviour of the spatial distribution of the light intensity in the waveguides changes qualitatively, i.e. spatial diffusion of the light is observed in the waveguides of arrays A and B, and the region of excited waveguides increases as the light propagates along the lattice. In this case, the light remains localised in the excited waveguides C_0 and C_1 of the second array (Fig. 4b). If $\alpha = \kappa$, then spatial diffusion of the light exists in all waveguides of both arrays, their excitation occurring at a closer distance from the end face of the lattice (Fig. 4c). When $\alpha < \kappa$, the light near the end face is localised in the zero waveguide of the first array. At some distance from the end face, there arises spatial diffusion of the light and successive excitation of the array waveguides is observed; however, the light intensity in the excited waveguides is very small and the waveguides of the arrays B and C exhibit strong spatial diffusion of light, which emerges at a very small distance from the end face. In this case, the waveguides of the array B are successively excited, the light intensity in which decreases at a certain distance and then increases again. The nature of the spatial distribution of the light intensity in all waveguides of both arrays is different (Fig. 4d).

Let us consider the case of complete excitation of a unit cell of an optical photonic lattice, when pumping is performed at the end face of all zero waveguides of arrays A, B, and C (22). We investigate the spatial distribution of the light intensity in the arrays of waveguides of the optical lattice for the same values of the coupling constants κ and α as in other cases. One can see from Fig. 5a that for $\alpha \gg \kappa$ both arrays of waveguides of the lattice are excited and the propagating light is strongly localised in the pumped waveguides of both arrays. The spatial distribution of the intensity in the waveguides is a sequence of intensity maxima separated by field zeros, and the spatial distribution of the light intensity in the zero waveguides of both arrays is periodic. For $\alpha > \kappa$ (Fig. 5b), the light in the waveguides of the first and second arrays A and B is still localised, and at a certain distance from the end face, spatial transverse diffusion of radiation arises symmetrically with respect to the waveguides A_0 and B_0. In the waveguides of the second array C, spatial diffusion arises asymmetrically with respect to the zero waveguide C_0 and waveguide C_1 (Fig. 5b).

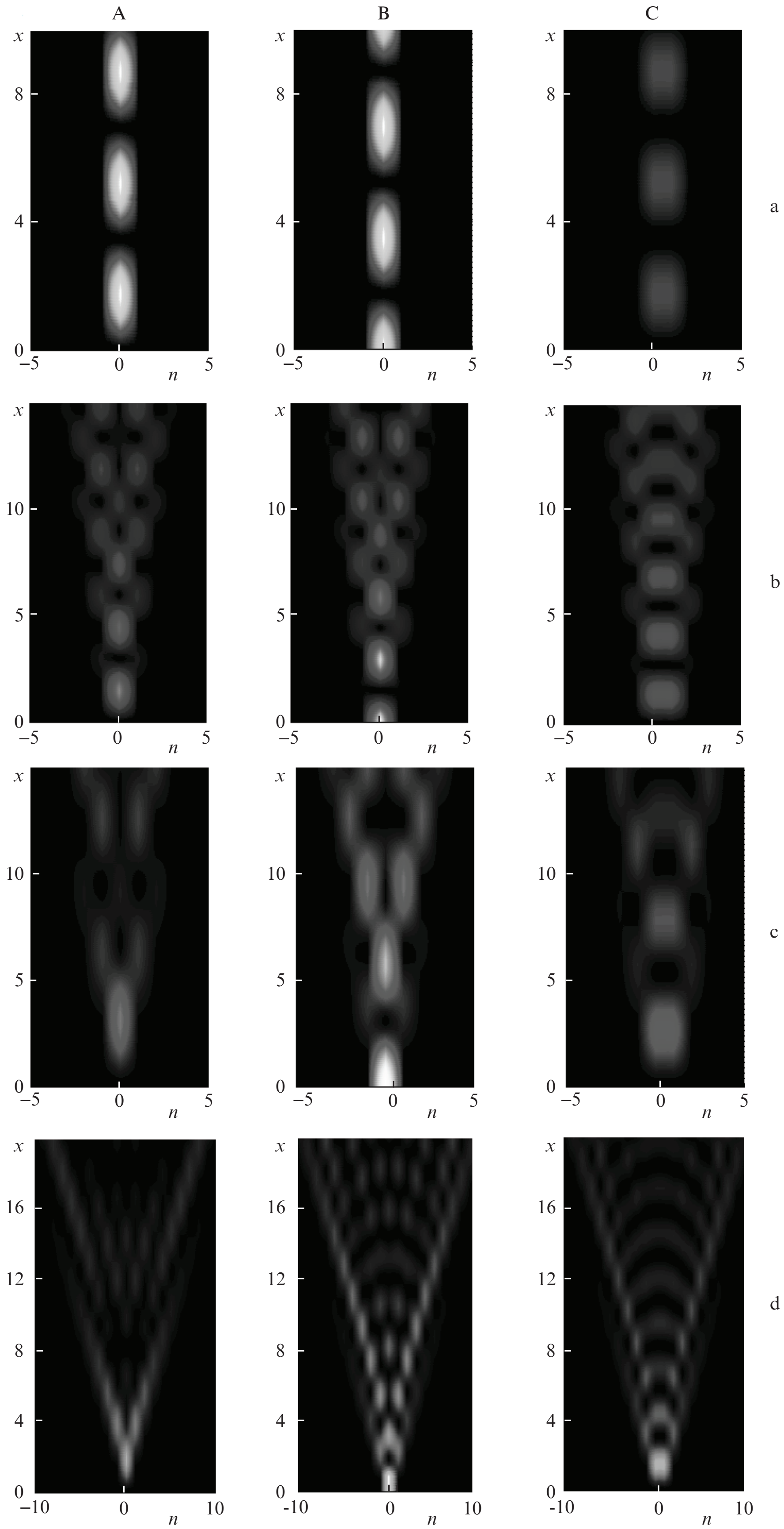

Figure 3. Spatial distributions of the light intensity in the first and second arrays of the lattice waveguides as functions of the coordinate x under pumping into the end face of the waveguide B_0 for different values of the constants: (a) $\alpha = 0.9$, $\kappa = 0.01$; (b) $\alpha = 0.9$, $\kappa = 0.3$; (c) $\alpha = 0.3$, $\kappa = 0.3$; and (d) $\alpha = 0.3$, $\kappa = 0.6$.

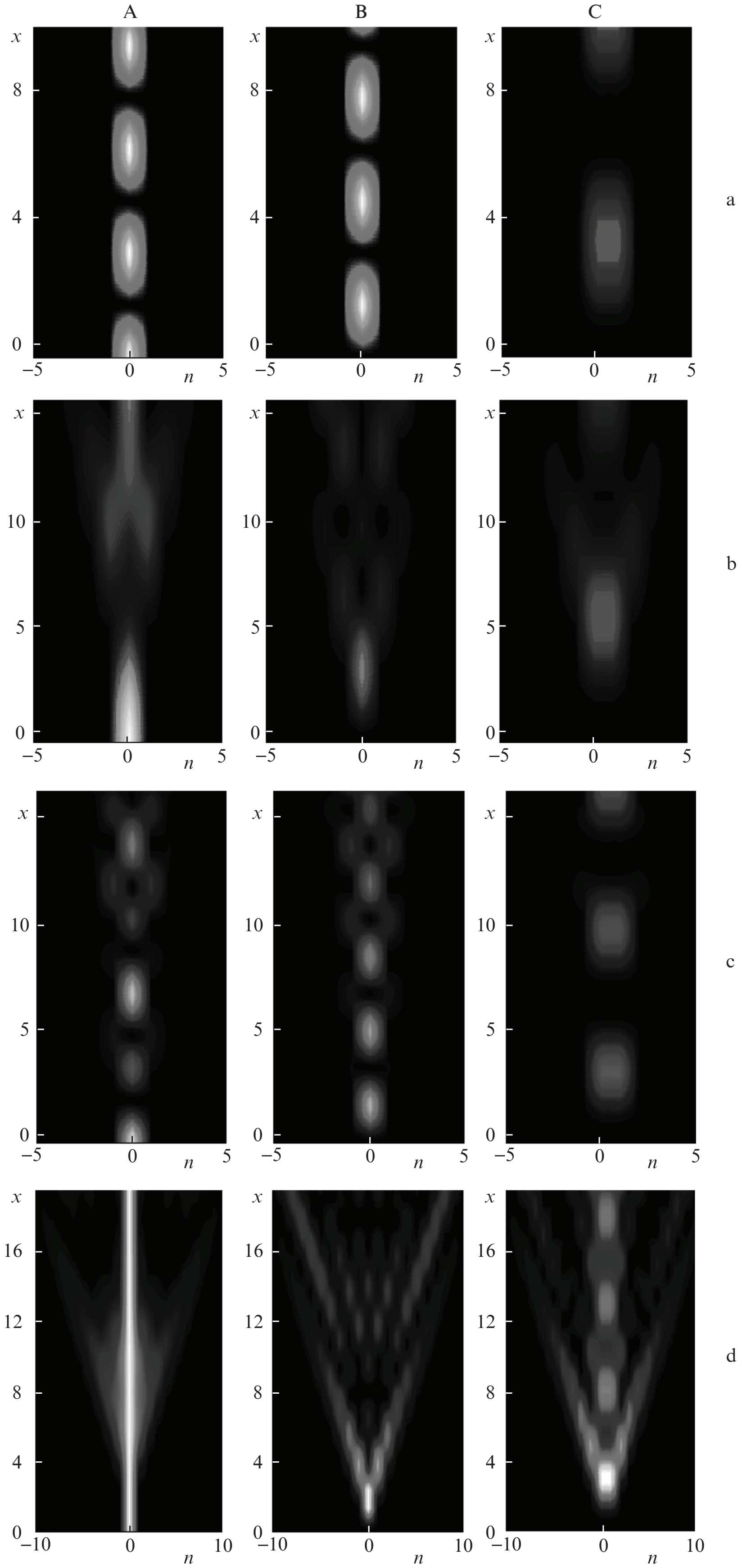

Figure 4. Spatial distributions of the light intensity in the first and second arrays of the lattice waveguides as functions of the coordinate x under pumping into the end face of the waveguide A_0 for different values of the constants: (a) $\alpha = 0.9$, $\kappa = 0.01$; (b) $\alpha = 0.9$, $\kappa = 0.3$; (c) $\alpha = 0.3$, $\kappa = 0.3$; and (d) $\alpha = 0.3$, $\kappa = 0.6$.

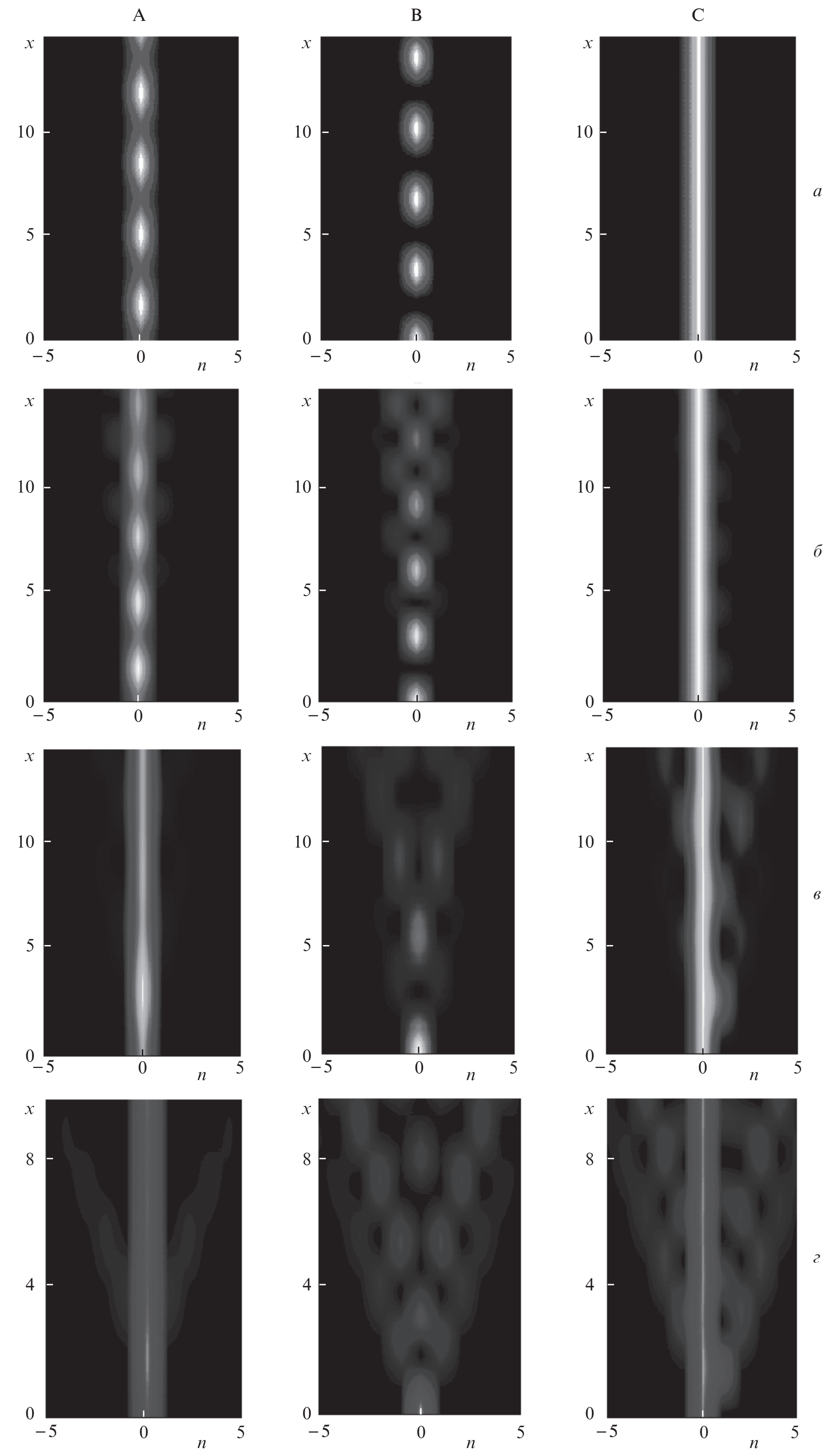

Figure 5. Spatial distributions of the light intensity in the first and second arrays of the lattice waveguides as functions of the coordinate x under complete excitation of the cell A, B, and C for different values of the constants: (a) $\alpha = 0.9$, $\kappa = 0.01$; (b) $\alpha = 0.9$, $\kappa = 0.3$; (c) $\alpha = 0.3$, $\kappa = 0.3$; and (d) $\alpha = 0.3$, $\kappa = 0.6$.

If $\alpha = \kappa$ (Fig. 5c), then the light is localised in the zero waveguide of the first array. The waveguides of the second array exhibit spatial diffusion of the light and are successively excited: symmetric with respect to B_0 in waveguides of the array B and asymmetric with respect to C_0 in the waveguides C; in this case, the region of the excited waveguides of the second array increases. In the case $\alpha < \kappa$ (Fig. 5d), spatial diffusion of the light is observed in all waveguides of both arrays, and the region of the excited waveguides of the arrays A and B increases, remaining symmetric with respect to the zero waveguides, while in the waveguides of the second array C, spatial diffusion is still asymmetric with respect to the waveguide C_0.

4. Conclusions

Using the coupled mode method, the spatial distribution of the intensity of the light propagating in an optical photonic lattice consisting of two parallel arrays of waveguides is theoretically studied under different initial conditions. The choice of the initial conditions significantly affects the intensity distribution in the waveguides of the optical photonic lattice. It is shown that this distribution is a periodic function of the coordinate and depends on the coupling constants in the lattice. The results of the theoretical study allow us to speak about the presence of a criterion that determines the possibility of localising the light propagating without transverse diffraction along the array in the waveguide arrays of the lattice. This fact makes it possible to conclude that localisation in the geometry under consideration can be experimentally observed only in an optical photonic lattice with strong coupling. It is also shown that for certain values of the coupling constants in the array, there is a complete periodic pumping of the light between the arrays. It is found that in the system in question one can observe spatially limited transverse diffraction of light, which substantially depends on the parameters of the system. In addition, the relationship between these parameters makes it possible to control the intensity of the pumped light propagating in the waveguides of the arrays. The obtained theoretical results and the results of numerical simulations are in complete agreement with the results of the experiment [33] and can find application in the development of optical devices in quantum electronics for controlling the propagation of light.

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