

Influence of saturable absorber saturation power, modulation depth and relaxation time on pulse parameters of a soliton fibre laser

A.A. Mastin, P.A. Ryabochkina

Abstract. The influence of the saturable absorber saturation power, modulation depth and relaxation time on the energy and shape of fibre soliton laser pulses has been studied by numerical simulation. The results suggest that the use of a saturable absorber with a lower saturation power in a fibre laser ensures pulsed lasing onset at a lower gain. The fibre laser pulse energy decreases as the saturation power of the absorber or the relaxation time of its open state increases at a constant gain coefficient. In addition, increasing the absorber saturation modulation depth at a constant gain coefficient has been shown to reduce the pulse peak power and energy.

Keywords: fibre laser, soliton energy, saturable absorber, time–bandwidth product.

1. Introduction

One advantage of fibre soliton lasers is that they allow sub-femtosecond pulses to be generated directly in the laser cavity, without external compression. Ultrashort pulses can be generated in passively mode-locked fibre lasers using saturable absorbers, such as SESAM structures [1–4] and films of carbon materials [5–7]. The effect of saturable absorbers reduces to introducing power-dependent loss into the laser cavity and modulating the laser power profile.

The peak power and energy of fibre laser pulses are determined by a combined effect of all laser cavity components. In a fibre laser operating in the soliton regime, its saturable absorber is responsible for pulse generation and maintains the pulse shape. The group velocity dispersion of a laser pulse and nonlinear effects in the laser cavity determine the pulse shape and limit the maximum pulse energy [8].

Saturable absorbers that are used for the mode locking of lasers can be characterised by the saturation power, modulation depth and relaxation time of the open state. The relaxation time of the open state of such an absorber determines how rapidly the loss introduced by it is restored after a laser pulse passes. As shown experimentally by Haus [9], the duration of a laser pulse obtained using saturable absorber mode locking can be considerably shorter than the relaxation time of the absorber's open state. Subsequently, this effect has

been the subject of extensive research. The key simulation studies were reported in Refs [9–13]. The theoretical results obtained in those studies are consistent with models of solid-state lasers within various approximations of real devices. Haus [9] considered an absorber whose relaxation time was infinitely long compared to the pulse duration. For weak absorber saturation, he obtained an analytical expression for the pulse profile. In the model he used, nonlinear and dispersion effects in the medium were left out of account.

Results on the effect of a weakly saturated absorber with a relaxation time considerably shorter than the pulse duration, studied by simulation using the Ginzburg–Landau equation, were presented in Refs [11–13]. The model thus obtained for a fibre laser exemplifies the use of the distributed approach in simulation. The basic principle of this approach is that all key physical effects responsible for optical pulse generation in mode-locked lasers are taken into account in a single equation. This approach to simulation reduces the computation time, which allows laser structure optimisation issues to be resolved rapidly and effectively. At the same time, the influence of particular parameters of laser cavity components can only be studied by solving the complex nonlinear Schrödinger equation. This equation is used to simulate pulse dynamics in passive and active optical fibres and to construct models for describing pulse propagation through discrete elements, such as a saturable absorber and coupler [14].

Paschotta and Keller [15] derived approximate relations for the pulse duration, identified mechanisms underlying the formation of pulses shorter than the relaxation time of the absorber and described pulse destabilisation mechanisms. The nonlinearity and dispersion of the medium were shown to influence pulse stability.

Studies of the role of the absorber relaxation time [9–13] employed a linearised representation of absorber opening dynamics, obtained in the case of weak absorber saturation. In such a case, nonlinear and dispersion effects in the laser cavity are left out of account. To obtain pulse stability criteria, one uses the concept of a distributed effect of laser cavity components. The results obtained in Refs. [9–13] describe pulse dynamics in solid-state lasers and are not fully applicable to fibre lasers, where nonlinear and dispersion effects have a significant effect on pulse stability, which shows up e.g. as the formation of a pedestal in the pulse profile [16].

Results obtained in experimental studies of how the finite nature of the relaxation time of an absorber's open state influences pulse generation conditions in fibre lasers were presented in Refs [17–19]. As shown in Refs [16–18] an absorber with a low saturation power ensures a stable onset of pulsed laser operation at low levels of pumping, whereas an absorber

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with a high saturation power allows lasing conditions to be stabilised. In a previous study [16], numerical simulation was used to assess the effect of the saturable loss relaxation time in a nonlinear absorber on the soliton pulse structure in fibre lasers differing in cavity length.

Summarising results obtained by different researchers on the soliton regime in fibre lasers, it is worth noting that the effect of the relaxation time of the open state of absorbers with different saturation powers on onset conditions for pulsed lasing and the feasibility of raising the maximum laser pulse energy has not yet been studied in sufficient detail.

The purpose of this work is to study, using numerical simulation, the influence of the saturation power, modulation depth and relaxation time of the open state of a saturable absorber on the pulse peak power and energy in a soliton fibre laser. Results of such studies are necessary in designing saturable absorbers with tailored parameters for improving the efficiency of pulsed fibre lasers.

2. Description of the numerical model

Using numerical simulation, we studied the influence of the saturation power, relaxation time and modulation depth of a saturable absorber on the launch and termination of single-pulse round trip operation.

In the general case, the dynamics of a pulse in active fibre of a laser cavity can be described by the nonlinear Schrödinger equation in the frequency domain [8]:

$$\frac{dU(z, \omega)}{dz} = i \sum_{n=2}^3 \beta_n \omega^n U(z, \omega) + i\gamma G(z, \omega) + \left(1 - \frac{\omega^2}{\Omega^2}\right) g(z) U(z, \omega), \quad (1)$$

where ω is the frequency shift between the spectral component of the laser pulse and the pulse carrier frequency ω_0 ; z is a coordinate along the fibre laser cavity axis;

$$U(z, \omega) = \int_{-\infty}^{\infty} \tilde{U}(z, t) \exp(-i\omega t) dt$$

is the spectral amplitude of the pulse field; $\tilde{U}(z, t)$ is the slowly varying pulse field amplitude (profile);

$$G(z, \omega) = \int_{-\infty}^{\infty} |\tilde{U}(z, t)|^2 \tilde{U}(z, t) \exp(-i\omega t) dt$$

is the Fourier transform of the self-phase modulation function of the pulse field; β_n is the group velocity dispersion coefficient of the pulse; γ is the Kerr nonlinearity coefficient of the medium; and Ω is a parabolic gain line parameter of the laser medium.

Because of the group velocity dispersion and self-phase modulation, the pulse frequency varies along the pulse profile. The frequency (wavelength) distribution along the pulse profile (chirp) can be found from the relation

$$\omega_{\text{ch}}(z, t) = \frac{\partial}{\partial t} \left(\arctan \frac{\text{Im } \tilde{U}(z, t)}{\text{Re } \tilde{U}(z, t)} \right). \quad (2)$$

The wavelength of the pulse along its profile (chirp) at time t is given by

$$\lambda_{\text{ch}}(z, t) = \frac{2\pi c}{\omega_{\text{ch}}(z, t) + \omega_0}, \quad (3)$$

where c is the speed of light.

The gain coefficient of the laser medium is

$$g(z) = g_0 \left(1 + \frac{E(z)}{P_g T_R} \right)^{-1}, \quad (4)$$

where g_0 is the small-signal gain coefficient; P_g is the saturation power of the active medium; T_R is the cavity round-trip time;

$$E(z) = \int P_A(z, t) dt \quad (5)$$

is the pulse energy; and

$$P_A(z, t) = |\tilde{U}(z, t)|^2 \quad (6)$$

is the pulse power profile.

The spectral pulse intensity has the form

$$P_\omega(z, t) = |U(z, t)|^2, \quad (7)$$

and the peak power is given by

$$U_p(z) = \max(P_A(z, t)). \quad (8)$$

Saturable loss dynamics in a passive nonlinear absorber can be described by the equation [8, 14]

$$\frac{\partial q(t)}{\partial t} = -\frac{q(t) - q_0}{\tau_a} - \frac{q(t) |\tilde{U}(z, t)|^2}{\tau_a P_a}, \quad (9)$$

where $q(t)$ is the absorption coefficient of the absorber at the signal wavelength; q_0 is the small-signal absorption coefficient; τ_a is the relaxation time of the saturated state of the absorber; and P_a is the loss saturation power of the absorber.

Diels and Rudolph [20] calculated the time–bandwidth product (the product of the spectral bandwidth and pulse duration). The bandwidth and pulse duration corresponded to the full width at half maximum (FWHM) of the $P_\omega(z, \omega)$ and $P_A(z, t)$ profiles. For Gaussian and sech pulses, the time–bandwidth product is 0.441 and 0.315, respectively [20].

In this work, pulsed lasing conditions were studied for a fibre ring laser schematised in Fig. 1. It comprises a multiplexer (1) for launching pump light, active fibre (2) of length L_1 , power divider (3), passive nonlinear absorber (4) and passive fibre (5) of length L_2 . Simulation was performed for a fibre laser which contained Tm³⁺-doped fibre as a gain medium. Frequency-filtering properties of cavity components were left out of account in the simulation. We assumed that a pulse in the laser cavity propagated clockwise and that 60% of its energy was outcoupled from the cavity by the power divider.

We simulated fibre lasers having saturable absorbers with the following parameters:

- (1) high saturation power ($P_a = 50$ W) and infinitely short relaxation time ($\tau_a = 0$);
- (2) high saturation power ($P_a = 50$ W) and finite relaxation time ($\tau_a = 5$ ps);
- (3) low saturation power ($P_a = 5$ W) and infinitely short relaxation time ($\tau_a = 0$);

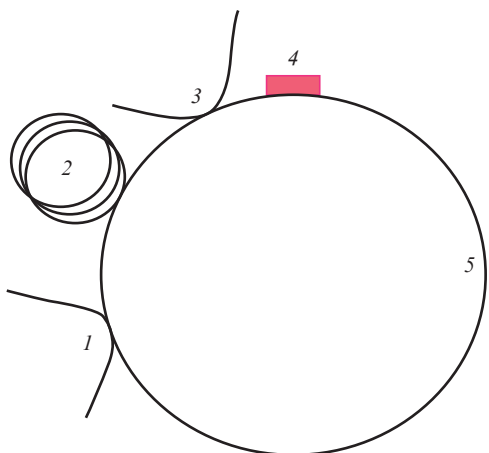


Figure 1. Schematic of the fibre ring laser for which the soliton pulse structure was simulated.

(4) low saturation power ($P_a = 5$ W) and finite relaxation time ($\tau_a = 10$ ps).

Hereafter, the fibre lasers with these parameters will be referred to as lasers 1, 2, 3 and 4, respectively. The other parameters of the cavities of lasers 1–4 were identical and are indicated below:

Pulse group velocity dispersion	
in the passive fibre, $\beta_2/\text{ps}^2 \text{ m}^{-1}$	-0.0695
Third-order group velocity dispersion	
in the active fibre, $\beta_3/\text{ps}^3 \text{ m}^{-1}$	0.0003
Pulse group velocity dispersion	
in the active fibre, $\beta_2/\text{ps}^2 \text{ m}^{-1}$	-0.012
Nonlinearity coefficient	
of the fibre, $\gamma/\text{W}^{-1} \text{ m}^{-1}$	0.0012
Saturation power of the gain	
medium, P_g/W02
Length of the active fibre, L_1/m	0.3
Length of the passive fibre, L_2/m	1
Gain line parameter of the laser	
medium [20], Ω/ps^{-1}36
Percent of the light outcoupled	
by the power divider (%)60

The above nonlinearity and dispersion values correspond to those of SMF 28 [21]. At $\Omega = 36 \text{ ps}^{-1}$, the width of the gain band peaking at $\lambda = 1930 \text{ nm}$ is 100 nm [21]. The gain profile of the laser medium was taken to be parabolic [8, 9, 12–14, 18, 19].

To find the fibre laser pulse structure, Eqn (1) was solved numerically in the MATLAB environment using the ode45 solver. The solution was sought during multiple pulse passes through the cavity. The optimal solution was chosen under the constraint of pulse profile stabilisation. As a rule, 100–150 passes of the pulse through the cavity were needed to obtain an optimal solution.

3. Simulation results and discussion

Figure 2 shows the pulse peak power and energy as functions of the gain coefficient of the active fibre, g_0 , for lasers 1–4 in the single-pulse round trip regime at an absorber modulation depth $q_0 = 0.1$. The symbols on the curves indicate the highest pulse

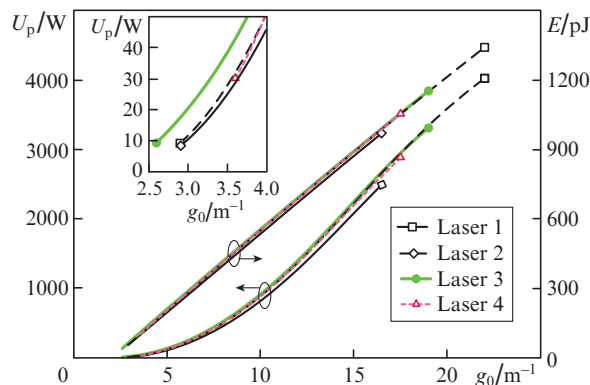


Figure 2. (Colour online) Pulse peak power U_p and pulse energy E as functions of the gain coefficient g_0 for the fibre lasers 1–4. The symbols on the curves indicate the highest values of U_p and E , corresponding to the termination of single-pulse round trip operation. Inset: initial portions of the $U_p(g_0)$ curves.

energy and peak power at the highest gain where single-pulse round trip operation persists. Further raising the gain leads to the termination of single-pulse round trip operation and generation of a second soliton for the cavity round-trip time.

It follows from Fig. 2 that the highest pulse peak power and energy are reached in the fibre laser 1, where the saturable absorber has the highest saturation power and the shortest saturated loss relaxation time. Increasing the relaxation time of the open state of the saturable absorber in laser 2 leads to a reduction in the highest achievable pulse peak power and energy, as does reducing the absorber saturation power in laser 3.

The inset in Fig. 2 shows initial portions of the pulse peak power vs active fibre gain coefficient curves of lasers 1–4. It is seen that the smallest gain coefficient needed for pulsed lasing onset is offered by laser 3, in which the saturable absorber has a lower saturation power than do the absorbers in lasers 1 and 2 and the shortest relaxation time of the saturated state. Increasing the relaxation time of the saturated state of the absorber in this laser led to an appreciable increase in the smallest gain coefficient needed for pulsed lasing onset.

The mechanism underlying the termination of single-pulse round trip operation with increasing gain coefficient can be understood using the data presented in Fig. 3, which shows semilog plots of the pulse power P_A (against time t) and pulse intensity P_ω (against wavelength λ) at the laser cavity output at the highest pulse energy of laser 4 (see Fig. 2).

It is seen from Fig. 3a that the pulse power profile comprises a central peak and a broad pedestal. The chirp wavelengths in the power profile correspond to the wavelengths of the Kelly peaks in the spectrum of the pulse [16]. The termination of single-pulse round trip operation is caused by instabilities in the power profile, which occur immediately behind the central peak and are due to the finite saturable loss relaxation time of the absorber, in agreement with previously reported results [12, 15]. The growth of instabilities causes an additional peak to emerge behind the central peak in the pulse profile. As a consequence of the gain dynamics in the active medium, the additional peak lags the central peak of the pulse by half the cavity round-trip time [15]. Thus, the mechanism underlying the termination of single-shot lasing is associated with the growth of instabilities behind the central peak in the pulse profile.

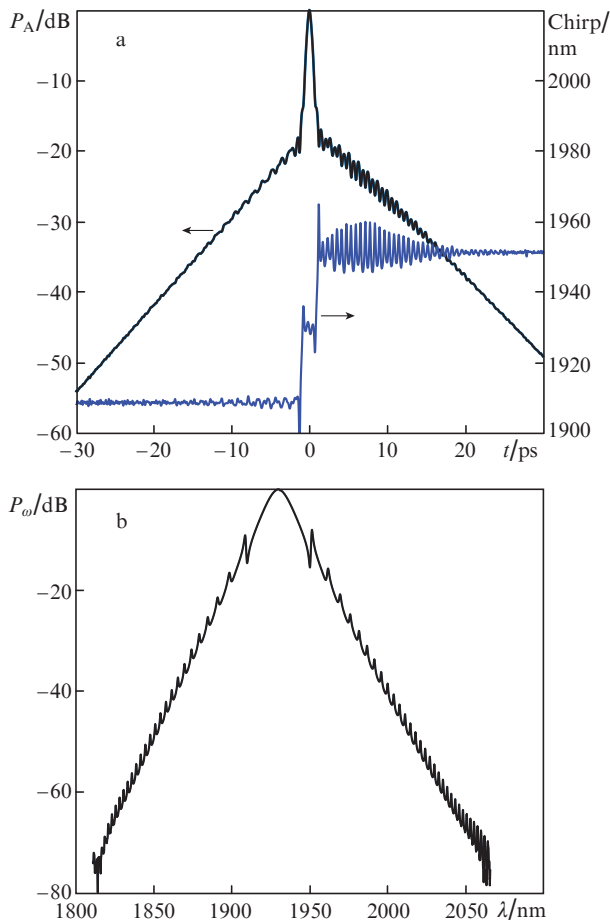


Figure 3. (Colour online) (a) Pulse power profile, pulse chirp and (b) pulse spectrum for laser 4 at the maximum pulse energy.

The finite loss relaxation time of the absorber causes the pulse spectrum to shift to longer wavelengths [16]. The shift is the result of the decrease in the loss introduced by the absorber into the intensity profile behind the central peak of the pulse as the loss relaxation time increases. In the case of a soliton pulse, this part of the profile is chirped with longer wavelengths than is the central peak (Fig. 3). Thus, the longer wavelength components of the pulse become amplified and the spectrum shifts to the longer wavelength part of the gain band. The shift accounts for the fact that the gain should be increased to initiate stable pulsed operation of laser 3. After the onset of pulsed lasing, the pulse spectrum has a small bandwidth because of the low pulse peak power. Under such conditions, the shift of the laser pulse spectrum to longer wavelengths, to the edge of the gain band of the active medium, leads to a decrease in pulse amplification efficiency and growth of instabilities in the centre of the gain band. This entails destabilisation of lasing conditions. To ensure conditions for stable lasing, it is necessary to raise the gain in order to increase the pulse peak power and extend the spectrum. A pulse with a broader spectrum, shifted to longer wavelengths, will be sufficiently amplified for maintaining stable lasing conditions.

Figure 4 shows the spectral width and FWHM as functions of gain coefficient for pulses of lasers 1–4. It follows from these data that increasing the gain coefficient leads to a decrease in pulse duration and increase in spectral width.

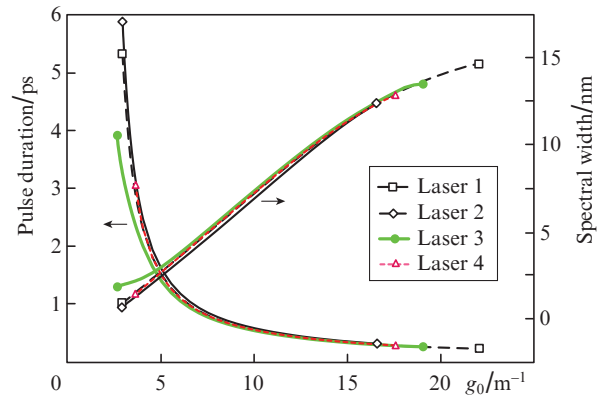


Figure 4. (Colour online) Spectral width and pulse duration as functions of gain coefficient for the fibre lasers 1–4.

Figure 5 shows the time–bandwidth product as a function of g_0 for lasers 1–4. It is seen that the curves have a local maximum. It also follows from Fig. 5 that the pulse profile of the fibre lasers 1–4 can be approximated by the sech function, for which the time–bandwidth product is 0.315.

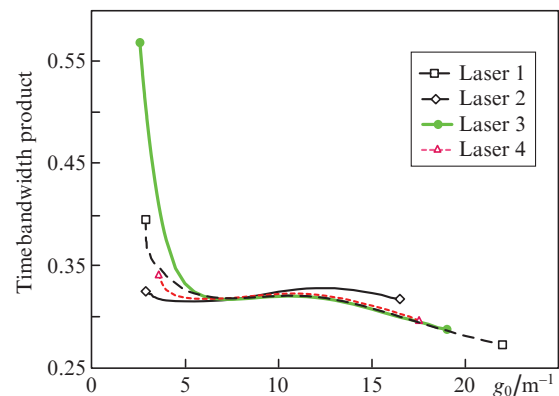


Figure 5. Time–bandwidth product as a function of gain coefficient for the fibre lasers 1–4.

Figure 6 shows the pulse peak power and energy as functions of the relaxation time of the open state of the absorbers for the fibre lasers at saturation powers of 50 and 5 W and a constant gain coefficient $g_0 = 14 \text{ m}^{-1}$. For each value of the saturation power of the absorbers, the corresponding curves are limited by the maximum relaxation time of its open state at which single-pulse round trip operation persists. At a longer absorber relaxation time, pulses are no longer stable, and a second soliton emerges after by the end of the cavity round trip time.

It is seen from Fig. 6 that a decrease in absorber saturation power at a constant gain coefficient leads to an increase in pulse peak power and energy. With increasing relaxation time, the pulse energy decreases, in agreement with results obtained by us previously [16]. Moreover, if the relaxation time of the open state of the saturable absorber exceeds a certain value, the pulse energy reaches a maximum at a given gain and single-pulse round trip operation ceases. Note also that the termination of lasing for a laser having an absorber with a high saturation power occurs at shorter relaxation times.

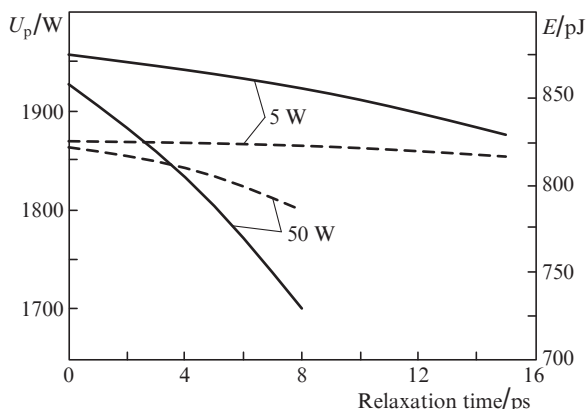


Figure 6. Pulse peak power (solid lines) and energy (dashed lines) as functions of the relaxation time of the open state of the absorbers for lasers 1–4 at saturation powers of 50 and 5 W and $g_0 = 14 \text{ m}^{-1}$.

Figure 7 presents simulation results illustrating the effect of the absorber modulation depth on the laser pulse energy and peak power at a constant g_0 of 10 m^{-1} . It is seen that, at a constant gain coefficient, the pulse peak power and energy decrease with increasing modulation depth. The initial modulation depths correspond to pulsed lasing onset at $g_0 = 10 \text{ m}^{-1}$. It follows from comparison of the data for lasers 1 and 3 with those for lasers 2 and 4 that increasing the relaxation time of the open state of the saturable absorber leads to an increase in the minimum modulation depth corresponding to pulsed lasing onset.

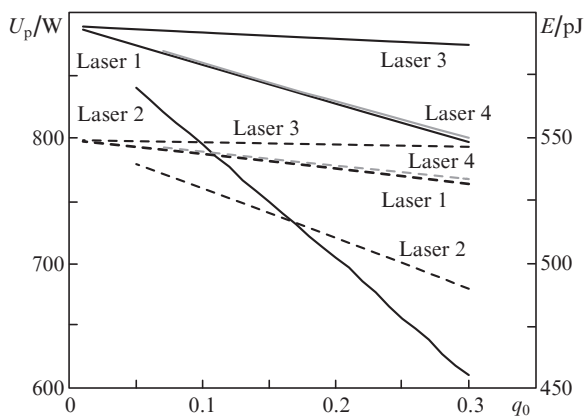


Figure 7. Pulse peak power (solid lines) and energy (dashed lines) as functions of the absorber modulation depth for lasers 1–4 at $g_0 = 10 \text{ m}^{-1}$.

Figure 8 shows the maximum pulse peak power and energy as functions of the relaxation time of the open state of the absorber at saturation powers of 50 and 5 W. It is seen that, at an infinitely short relaxation time of the open state of the absorber, the laser having the absorber with the high saturation power has a high pulse energy. As the relaxation time of the open state of the absorber increases, the maximum pulse energy in the laser having the absorber with the high saturation power decreases more rapidly than that in the laser whose cavity has the absorber with the lower saturation power. As a result, the maximum possible pulse energy in the

laser having the absorber with the lower saturation power and a particular relaxation time of the open state exceeds the maximum possible pulse energy in the laser having the absorber with the higher saturation power and the same relaxation time of the open state. Thus, starting at a certain relaxation time of the open state, which corresponds to the intersection of the curves in Fig. 8, the absorber with the lower saturation power allows a higher pulse energy to be reached than does the absorber with the higher saturation power.

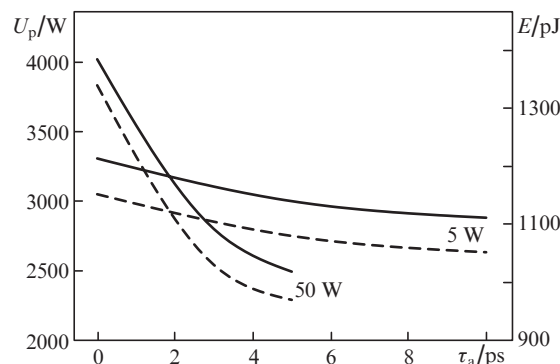


Figure 8. Maximum pulse peak power (solid lines) and maximum pulse energy (dashed lines) as functions of the relaxation time of the open state of the saturable absorber at saturation powers of 50 and 5 W.

4. Conclusions

The efficiency of using a saturable absorber with a high saturation power for maximising the fibre laser pulse energy has been shown by numerical simulation to increase as the relaxation time of its open state decreases. Increasing the relaxation time of the open state of the saturable absorber at a constant gain coefficient leads to a decrease in the maximum achievable pulse energy.

Reducing the saturation power and loss relaxation time of the absorber allows for pulsed lasing onset at a smaller gain coefficient. Increasing the absorber saturation modulation depth at a constant gain coefficient leads to a decrease in laser pulse peak power and energy.

The present results suggest that, for effective initiation of pulsed fibre laser operation, it is necessary to use a combined saturable absorber comprising an absorber with a low saturation power and the minimum possible relaxation time, which allows for pulsed lasing onset at the minimum possible gain, and an absorber with a high saturation power and the minimum possible relaxation time of the open state – in order to raise the maximum pulse energy.

To ensure stable single-pulse round trip operation at a particular pulse energy, it is reasonable to use a combination of saturable absorbers with characteristic parameters.

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