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Numerical simulation of the beam self-cleaning process in a multimode graded-index fibre during propagation of a pump wave and a Stokes component

O.S. Sidelnikov, E.V. Podivilov, S.A. Babin, S. Wabnitz, M.P. Fedoruk

Abstract. **A model of coupled modes is proposed, which makes it possible to describe the propagation of a pump wave and a Stokes component in a multimode graded-index fibre. A decrease in the depression in the centre of the profile of the pump wave intensity distribution as a result of the influence of Kerr effects and random linear coupling of modes is demonstrated.**

Keywords: *multimode optical fibre, nonlinear effects, numerical simulation, stimulated Raman scattering.*

1. Introduction

The spatiotemporal dynamics of the intensity of a light beam profile in multimode fibres is currently of interest from the point of view of both fundamental physics and practical applications [1 – 3]. It is known that when a light pulse propagates through a multimode fibre, it is affected by random linear coupling caused by stress, bending, or various defects in fibre. This leads to a rapid distortion of the spatial profile of the input beam intensity and to the appearance of spotty structures at the fibre output. Recent experiments [3 – 5] showed that due to the nonlinear Kerr effect in multimode graded-index fibres, beam self-cleaning can be observed, resulting in a stable bell-shaped beam, the diameter of which at the fibre output is close to that of the fundamental mode.

Physical experiments in nonlinear optics often require complex and expensive equipment; therefore, new methods must be developed for predictive mathematical modelling of various optical systems. In this work, using numerical simulations, we study the beam self-cleaning process with the simultaneous propagation of a Stokes wave and a pump wave in a multimode parabolic-index fibre. A model of coupled modes is proposed, which makes it possible to take into account stimulated Raman scattering (SRS), the Kerr effect, fibre losses, and random linear mode coupling.

O.S. Sidelnikov Novosibirsk State University, ul. Pirogova 2, 630090 Novosibirsk, Russia; e-mail: o.sidelnikov@g.nsu.ru;
E.V. Podivilov, S.A. Babin Novosibirsk State University, ul. Pirogova 2, 630090 Novosibirsk, Russia; Institute of Automation and Electrometry, Siberian Branch, Russian Academy of Sciences, prosp. Akad. Koptyuga 1, 630090 Novosibirsk, Russia; e-mail: podivilov@iae.nsk.su, babin@iae.nsk.su;
S. Wabnitz Novosibirsk State University, ul. Pirogova 2, 630090 Novosibirsk, Russia; Sapienza University of Rome, 00185, Italy, Rome, Piazzale Aldo Moro, 5; e-mail: stefan.wabnitz@uniroma1.it;
M.P. Fedoruk Novosibirsk State University, ul. Pirogova 2, 630090 Novosibirsk, Russia; Institute of Computational Technologies, Siberian Branch, Russian Academy of Sciences, prosp. Akad. Lavrent'eva 6, 630090 Novosibirsk, Russia; e-mail: rector@nsu.ru

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2. Balance equations

We consider a multimode graded-index fibre 1 km in length:

$$n(r) = \begin{cases} n_0\sqrt{1-[2\Delta(r/a)]^2}, & r < a, \\ n_{cl}, & r \geqslant a, \end{cases}$$

where $n_0 = 1.47$ is the refractive index at the centre of the fibre; $n_{\rm cl} = 1.457$ is the refractive index of the cladding; a = 50 μm is the fibre radius; and $\Delta = (n_0^2 - n_{\rm cl}^2)/(2n_0^2)$. A pump wave with a wavelength of $\lambda_{\rm p}$ = 940 nm and a Stokes component with $\lambda_{\rm S}$ = 980 nm propagated simultaneously along the fibre. A pump wave with a parabolic intensity profile was applied to the fibre input, filling the entire fibre core. The aim of this work was mathematical modelling of the Raman gain of a Stokes wave in the fundamental mode. With increasing pump power, the power of the Stokes wave will also increase, which will lead to depletion of the pump. It is expected that due to this, a depression will be formed in the centre of the transverse distribution of the pump wave intensity, the radius of which will be equal to the radius of the fundamental mode.

To simulate the propagation of a Stokes wave and a pump wave through a multimode fibre, we first considered a model based on the balance equations [6]:

$$\frac{dI_S}{{\rm d}z} = g_{\rm R} k_{\rm S} I_{\rm p} I_{\rm S} - \alpha_{\rm S} I_{\rm S},$$

$$\frac{dI_{\rm p}}{{\rm d}z} = -g_{\rm R} k_{\rm p} \frac{\lambda_{\rm S}}{\lambda_{\rm p}} I_{\rm S} I_{\rm p} - \alpha_{\rm p} I_{\rm p},$$

where $I_{\rm p}$ and $I_{\rm S}$ are the intensities of the pump wave and the Stokes component; $\alpha_{\rm S,p}$ are the loss factors; $k_{\rm S,p} = 2\pi n_0/\lambda_{\rm S,p}$ are the wave numbers; and $g_{\rm R}$ is the Raman gain estimated by the pump power threshold. A radial dependence of the intensities of all waves was assumed for this model, and the 'multipass' problem was solved until the output distribution of the pump wave and the Stokes component was established. At each passage of the Stokes wave, 153-W pump radiation was supplied to the fibre input. At the output, the Stokes component passed through a filter that reflected 4% of the power of the fundamental mode and 0.4% of the power of each of the other modes, similar to the experimental conditions from [7].

Figure 1 shows the output distributions of the intensities of the pump wave and the Stokes component, obtained using the balance equations. One can see that the pump wave is depleted at the centre, and a large depression is formed in its distribution. This is due to the fact that the Stokes component has a small radius, and within the framework of this model, energy exchange occurs only between the regions of spatial distributions of the Stokes wave and pump wave intensities, which have the same radial coordinates. However, this model is rather crude, since the interaction of the pump wave and the Stokes component is not pointwise, because the modes have a finite size. In addition, the balance equations do not allow the effects associated with the Kerr nonlinearity and random coupling of spatial modes to be described, which can be significant in experiments with Raman amplification of a Stokes wave due to a pump wave in a graded-index fibre [7] and in the case of propagation of only one pump wave [3, 4]. Therefore, to describe the experiment, it is necessary to construct a more detailed model of coupled modes.

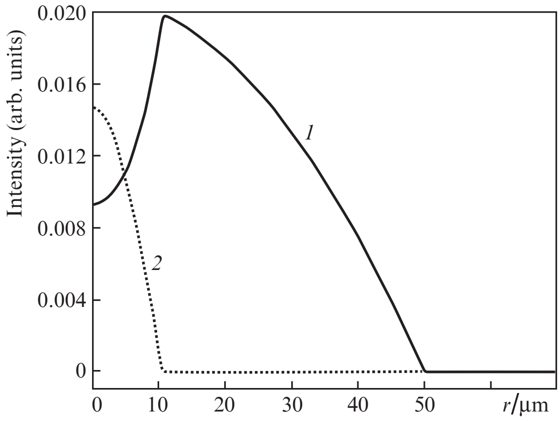

Figure 1. Output distributions of the intensities of (*1*) the pump wave and (*2*) the Stokes component, obtained using the balance equations.

3. Model of coupled modes

For a more accurate description of the process of propagation of the pump wave and the Stokes component in a multimode graded-index fibre, we used in this work a system of equations based on the coupled mode model:

$$\frac{\partial A_{p,m}}{\partial z} = \sum_{m1,p1} C^{m1,p1}_{m,p} A_{m1,p1} - \frac{\alpha_{\rm S}}{2} A_{p,m} - {\rm i}\frac{k_{\rm S} n_2 a}{n_0 \sqrt{2\Delta}}$$

$$\times \sum_{m1,m2,m3} \sum_{p1,p2,p3} \Big[q^{m1,m2,m3,m}_{p1,p2,p3,p} P_{\rm S} A^*_{p1,m1} A_{p2,m2} + g^{m1,m2,m3.m}_{p1,p2,p3,p}$$

$$\times P_{\rm p} (2 + {\rm i} f) B^*_{p1,m1} B_{p2,m2} \Big] A_{p3,m3},$$

$$\frac{\partial B_{p,m}}{\partial z} = \sum_{m1,p1} C^{m1,p1}_{m,p} B_{m1,p1} - \frac{\alpha_{\rm p}}{2} B_{p,m} - {\rm i}\frac{k_{\rm p} n_2 a}{n_0 \sqrt{2\Delta}}$$

$$\times \sum_{m1,m2,m3} \sum_{p1,p2,p3} \Big[h^{m1,m2,m3,m}_{p1,p2,p3,p} P_{\rm p} B^*_{p1,m1} B_{p2,m2} + l^{m1,m2,m3.m}_{p1,p2,p3,p}$$

$$\times P_{\rm S} (2 - {\rm i} f) A^*_{p1,m1} A_{p2,m2} \Big] B_{p3,m3}.$$

Here $A_{p,m}$ and $B_{p,m}$ are the amplitudes of the mode components of the Stokes wave and the pump wave, respectively; p and m are the radial and azimuthal orders of the mode; $P_{\rm S}$ and $P_{\rm p}$ are the powers of the Stokes wave and the pump wave; n_2 is the nonlinear refractive index; and f is the ratio of the Raman and Kerr constants.

The coefficients $q^{m1,m2,m3,m}_{p1,p2,p3,p}$, $g^{m1,m2,m3,m}_{p1,p2,p3,p}$, $h^{m1,m2,m3,m}_{p1,p2,p3,p}$, and $l^{m1,m2,m3,m}_{p1,p2,p3,p}$ are obtained by calculating the overlap integrals of the spatial modes of the pump wave and the Stokes component. Thus, this model allows one to take into account the effects of self-phase modulation, cross-phase modulation, Raman effect, and fibre loss. In addition, this system of equations also takes into account the random linear coupling between all spatial modes due to various fibre imperfections, bends and stresses. Coefficients $C^{m1,p1}_{m,p}$ are random variables with normal distribution with zero mean and variance σ^2. During propagation, 496 modes with numbers $n = 2p + |m| \leqslant 30$, where p and m are integers and $p \geqslant 0$.

As the initial data for such a model in the case of the Stokes component, we used the expansion of a Gaussian beam with a radius of 12 μm into spatial modes. For the pump wave, all modes initially had equal intensities and random phases. The calculations were performed for different realisations of random phases, and then the resulting output spatial distributions of the pump wave and Stokes component intensities were averaged. This model also made use of a filter that passed 4% of the power of the fundamental mode and 0.4% of the power of each of the other modes.

For a numerical solution, the system of equations was written in matrix form:

$$\partial A / \partial z = M_A A,$$

$$\partial B / \partial z = M_B B,$$

where A and B are vectors containing the amplitudes of all Stokes and pump modes, respectively, and the matrices M_A and M_B correspond to the right-hand side of the coupled mode model. The solution at each next step of integration can be found using the following numerical scheme:

$$A^{n+1} = \exp(h M_A^n) A^n,$$

$$B^{n+1} = \exp(h M_B^n) B^n,$$

where A^n and B^n are the solution at the nth step; and h is the integration step. To calculate the matrix exponent, it is proposed to use the Padé approximation [8, 9]:

$$e^X \approx \left[\sum_{j=0}^{n} \frac{(2n-j)!\,n!\,X^j}{2n!\,(n-j)!\,j!} \right]^{-1} \sum_{j=0}^{n} \frac{(2n-j)!\,n!\,(-X)^j}{2n!\,(n-j)!\,j!}.$$

In the calculations we used the expansion of the exponent up to the third term inclusive.

Figure 2 shows the input and output distributions of the intensities of the pump wave and the Stokes component, obtained using the coupled mode model for $\sigma^2 = 3 \times 10^{-7}$. One can see that in this case, due to the allowance for the random linear coupling of the modes and the Kerr nonlinearity during wave propagation, energy flows into the fundamental pump-wave mode, and as a result, only a small depression appears in its distribution profile. In addition, part of the energy of the Stokes component flows into high-

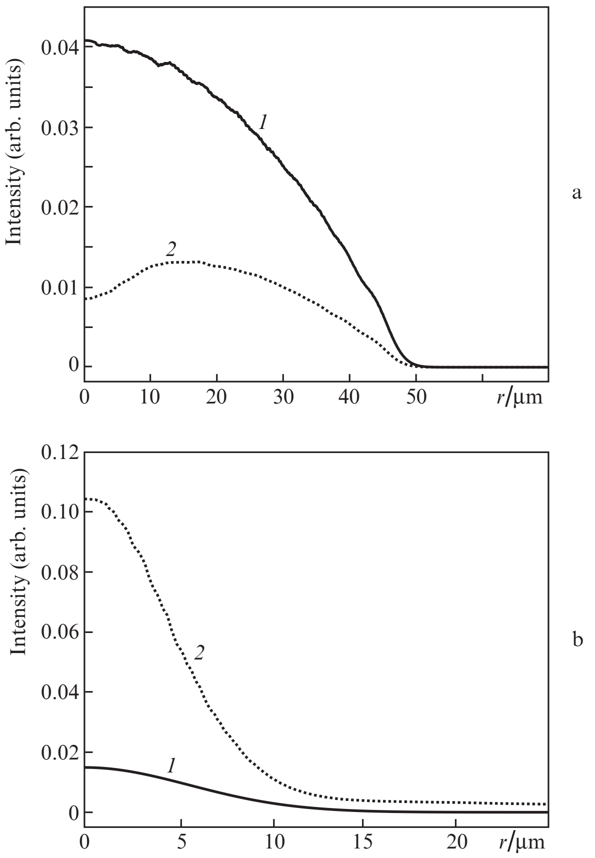

Figure 2. (*1*) Input and (*2*) output spatial distributions of the intensities of (a) the pump wave and (b) the Stokes component for the coupled mode model.

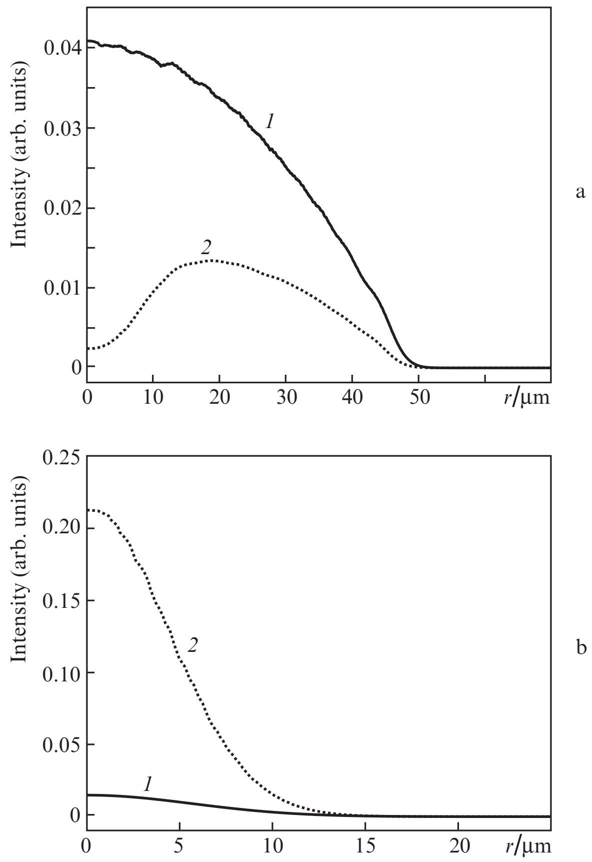

Figure 3. (*1*) Input and (*2*) output spatial distributions of the intensities of (a) the pump wave and (b) the Stokes component for the coupled mode model, which takes into account only the losses in the fibre and the SRS effect.

order modes, which leads to a wider spatial distribution profile at the output.

Figure 3 shows the input and output distributions of the intensities of the pump wave and the Stokes component, obtained using the coupled mode model, in which only the terms corresponding to the losses in the fibre and the SRS effect were retained.

In this case, a large depression is formed at the centre of the spatial distribution of the pump wave intensity; in contrast to the balance model (see Fig. 1), the boundaries of the dip are smoothed in accordance with the finite sizes of the modes. The output profile of the Stokes component intensity for this model has a Gaussian distribution and corresponds to the fundamental mode.

Thus, the obtained spatial intensity distributions are in qualitative agreement with the results of the model based on the balance equations; however, significant quantitative differences are observed that are important for describing the experiment.

It should also be noted that the random linear coupling of modes generally leads to an increase in the efficiency of amplification of the Stokes component due to the mixing of pump wave modes. However, at the same time, due to the linear coupling, part of the energy of the Stokes wave flows from the fundamental mode to higher-order modes. And since the filter used in the calculations passes only 0.4% of the power of high-order modes, this leads to the fact that the model of coupled modes, which takes into account their random linear coupling, demonstrates a lower intensity of the Stokes wave at the output from the fibre compared to the model containing only the terms, corresponding to SRS.

Thus, using numerical simulations, we have studied the propagation of a Stokes wave and a pump wave in a multimode parabolic-index fibre. A system of equations based on the coupled mode model is proposed that allows one to take into account stimulated Raman scattering, the Kerr effect, fibre losses, and random linear mode coupling. The simulation results show that taking these effects into account leads to significant differences between the intensity profiles of the pump waves and the Stokes component from those calculated on the basis of a rather crude model of the balance equations, which gives only a qualitative picture. The proposed model makes it possible to quantitatively compare the calculation results with the experimental results.

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