Quantum Electronics **50** (12) 1110 – 1114 (2020)

https://doi.org/10.1070/QEL17457

Nonlinear waves in a thin dielectric film on the surface of a topological insulator

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Abstract. **Based on the dispersion relation for a wave localised in a thin film of a nonlinear dielectric, which is located on the surface of a topological insulator, we have derived a system of equations that describes the propagation of a surface wave. It is shown that the longitudinal and transverse tangential components of the electric field vector are related due to the nonlinearity of the film and change periodically during propagation. It is found that the rotation period of this vector is determined by the axion charge of the topological dielectric and the nonlinear susceptibility of the thin film.**

Keywords: *thin film, topological insulator, dispersion relation, wave localisation, travelling waves, nonlinear waves.*

1. Introduction

A number of phenomena of nonlinear optics are being investigated in thin films, which represent a layer of a polarisable material with a thickness significantly less than the wavelength of electromagnetic radiation. The first works in this direction were devoted to the study of optical bistability [1 – 7] and related issues, such as dynamic chaos [8 – 10]. Many nonlinear optical processes occurring at the interface between media have been studied on the basis of the thin film model. We should mention here the investigations of coherent transient processes, in particular superradiance [11 – 14], photon echo [15], reflection and refraction of ultrashort pulses [1, 2, 6, 16 – 24], and parametric processes [25 – 28]. It was most often assumed that the nonlinear properties of a thin film are due to the presence of resonant (two-level or three-level) atoms in it. However, a film can exhibit the nonlinear properties, for example, in the case of ferroelectric media [29], semiconductors [30], polymers [23, 31], and metals [32, 33].

For a surface wave to exsits, the dielectric constant of one of the media must be negative. This condition is fulfilled for the metal – dielectric interface. Artificially created media, i.e. metamaterials [34, 35] and hyperbolic media [36, 37], have a negative dielectric constant. However, if a thin film of a polarisable material is located at the interface between two dielectrics with positive dielectric constants, then the displacement currents induced in it can also ensure the existence of a surface wave. Thus, the use of thin films expands the region of existence of surface waves.

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Received 9 October 2020
Kvantovaya Elektronika **50** (12) 1110 – 1114 (2020)
Translated by I.A. Ulitkin

It is known that when a wave passes through an interface between various topological insulators (with conventional dielectrics), the vectors of the magnetic and electric fields rotate [38 – 40]. For this reason, there are no separate surface TE or TM waves and the surface wave is hybrid, i.e. all components of its fields are nonzero. This is the peculiarity of surface waves at the dielectric – topological insulator interface.

In this work, we investigated the propagation of an electromagnetic wave localised in a thin film of a nonlinear dielectric located at the interface between a topological insulator and an ordinary dielectric. Because the film thickness is less than the radiation wavelength, a macroscopic description of the fields inside the film is impossible, and the presence of the film manifests itself through the conditions of continuity/discontinuity of the field components and inductions [1, 2, 41]. These conditions in relation to the considered situation are presented in Section 2. The nonlinear dispersion relation for the surface wave is obtained in Section 3. The truncated wave equations for the tangent components of the electric field slowly varying in space and time are derived in Section 4. The solution to the wave equations is presented in Section 5. It is shown that the tangential transverse and longitudinal components of the electric field envelope vary periodically in time and space so that the tangential vector of the electric field rotates in the interface plane with a certain frequency. The normal component of the electric field vector changes synchronously with the longitudinal component.

2. Role of surface currents at the interface

The propagation of an electromagnetic wave in a dielectric or in a topological insulator is described by the system of Maxwell's equations (there are no free charges and currents) [42]

$$\mathrm{rot}\boldsymbol{E} = -\frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t},\ \ \mathrm{div}\boldsymbol{B} = 0, \tag{1}$$

$$\mathrm{rot}\boldsymbol{H} = \frac{1}{c}\frac{\partial \boldsymbol{D}}{\partial t},\ \ \mathrm{div}\boldsymbol{D} = 0. \tag{2}$$

In this case, inductions and fields in a dielectric are related by the relations $\boldsymbol{H} = \boldsymbol{B}$ and $\boldsymbol{D} = \boldsymbol{E} + 4\pi\boldsymbol{P}$, and in a topological insulator , by the relations $\boldsymbol{H} = \boldsymbol{H}_{\mathrm{a}} \equiv \boldsymbol{B} - \alpha\theta\boldsymbol{E}$ and $\boldsymbol{D} = \boldsymbol{D}_{\mathrm{a}} \equiv \boldsymbol{E} + 4\pi\boldsymbol{P} + \alpha\theta\boldsymbol{B}$. Here α is the fine structure constant, and the parameter θ, called the axion charge [42], is zero in a conventional dielectric and $\pi(\mathrm{mod}2\pi)$ in a topological insulator [40]. When a wave passes from one medium to another, the field strengths and inductions change according to the conditions of continuity at the interface.

Let the coordinate axes be chosen so that the normal to the interface plane $\boldsymbol{n}$ is directed along the x axis, and the unit vectors $\boldsymbol{e}_z$ and $\boldsymbol{e}_y$ of the z and y axes lie in the interface plane, and the z axis is directed along the wave vector of the surface wave, while the y axis is perpendicular to it. Continuity conditions are derived from equations (1) and (2). If on both sides of the interface there are media with different values of θ, then the following conditions are satisfied [43, 44]:

$$(\boldsymbol{D}^{(1)} - \boldsymbol{D}^{(2)})\boldsymbol{n} = \alpha(\theta^{(2)} - \theta^{(1)})\boldsymbol{B}^{(1)}\boldsymbol{n},$$

$$(\boldsymbol{H}^{(1)} - \boldsymbol{H}^{(2)})\boldsymbol{e}_{z,y} = \alpha(\theta^{(1)} - \theta^{(2)})\boldsymbol{E}^{(1)}\boldsymbol{e}_{z,y}, \quad (3)$$

$$(\boldsymbol{B}^{(1)} - \boldsymbol{B}^{(2)})\boldsymbol{n} = 0, \ \ (\boldsymbol{E}^{(1)} - \boldsymbol{E}^{(2)})\boldsymbol{e}_{z,y} = 0.$$

Hereinafter, the superscript indicates the medium number. Expressions (3) mean that a current flows along the interface

$$\boldsymbol{j}_{\mathrm{a}} = \frac{c\alpha}{4\pi}(\theta^{(1)} - \theta^{(2)})\boldsymbol{E}^{(1)} \times \boldsymbol{n},$$

having a topological nature. Its direction is determined by the orientation of the interface and the difference in numbers $\theta^{(1,2)}$. As in the Hall effect, the current $\boldsymbol{j}_{\mathrm{a}}$ is perpendicular to the electric field.

If a thin (less than a wavelength) film of a substance is located at the interface, which is characterised by polarisation $\boldsymbol{P}_{\mathrm{f}}$, then continuity conditions (3) (their derivation at $\theta = 0$ and in the absence of a thin film can be found in [45], and when the thin film is taken into account, in [1–10]) are modified and take the form:

$$(\boldsymbol{D}^{(1)} - \boldsymbol{D}^{(2)})\boldsymbol{n} = \alpha(\theta^{(2)} - \theta^{(1)})\boldsymbol{B}^{(1)}\boldsymbol{n},$$

$$(\boldsymbol{B}^{(1)} - \boldsymbol{B}^{(2)})\boldsymbol{n} = 0, \ \ (\boldsymbol{E}^{(1)} - \boldsymbol{E}^{(2)})\boldsymbol{e}_{z,y} = 0,$$

$$(\boldsymbol{H}^{(1)} - \boldsymbol{H}^{(2)})\boldsymbol{e}_z = \alpha(\theta^{(1)} - \theta^{(2)})\boldsymbol{E}^{(1)}\boldsymbol{e}_z + \frac{4\pi}{c}\frac{\partial}{\partial t}\boldsymbol{P}_{\mathrm{f}}\boldsymbol{e}_y, \quad (4)$$

$$(\boldsymbol{H}^{(1)} - \boldsymbol{H}^{(2)})\boldsymbol{e}_y = \alpha(\theta^{(1)} - \theta^{(2)})\boldsymbol{E}^{(1)}\boldsymbol{e}_y - \frac{4\pi}{c}\frac{\partial}{\partial t}\boldsymbol{P}_{\mathrm{f}}\boldsymbol{e}_z.$$

According to (4), magnetic inductions experience a discontinuity determined by surface currents: topological current $\boldsymbol{j}_{\mathrm{a}}$ and displacement current in a thin film.

3. Fields outside the interface and the dispersion relation

In the chosen coordinate system, the strengths of all fields depend only on the variables x, z and time. In this case, for the components of the electric field E_y and the magnetic field H_y, we can write the wave equations, and express the remaining components in terms of them. Then the system of Maxwell's equations is reduced to the system of equations for the Fourier components of the field strengths:

$$\frac{\partial^2 E_y}{\partial z^2} + \frac{\partial^2 E_y}{\partial x^2} + k_0^2 D_y = 0, \ \ H_x = \frac{\mathrm{i}}{k_0}\frac{\partial E_y}{\partial z}, \ \ H_z = -\frac{\mathrm{i}}{k_0}\frac{\partial E_y}{\partial x},$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = \mathrm{i}k_0 H_y, \ \ \mathrm{i}k_0 D_x = \frac{\partial H_y}{\partial z}, \ \ \mathrm{i}k_0 D_z = -\frac{\partial H_y}{\partial x},$$

where $k_0 = \omega/c$. Since the media are homogeneous in the direction of the z axis, all fields and induction can be represented as $\boldsymbol{E}(x, z; \omega) = \boldsymbol{e}(x; \omega)\exp(\mathrm{i}\beta z)$, $\boldsymbol{H}(x, z; \omega) = \boldsymbol{h}(x; \omega)\exp(\mathrm{i}\beta z)$ and $\boldsymbol{D}(x, z; \omega) = \boldsymbol{d}(x; \omega)\exp(\mathrm{i}\beta z)$, where β is the propagation constant. This allows us to reduce the system of the above equations to two ordinary equations for the components $e_y(x)$ and $e_z(x)$. The rest of the components are expressed through them. Let the media be isotropic and the dielectric constant be $\varepsilon(\omega) = \varepsilon_1$ at $x < 0$ and $\varepsilon(\omega) = \varepsilon_2$ at $x > 0$.

Outside the thin film, the media are linear, and the equations for $e_y(x)$ and $e_z(x)$ have the form

$$\frac{\partial^2 e_{y,z}}{\partial x^2} - p^2 e_{y,z} = 0 \ \text{for} \ x > 0,$$

$$\frac{\partial^2 e_{y,z}}{\partial x^2} - q^2 e_{y,z} = 0 \ \text{for} \ x < 0,$$

where $p^2 = \beta^2 - k_0^2\varepsilon_2$ and $q^2 = \beta^2 - k_0^2\varepsilon_1$. Solutions for all fields in linear media that satisfy the boundary conditions (for surface waves, the fields vanish far from the interface) are written in the form:

$$e_x^{(1)}(x) = [\beta/(\mathrm{i}q)]B_1\exp(qx), \ \ e_y^{(1)}(x) = A_1\exp(qx),$$

$$e_z^{(1)}(x) = B_1\exp(qx),$$

$$h_x^{(1)}(x) = -(\beta/k_0)A_1\exp(qx), \ \ h_y^{(1)}(x) = [k_0\varepsilon_1/(\mathrm{i}q)]B_1\exp(qx),$$

$$h_z^{(1)}(x) = -(\mathrm{i}q/k_0)A_1\exp(qx)$$

for $x < 0$ and

$$e_x^{(2)}(x) = (\mathrm{i}\beta/p)B_2\exp(-px), \ \ e_y^{(2)}(x) = A_2\exp(-px),$$

$$e_z^{(2)}(x) = B_2\exp(-px),$$

$$h_x^{(2)}(x) = -(\beta/k_0)A_2\exp(-px), \ \ h_y^{(2)}(x) = (\mathrm{i}k_0\varepsilon_2/p)B_2\exp(-px),$$

$$h_z^{(2)}(x) = (\mathrm{i}p/k_0)A_2\exp(-px)$$

for $x > 0$.

The continuity conditions at $x = 0$ allow one to determine the amplitudes $A_{1,2}$ and $B_{1,2}$. In the case under consideration, this leads to the equations

$$A_1 = A_2, \ \ B_1 = B_2,$$

$$\left(\frac{\varepsilon_1}{q} + \frac{\varepsilon_2}{p}\right)B_1 = \mathrm{i}\frac{\kappa}{k_0}A_1 - 4\pi p_z, \quad (5)$$

$$(q + p)A_1 = \mathrm{i}\kappa k_0 B_1 + 4\pi k_0^2 p_y, \quad (6)$$

where $\kappa = \alpha\Delta\theta$; $\Delta\theta = \theta^{(1)} - \theta^{(2)}$; and p_z and p_y are the tangential components of the thin film polarisation.

For film polarisation, we employ the model used in [46, 47]:

$$p_{y,z} = \chi e_{y,z}(0) + \chi_3[|e_y(0)|^2 + |e_z(0)|^2]e_{y,z},$$

where $e_{y,z}(0)$ are the tangential components of the wave electric field inside the film, that is, at point $x = 0$; χ and χ_3 are linear and nonlinear third-order susceptibilities. Due to the continuity of the tangent components, we have $e_y(0) = A_1$ and $e_z(0) = B_1$. Consequently, $p_y = R(|A|^2,|B|^2)A$ and $p_z = R(|A|^2,|B|^2)B$, where $R(|A|^2,|B|^2) = \chi + \chi_3(|A|^2 + |B|^2)$. The indices for variables A and B are omitted hereinafter.

Taking into account the expressions for polarisation, equations (5) and (6) can be rewritten as

$$a_{11}A - a_{12}B = g_1 A,\ \ a_{21}A + a_{22}B = -g_2 B, \tag{7}$$

where the following notations are introduced

$$a_{11} = p + q - 4\pi k_0^2\chi,\ \ a_{12} = k_0\kappa,\ \ a_{22} = \varepsilon_1 p + 4\pi pq\chi,$$

$$a_{21} = (\kappa/k_0)pq,\ \ g_1 = 4\pi k_0^2\chi_3(|A|^2 + |B|^2),$$

$$g_2 = 4\pi pq\chi_3(|A|^2 + |B|^2).$$

For the case of a linear thin film ($\chi_3 = 0$) , Eqns (7) become a homogeneous system of linear equations, a nontrivial solution of which is possible if its determinant vanishes: $f(\omega,\beta) \equiv a_{11}a_{22} + a_{12}a_{21} = 0$. This is the dispersion relation for linear surface waves. In the nonlinear case, waves with amplitudes A and B are coupled due to the nonlinearity of the film polarisation. Equations (7) can be considered a nonlinear generalisation of the dispersion relation. To clearly determine the expression that defines the dispersion law of linear waves, we rewrite (7) in an equivalent form:

$$(a_{11}a_{22} + a_{12}a_{21})A = f(\omega,\beta)A$$
$$= R_{\rm nl}(|A|^2,|B|^2)(a_{22}k_0^2 A - a_{12}pqB), \tag{8}$$

$$(a_{11}a_{22} + a_{12}a_{21})B = f(\omega,\beta)B$$
$$= R_{\rm nl}(|A|^2,|B|^2)(a_{21}k_0^2 A + a_{11}pqB), \tag{9}$$

where $R_{\rm nl}(|A|^2,|B|^2) = 4\pi\chi_3(|A|^2 + |B|^2)$ reflects the nonlinear properties of a thin film in the Agranovich–Babichenko–Chernyak model [46, 47].

Below, we will consider the propagation of a quasi-harmonic wave, which can be represented as a wave with a slowly varying envelope.

4. Equations describing a surface quasi-harmonic wave

If we determine the normalised envelopes of the tangential components of the electric field of the surface wave inside a thin film ψ_1 and ψ_2 by the formulas

$$A\exp({\rm i}\beta z) = (k_0\kappa pq)^{1/2}\psi_1,\ \ B\exp({\rm i}\beta z) = (k_0\kappa pq)^{1/2}\psi_2,$$

then equations (8) and (9) can be written in the form:

$$f(\omega,\beta)\psi_1 = 4\pi\chi_3(|\psi_1|^2 + |\psi_2|^2)(\delta_1\psi_1 - \psi_2), \tag{10}$$

$$f(\omega,\beta)\psi_2 = -4\pi\chi_3(|\psi_1|^2 + |\psi_2|^2)(\delta_2\psi_2 + \psi_1). \tag{11}$$

Here we introduced the parameters

$$\delta_1 = \frac{k_0 a_{22}}{\kappa pq},\ \ \delta_2 = \frac{a_{11}}{\kappa k_0}.$$

Let the tangential components of the electric field of the surface wave be described by quasi-harmonic waves $\psi_{1,2}(z,t) = u_{1,2}(z,t)\exp({\rm i}\beta_{\rm c}z - {\rm i}\omega_{\rm c}t)$, where $\omega_{\rm c}$ is the carrier wave frequency, and $\beta_{\rm c}$ is its propagation constant related to frequency by the equation $f(\omega_{\rm c},\beta_{\rm c}) = 0$. Envelopes $u_{1,2}(z,t)$ are assumed to be slowly varying in space and time in comparison with changes in the carrier wave. Let the Fourier components of the quasi-harmonic wave satisfy the nonlinear dispersion relation $f(\omega,\beta)\psi_{1,2}(\omega,\beta) = h_{1,2}(\omega,\beta;|\psi_{1,2}|^2)$, in which the right-hand side is a small quantity. In the case when $h_{1,2} = 0$, the dispersion relation for the envelope follows from the dispersion relation for the quasi-harmonic wave: $f(\omega_{\rm c} + \omega, \beta_{\rm c} + \beta)u_{1,2}(\omega,\beta) = 0$. If $h_{1,2} \neq 0$, the problem can become very complex. There is a method for obtaining an approximate dispersion relation for $u_{1,2}(z,t)$ [48, 49]. If $h_{1,2}$ is considered as a small perturbation, then in the first order of smallness we have

$$f(\omega_{\rm c} + \omega, \beta_{\rm c} + \beta)u_{1,2}(\omega,\beta) = h_{1,2}(\omega_{\rm c} + \omega, \beta_{\rm c} + \beta; |u_{1,2}|^2),$$

or

$$f(\omega_{\rm c} + \omega, \beta_{\rm c} + \beta)u_{1,2}(\omega,\beta) = h_{1,2}(\omega_{\rm c}, \beta_{\rm c}; |u_{1,2}|^2). \tag{12}$$

For a quasi-harmonic wave, the functions $u_{1,2}(\omega,\beta)$ are nonzero in the region where $\omega \ll \omega_{\rm c}$ and $k \ll k_0$. Therefore, we can expand $f(\omega_{\rm c} + \omega, \beta_{\rm c} + \beta)$ in a Taylor series and restrict ourselves to the first (or first and second, if we take into account second-order dispersions) order of derivatives. Since $f(\omega_{\rm c},\beta_{\rm c}) = 0$, expression (12) is written as

$$\left(\frac{\partial f}{\partial\omega}\omega + \frac{\partial f}{\partial\beta}\beta + \dots\right)u_{1,2}(\omega,\beta) = h_{1,2}(\omega_{\rm c},\beta_{\rm c};|u_{1,2}|^2).$$

The derivatives in brackets are calculated at point ($\beta_{\rm c}$, $\omega_{\rm c}$), and the ellipsis denote the higher orders of the expansion in β and ω. Since β and ω in the vicinity of point ($\beta_{\rm c}$, $\omega_{\rm c}$) are related by the dispersion relation $f(\omega,\beta) = 0$, we can use the theorem on the differentiation of an implicit function, which yields $\partial f/\partial\beta = -({\rm d}\omega/{\rm d}\beta)\partial f/\partial\omega$, and determine the group velocity $v_{\rm g} = (\partial\omega/\partial\beta)$ [48, 49]. The substitutions $\omega \to {\rm i}\partial/\partial t$, $\beta \to -{\rm i}\partial/\partial z$ and $u_{1,2}(\omega,\beta) \to u_{1,2}(z,t)$ lead to an evolutionary equation for slowly varying envelopes. By applying this procedure to the system of equations (10) and (11), one can obtain a system of equations describing the propagation of a surface wave in a nonlinear thin film on the surface of a topological insulator:

$${\rm i}\left(\frac{\partial}{\partial t} + v_{\rm g}\frac{\partial}{\partial z}\right)u_1 = \mu(|u_1|^2 + |u_2|^2)(\delta_1 u_1 - u_2), \tag{13}$$

$${\rm i}\left(\frac{\partial}{\partial t} + v_{\rm g}\frac{\partial}{\partial z}\right)u_2 = -\mu(|u_1|^2 + |u_2|^2)(\delta_2 u_2 + u_1), \tag{14}$$

where $\mu = 4\pi\chi_3(\partial f/\partial\omega)^{-1}$ and derivatives are calculated for $\omega = \omega_{\rm c}$. Equations (13) and (14) describe the change in the envelopes of the tangent components of the electric fields of the surface wave without taking into account the dispersion of group velocities and in this sense can be called truncated (reduced) wave equations. Similar equations were obtained when considering surface waves at the interface between a nonlinear dielectric and a topological insulator [50]. The differences are in the form of the right-hand sides of equations (13) and (14) and are caused by a different type of nonlinearity in the problem considered in [50].

5. Stationary nonlinear surface wave

Since the group velocity dispersion is absent, one can use the derivative along the characteristic

$$\frac{\partial}{\partial \zeta} = \left(\frac{\partial}{\partial t} + v_{\rm g}\frac{\partial}{\partial z}\right)$$

and pass from (13) and (14) to a system of ordinary differential equations. It is also useful to pass to real variables $u_1(z, t) = a\exp(i\varphi_1)$, $u_2(z, t) = b\exp(i\varphi_2)$ and rewrite the resulting system of equations in real form:

$$\frac{\partial a}{\partial \zeta} = -\mu(a^2 + b^2)b\sin\Phi,\ \frac{\partial b}{\partial \zeta} = \mu(a^2 + b^2)a\sin\Phi, \qquad (15)$$

$$\frac{\partial \Phi}{\partial \zeta} = \mu(a^2 + b^2)\left[\delta + \left(\frac{a}{b} - \frac{b}{a}\right)\cos\Phi\right], \qquad (16)$$

where the phase difference $\Phi = \varphi_2 - \varphi_1$ and the parameter $\delta = \delta_1 + \delta_2$ are introduced. Equations (15) and (16) resemble the equations arising in the description of the parametric interaction of waves. For this reason, it can be expected that the transformation of the transverse tangent component of the electric field $e_y(z; t)$ into the longitudinal component $e_z(z; t)$ and vice versa will be periodic.

The system of equations (15) and (16) has two integrals of motion:

$$a^2 + b^2 = a_0^2,\ ab\cos\Phi - \frac{\delta}{2}a^2 = I_0.$$

If we exclude b from these expressions, then there remains one equation,

$$\frac{\partial w}{\partial \zeta} = -\sqrt{1 - w^2}\sin\Phi, \qquad (17)$$

$$2w\sqrt{1 - w^2}\cos\Phi - \delta w^2 = J_0 = I_0/a_0^2,$$

for the variable $w = a/a_0$, which depends on $\xi = \mu a_0^2\zeta$. The constant J_0 is determined by the initial conditions. In the general case, the solution to equation (17) is expressed in terms of elliptic functions and is cumbersome. However, if we assume that at the initial moment the electric field had no transverse component, i.e. $e_y(z; 0) = 0$, then at subsequent times, the transverse component will change according to the formula

$$a(\zeta) = a_0\sqrt{\frac{4}{4 + \delta^2}}\sin\left(\mu a_0^2\sqrt{\frac{4 + \delta^2}{4}}\zeta\right). \qquad (18)$$

The phase difference Φ is determined by the ratio

$$\cos^2\Phi = \frac{\delta^2\sin^2\phi}{\delta^2 + 4\cos^2\phi},\ \phi = \mu a_0^2\sqrt{\frac{4 + \delta^2}{4}}\zeta.$$

It follows from the expression for the first integral of motion that the envelope of the longitudinal field component $e_z(z; t)$ has the form

$$b(\zeta) = a_0\sqrt{\frac{4}{4 + \delta^2}}\cos\left(\mu a_0^2\sqrt{\frac{4 + \delta^2}{4}}\zeta\right). \qquad (19)$$

It can be seen from (18) and (19) that the components $e_y(z; t)$ and $e_z(z; t)$ change harmonically with a shift by $\pi/2$. This means that the nonlinear properties of a thin film manifest themselves during the rotation of the tangential vector of the electric field in the plane of the dielectric–topological insulator interface.

From (18), it is possible to determine the period T of oscillations (changes in space and time) of the electric field envelope $e_y(z; t)$ or $e_z(z; t)$. If the measured quantity is the squared envelope (i.e. the intensity of the wave), then

$$\mu a_0^2\sqrt{\frac{4 + \delta^2}{4}}T = \pi. \qquad (20)$$

Since $\kappa \approx 10^{-2}$, the parameter

$$\delta = \frac{1}{\kappa}\left(\frac{a_{11}}{k_0} + \frac{k_0 a_{22}}{pq}\right) \approx 10^2.$$

Consequently, the expression under the root in (20) is much greater than unity. This allows one to find an estimate for the period T:

$$T \approx \frac{\kappa}{a_0^2\chi_3}.$$

Thus, the oscillation period can vary as a function of the surface wave intensity.

6. Conclusions

We have considered a surface wave propagating along the interface between a topological insulator and a conventional dielectric. A thin film of nonlinear dielectric is located at the interface, due to which the electromagnetic wave is localised at this interface. In the absence of a film, a surface wave can exist only if the signs of the dielectric constants of the media are opposite or the dielectric is nonlinear. The magnetoelectric effect inherent in a topological insulator leads to mixing (hybridisation) of transverse electric (TE) and transverse magnetic (TM) waves, and the nonlinearity of the film leads to their interaction. As a result, the tangential vector of the electric field of the surface wave rotates in the plane of separation of the media during wave propagation. In this case, the normal component of the electric field oscillates with the same frequency. By varying the intensity of the surface wave, the oscillation period can be changed.

To take into account the nonlinear properties of a thin film, a simple model was used [46, 47], which allows the processes occurring in the film to be described qualitatively. Of fundamental importance is the magnetoelectric effect that is typical of a topological insulator.

Acknowledgements. The work was supported by the Russian Foundation for Basic Research (Grant No. 18-02-00921).

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