Quantum Electronics **50** (4) 361 – 365 (2020)

https://doi.org/10.1070/QEL17280

Self-compression of laser pulses in a discrete medium

A.O. Sofonov, V.A. Mironov

Abstract. **Using the discrete nonlinear Schrödinger equation, we study the laser radiation self-action dynamics in a nanostructured waveguide system. It is shown that for a laser pulse energy exceeding the critical value, the nonlinear evolution of the wave field markedly differs from the corresponding process in a continuous medium. While the pulse propagates in a discrete medium, its length decreases to values comparable to the scale of the structure and the velocity decreases to zero. The system parameters are determined at which the wave field after the laser pulse termination begins to propagate in the opposite direction in a compressed form. The process of slowing down and stopping the pulse is accompanied by strong radiation losses, so that about a third of the energy of the initial field distribution remains in the final compressed state.**

Keywords: *self-compression, nanostructured waveguide medium, laser pulses.*

1. Introduction

One of the main tasks of laser physics is the elaboration of methods for increasing the intensity of electromagnetic radiation. At first, success in this direction was largely due to the development of active laser elements and, then, due to technological progress, with approaches related to pulse compression turning out to be more promising [1, 2]. In this case, wide use was made of the approaches based on the spectral and temporal transformation of pulses during their propagation through a nonlinear media, which was facilitated, especially recently, by the development of fibre technology and nanooptics. Optical fibres are widely used as nonlinear media. They ensure high nonlinearity and make it possible to control dispersion due to the waveguide contribution (spatial boundedness of the system). With the development of nanotechnology, progress in this direction has continued. A technique is being developed for fabricating waveguide systems from a set of microresonators [3, 4]. A review of modern methods for the synthesis of plasma nanoparticles is presented in book [4]. Peculiarities of optical microresonators are considered in [3]. Various artificial nanostructures are produced to use their unique optical properties.

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Received 17 February 2020
Kvantovaya Elektronika **50** (4) 361 – 365 (2020)
Translated by I.A. Ulitkin

Periodic systems of coupled nanophotonic structures are a special class of systems in which wave excitations of new types propagate. The peculiarities of their theoretical description based on the model of discrete systems are considered, for example, in [5]. In particular, within the framework of this model, the possibility of using a waveguide system of microresonators to slow down the light and control the velocity of its propagation was studied [6, 7]. In this paper, we continue to study nonlinear processes in similar systems based on the nonlinear Schrödinger equation. Section 2 presents the statement of the problem for studying the characteristic features of the laser pulse self-action in systems that are highly nonuniform in the direction of the pulse propagation. They are due to the fact that in a discrete system the pulse propagation velocity is not a constant value, but changes during the evolution of the wave packet in a self-consistent regime. In the following sections, it is analytically and numerically shown that, as a result of nonlinear control of the pulse velocity, one can not only stop it, but also force it to move in the opposite direction. In this case, it is natural to expect self-compression of a laser pulse as it propagates in a medium.

2. Statement of the problem

Let us consider the structural changes of a laser pulse in the process of its self-action in nanostructured media. One of the simplest nanostructures is a linear cluster of identical (for example, spherical) nanoparticles [4, 8, 9], the geometry of which is shown in Fig. 1.

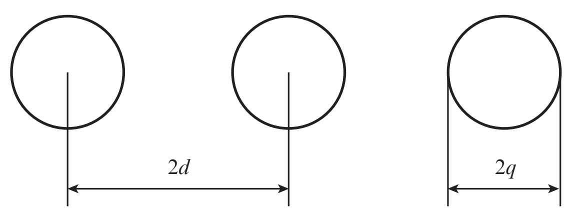

Figure 1. Geometry of a linear cluster consisting of identical nanospheres.

Due to the interaction of nanoparticles in a linear metal cluster, there is one mode with a longitudinal orientation of dipole moments and two modes with a transverse one. The propagation of linear excitations in such systems in various approximations was studied in [4, 8]. It was shown that the processes occurring in the system can be described with fairly good accuracy by a model in which only the dipole moment is associated with each particle, and higher moments are not taken into account. Thus, under the conditions where the interaction of only neighbouring dipoles dominates, we consider the self-action dynamics of the transverse mode in question by using the equation

$$\frac{\partial^2 \phi_n}{\partial t^2} = -w_1^2(\phi_{n+1} + \phi_{n-1}) - w_0^2 \phi_n + \alpha |\phi_n|^2 \phi_n. \quad (1)$$

Here, the second term on the right-hand side describes oscillations of the field ϕ_n of the nth dipole of a single spherical nanoparticle with a frequency w_0, and the first term is responsible for the propagation of excitation in the system. The intensity of interaction is determined by

$$w_1^2 = \frac{1}{3}\left(\frac{q}{d}\right)^3 w_0^2. \quad (2)$$

The conditions of the linear model applicability were discussed in [4]. In the quasi-static approximation under consideration, we do not take into account the radiation losses associated with the excitation of a synchronous electromagnetic wave in a vacuum. An analysis of the dispersion relation obtained in [8] with allowance for the finite speed of light propagation shows that the losses are small at $w_0 d/c \ll 1$ on the paths in question.

To study the nonlinear dynamics of the system, the last term is additionally introduced into equation (1). The mechanisms of cubic nonlinearity of a plasma cluster were discussed in [10]. In this case, we can make use of the value of the characteristic nonlinear field $E_{\rm nl} = m w_0^2 q/e$, where m is the mass of the electron, and e is its charge. As a result, for the parameter α in equation (1) we can give the following estimate: $\alpha = -w_0^2 e^2/(m^2 w_1^4 q^2)$.

For $w_0 \gg w_1$, which is easily satisfied for $q < d$, as a result of truncation of Eqn (1), we arrive at the discrete nonlinear Schrödinger equation (DNSE) [5], well known in the theory of nonlinear waves, in dimensionless variables for the wave field envelope ψ_n ($\phi_n \propto \psi_n \exp(\mathrm{i} w_0 t)$):

$$\mathrm{i}\frac{\partial \psi_n}{\partial t} + (\psi_{n+1} + \psi_{n-1}) + |\psi_n|^2 \psi_n = 0. \quad (3)$$

Here, the evolutionary variable is normalised to $w_1^2/(2w_0)$.

A somewhat more complex system of equations was used in the problem of the maximum possible light slowdown in coupled optical microresonators (Fig. 2) [6, 7]:

$$\mathrm{i}\frac{\partial \psi_n}{\partial t} = -(\psi_{n+1} + \psi_{n-1}) - \delta b_n - |\psi_n|^2 \psi_n, \quad (4)$$

$$\mathrm{i}\frac{\partial b_n}{\partial t} = -\Delta b_n - \delta \psi_n. \quad (5)$$

In this case, the field envelope ψ_n of the central chain of oscillators (microresonators) is related to the field envelope b_n of lateral oscillators, detuned in frequency by Δ. In the absence of coupling ($\delta = 0$), the nonlinear evolution of the laser pulse in the chain is described by DNSE (3). At $t = 0$, a wave pulse described by the complex function $\psi_n(n, t = 0)\exp(\mathrm{i}\gamma n)$ is defined, which then propagates along the chain in one direction with a group velocity determined by the dispersion relation in the linear case. In contrast to [6, 7], in the nonlinear self-action regime under consideration, we have an additional possibility of controlling the speed of the light pulse by means of a nonlinear shift of the field frequency ϕ_n in the central waveguide system.

This possibility is best explained by equation (3). It has an integral

$$W = \sum_{n=-\infty}^{+\infty} |\psi_n|^2, \quad (6)$$

which describes the conservation of energy of a localised wave packet. Below, we will consider it as a control parameter. The velocity of the energy centre of the wave packet in the discrete case has the form

$$v = \frac{1}{W}\frac{\partial}{\partial t}\sum_{n=-\infty}^{+\infty} |\psi_n|^2. \quad (7)$$

Using the continuity equation, expression (7) can be represented as

$$v = -\frac{\mathrm{i}}{W}\sum_{n=-\infty}^{+\infty}(\psi_{n+1}\psi_n^* - \psi_n \psi_{n+1}^*). \quad (8)$$

Using the Poisson summation formula, expression (8) is conveniently written in the form:

$$v = -\frac{\mathrm{i}}{W}\sum_{n=-\infty}^{+\infty}\int_{-\infty}^{+\infty}[\psi(x+1)\psi^*(x) - \psi^*(x+1)\psi(x)] \times \exp(2\pi \mathrm{i} n x)\,\mathrm{d}x. \quad (9)$$

Consider the wave packet of a Gaussian shape:

$$\psi(x,t) = \sqrt{\frac{W}{a\sqrt{\pi}}} \times \exp\left[-\frac{(x-x_0)^2}{2a^2} + \mathrm{i}\beta(x-x_0)^2 + \mathrm{i}\gamma(x-x_0)\right], \quad (10)$$

where a is the effective width of the wave packet; x_0 is the position of the energy centre; and γ and β describe linear and quadratic phase front corrections, respectively. Then for the group velocity we find the expression

$$v_{\rm g} = 2\sin\gamma \exp\left(-\frac{1}{4a^2} - a^2\beta^2\right). \quad (11)$$

In deriving it, we actually used relation (9) for $n = 0$. Estimates show that this is justified for pulses with a length $a \gg \sqrt{2}/\pi$, i.e., even for wave packets with a width comparable to the cell size.

It follows from expression (11) that, in the nonlinear regime, the pulse velocity in a discrete medium is not a constant value. Its change is determined by the behaviour of the

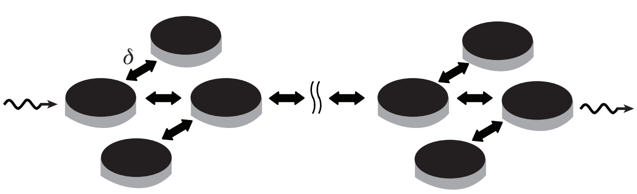

Figure 2. Geometry of coupled optical microresonators.

wave beam parameters (a, β, γ) during the system evolution. We obtain the corresponding equations in the next section. Based on them, we will perform a qualitative study of the characteristic features of the laser pulse self-action in the discrete case. The results of a numerical study of processes using the original equation (3) are presented in Section 4.

3. Qualitative study of the self-action dynamics

To describe the dynamics of a self-consistent system, we use the aberrationless approximation. The original equation (3) was studied in relation to the spatial evolution of wave beams in a coupled system of nonlinear fibres [11]. Based on the spatio-temporal analogy, i.e., replacing the evolution variable z by a time variable ($z \to t$), we can present the system of equations describing the dynamic self-action of Gaussian laser pulses (10) in a discrete medium as

$$\frac{\mathrm{d}a}{\mathrm{d}t} = 4\beta a\cos\gamma\exp\left(-\frac{1}{4a^2} - a^2\beta^2\right), \tag{12a}$$

$$\frac{\mathrm{d}\beta}{\mathrm{d}t} = \frac{\cos\gamma}{a^2}\left(\frac{1}{a^2} - 4a^2\beta^2\right)\exp\left(-\frac{1}{4a^2} - a^2\beta^2\right) - \frac{W}{\sqrt{2\pi}\,a^3}, \tag{12b}$$

$$\frac{\mathrm{d}x_0}{\mathrm{d}t} = 2\sin\gamma\exp\left(-\frac{1}{4a^2} - a^2\beta^2\right), \tag{12c}$$

$$\frac{\mathrm{d}\gamma}{\mathrm{d}t} = 0. \tag{12d}$$

It should be noted that the system of equations (12a)–(12d) was obtained in the framework of an approximate variational approach, when only the terms with $n = 0$ are taken into account in the Lagrangian of the original equation (1) written using the Poisson summation formula. This greatly simplifies the situation, since the coefficient γ responsible for the linear phase correction becomes the integral of the problem ($\gamma = \gamma_0$). As a result, the displacement of the position of the energy centre x_0 does not affect the time evolution of the internal structure of the Gaussian pulse, defined by equations (12a) and (12b) with the initial value of γ equal to γ_0:

$$\frac{\mathrm{d}a}{\mathrm{d}t} = 4\beta a\cos\gamma_0\exp\left(-\frac{1}{4a^2} - a^2\beta^2\right), \tag{13a}$$

$$\frac{\mathrm{d}\beta}{\mathrm{d}t} = \frac{\cos\gamma_0}{a^2}\left(\frac{1}{a^2} - 4a^2\beta^2\right)\exp\left(-\frac{1}{4a^2} - a^2\beta^2\right) - \frac{W}{\sqrt{2\pi}\,a^3}. \tag{13b}$$

No less important is another fact. The group velocity of the laser pulse, which is described by expression (12c) that coincides with (11) for $\gamma = \gamma_0$, substantially depends on the behaviour of $a(t)$ and $\beta(t)$. Here we come across an unusual situation when the evolution of the internal structure of the wave packet controls its propagation velocity. Finally, this affects the propagation path of a laser pulse in a discrete medium. We will show below that the length of the path can be finite.

A detailed analytical study of the system of equations (13a) and (13b) was performed in [11, 12]. It should be noted that in the case of self-action of laser pulses in question, the system of equations (12) has a much wider range of applicability. The description of the evolution of wave beams is limited by the approximation of paraxial optics ($\gamma \ll 1$). The interval of variation of γ in this problem is determined by the Brillouin zone ($0 \le \gamma \le \pi$).

Based on the results obtained in [11, 12] and the spatio-temporal analogy, we can argue that the field evolution and the propagation of a pulse with a constant velocity, characteristic of a continuous medium, are realised only when the wave packet energy W is less than the critical value W_c. When the laser pulse energy exceeds the critical value ($W > W_c$), the spatial length of the wave packet decreases as it propagates in the chain up to values of the order of the characteristic scale of a discrete medium. In this case, the self-action dynamics leads to a strong spatial localisation of the laser pulse.

To estimate the critical energy of the wave packet that is initially smooth on the scale of the structural element of the medium, we proceed as follows. Note that the limiting passage from equations (13a) and (13b) to their analogues for a continuous medium is carried out under the simultaneous fulfilment of two conditions: $a \to \infty$ and $a\beta \to 0$. As a result, to describe the behaviour of the pulse length, we arrive at the equation

$$\frac{\mathrm{d}^2 a}{\mathrm{d}t^2} = \frac{4\cos^2\gamma_0}{a^3} - \frac{4W\cos\gamma_0}{\sqrt{8\pi}\,a^2}. \tag{14}$$

This equation, and even more so equations (12) show that the self-action dynamics in a discrete system differs significantly from the situation described by the continuous nonlinear Schrödinger equation. Even in the case of wide wave packets ($a \to \infty$), equation (14) 'remembers' ($\gamma_0 \ne 0$) the dispersion relation of waves in a discrete medium.

Equation (14) is well known and is used to describe self-focusing in the aberrationless approximation. Its solution has been analysed in many books on the theory of nonlinear waves. A phase portrait of the system is given, for example, in [13]. The equilibrium state of the system on the phase plane has the coordinates $a' = \sqrt{8\pi}\cos\gamma_0/W$ and $\mathrm{d}a/\mathrm{d}t = 0$. The integral of equation (14) for fields with an initial plane phase front ($\mathrm{d}a/\mathrm{d}t = 0$ at $t = 0$),

$$\left(\frac{\mathrm{d}a}{\mathrm{d}t}\right)^2 = -\frac{8\cos^2\gamma_0}{a^2} + \frac{8W\cos\gamma_0}{\sqrt{8\pi}\,a} + \frac{8\cos^2\gamma_0}{a_0^2} - \frac{8W\cos\gamma_0}{\sqrt{8\pi}\,a_0}, \tag{15}$$

describes a periodic change in the spatial length of the pulse from the initial value $a(t = 0) = a_0 \ll a'$ to $a_{\min} < a'$. Based on the fact that $a_{\min}$ becomes equal to the size of the structural element of the medium ($a_{\min} = 1$), we find

$$W_c = \sqrt{8\pi}\cos\gamma_0 \approx 5\cos\gamma_0. \tag{16}$$

At this value of the critical pulse energy W_c, the coordinate corresponding to the equilibrium state becomes equal to the size of the structural element of the medium ($a' = 1$). For this reason, expression (16) gives a somewhat overestimated estimate for W_c compared to the value obtained as a result of a rather sophisticated analysis of the dynamics of the system based on equations (13) [11, 12]. The analysis shows that at a pulse energy $W > W_c$, a periodic change in the wave packet width a is replaced by the localisation of the wave field in the structural element of the medium ($a \approx 1$).

From expression (15), one can estimate the self-compression time. In the supercritical regime, to change the spatial pulse length, we have the relation

$$\left(\frac{\mathrm{d}a}{\mathrm{d}t}\right)^2 \approx \frac{8W\cos\gamma_0}{\sqrt{8\pi}\,a}. \quad (17)$$

Based on Eqn (17), for the pulse collapse time we can obtain the expression

$$t_s = \frac{2}{3}\frac{a_0^{3/2}\sqrt[4]{8\pi}}{\sqrt{8W\cos\gamma_0}}. \quad (18)$$

Another important estimate can be found from (12c). The distance travelled by the pulse during self-localisation till it stops is described by the expression

$$x_s = 2t_0\sin\gamma_0 = \frac{2}{3}\frac{a_0^{3/2}\sqrt[4]{8\pi}\sin\gamma_0}{\sqrt{2W\cos\gamma_0}}. \quad (19)$$

Thus, a laser pulse with an energy exceeding the critical value (16) propagates in a discrete medium to a finite distance (19). It is maximum at $W \approx W_c$ and increases in proportion to the initial pulse length to the power of 3/2. In the region of zero discrete dispersion ($\gamma_0 \to \pi/2$), the critical value of energy (16) decreases ($W_c \to 0$), and the distance x_s travelled by the pulse increases infinitely ($x_s \to \infty$).

It is also interesting to note that the length of the pulse propagation path at $W > W_c$ decreases with increasing energy. This means that there is another critical energy value:

$$W_c = \frac{2}{9}\frac{a_0^{4/3}\sqrt{8\pi}\sin^2\gamma_0}{\cos\gamma_0}, \quad (20)$$

above which the field is localised near the boundary of the discrete medium.

4. Numerical simulation

The above picture of dynamic self-action in discrete systems is generally confirmed by the results of a numerical solution of the original equation (3). Next, we graphically illustrate those new processes that accompany spatial self-compression of laser pulses using the DNSE. Under the conditions we are discussing, it is convenient to use the initial field distributions in the form

$$\psi_n = \frac{\sqrt{2}\,N\sigma}{\cosh\sigma(n-n_0)}\exp[\mathrm{i}\gamma_0(n-n_0)], \quad (21)$$

where $1/\sigma$ defines the characteristic size of the field localisation region; n_0 is the number of the central cell; and N is the estimate of the number of solitons contained in the initial field distribution. The quantity γ_0 characterises the initial propagation velocity of the pulse. In these notations, the energy of the wave packet is expresses as

$$W = 4\sigma N^2. \quad (22)$$

Comparing (22) and (16), we can estimate the critical value of the number of solitons:

$$N_c = \left(\sqrt{\frac{\pi}{2}}\frac{\cos\gamma_0}{\sigma}\right)^{1/2}, \quad (23)$$

above which the discreteness of the medium significantly affects the self-action dynamics of pulses.

In contrast to [11], we study in more detail the characteristic features of the self-action dynamics in the case of finite γ.

Figure 3 shows one of the variants of numerical calculation (for $\gamma < \pi/2$) of the radiation self-action dynamics in the supercritical regime. One can see that the process of rather strong spatial self-compression (the pulse length decreased by an order of magnitude) is accompanied by radiation losses. As a result, more than a third of the energy of the initial pulse remains in the field localisation region. It is important to note the following. In a continuous medium, the possibility of self-localisation is limited by the development of modulation instability, which leads to the splitting of the field distribution into solitons. In a discrete medium, despite the existence of this instability [8, 14], we did not notice its manifestation. The process of spatial localisation of the field at nine solitons in its initial distribution proceeds in the same way as with a smaller number of them (the field is localised on one cell and loses two thirds of its energy).

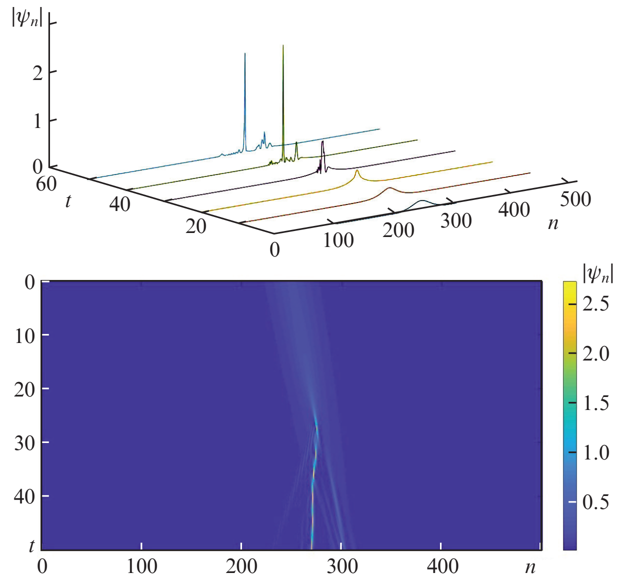

Figure 3. Evolution of the wave packet (21) ($\sigma = 0.05$, $\gamma_0 = 0.5$, $N = 6$), calculated using equation (3).

Another important characteristic of the self-action dynamics in a discrete medium is that due to the asymmetry of radiation losses (along the pulse propagation and in the opposite direction), the compressed pulse (as a rule) begins to move in the opposite direction. This effect manifests itself more pronouncedly for the parameter γ related to the region of zero discrete dispersion ($\gamma \approx \pi/2$). One can see from Fig. 4 ($\gamma = 1.5$) that the compressed pulse returns to the input of the waveguide system. At the same time, its temporary duration decreases three times. Radiation losses of the wave field in the region of laser pulse reflection occur mainly in the direction of its initial propagation.

In a more complex system of coupled optical microresonators, such a simple analytical study of the processes, with the exception of the case $\delta = 0$, cannot be carried out. However, under conditions when the central waveguide–resonator structure is dominant, and the side structure at $\delta \neq 0$ determines only an additional delayed response, a numerical study shows that the self-action dynamics proceeds in a similar way. For the same parameters of the wave packet as in Fig. 4 and coefficients $\delta = 0.03$, $\Delta = -1$ in equations (4) and (5), the picture is almost the same. The main difference from

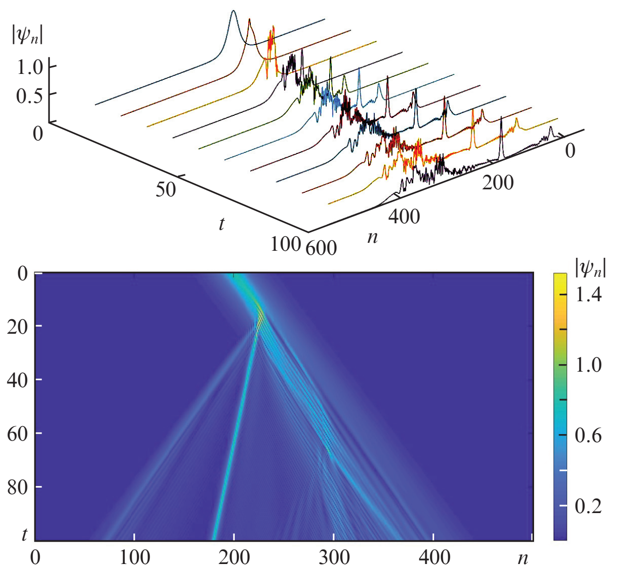

Figure 4. Evolution of the wave packet (21) ($\sigma = 0.1, \gamma_0 = 1.5, N = 5.5$), calculated using equation (3).

the results obtained above is a slightly higher level of radiation losses. This reduces the efficiency of self-compression of laser pulses in this system.

5. Conclusions

We have studied the self-action dynamics of laser pulses in two systems belonging to different frequency ranges. It is assumed that the scheme in Fig. 2 consists of nonlinear optical microresonators [6, 7], and the system of coupled metal clusters (Fig. 1) is waveguiding for radiation with a frequency close to the field oscillation frequency of 5×10^{15} s^{-1} in a metal cluster [4, 8, 9]. It has been shown that a laser pulse with an energy exceeding the critical value (16) propagates in the system with a velocity decreasing to zero. In the process of stopping the pulse, it is localised in the structural element of the medium with an energy efficiency of about 30%. Moreover, when setting the initial parameters of the system near the zero group velocity dispersion, the laser pulse not only stops, but also returns back in a compressed form. Here we are dealing with a rather specific radiation self-action dynamics. An initially smooth (on a characteristic scale of the medium) laser pulse is reflected from a discrete medium. The nonlinear process is controlled by means of a self-consistent strong change in the group velocity of the wave packet. As a result, the laser pulse self-action dynamics at an energy exceeding the critical value becomes spatio-temporal. It is premature to draw more definite conclusions because of insufficient data on system parameters (especially nonlinear ones).

Acknowledgements. The authors are grateful to A.V. Andrianov for useful discussions.

This work was supported by the Presidium of the Russian Academy of Sciences under the Fundamental Research Programme 'Extreme Light Fields and Their Interaction with Matter' (Section 2) and megagrant (Contract No. 14. W03.31.0032) (Section 3).

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