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Possibility of reducing nonlinear distortions of a frequency response of a gas ring laser with a periodic alternating frequency bias

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Abstract. **We report a theoretical study of the dependence of the beat frequency of counterpropagating waves on the rotation velocity in a gas ring laser with a periodic alternating frequency bias. Rectangular biases of two types are considered: with constant amplitude and with periodic amplitude modulation. We have found dynamic zones for locking-in the counterpropagating wave frequencies and determined deviations of the frequency response from the ideal one that arise when measuring frequency nonreciprocities, close in magnitude to the amplitude of an alternating frequency bias. It is shown that in the case of a rectangular bias with periodic amplitude modulation, nonlinear distortions of the frequency response can be reduced by an order of magnitude.**

Keywords: *ring laser, laser gyroscope, frequency bias, dynamic lock-in zones, frequency response.*

1. Introduction

Intracavity scattering of light in a gas ring laser (GRL) leads to the emergence of a zone of insensitivity to rotation and to a nonlinear dependence of the GRL frequency response on rotation. To exit the insensitivity zone and to reduce nonlinear distortions of the frequency response, an alternating frequency bias is used [1 – 4]. In Zeeman laser gyroscopes, when a magnetic field is applied to an active medium, a rectangular frequency bias (meander) is formed. The frequency response of a GRL with an alternating frequency bias was theoretically and experimentally investigated in a number of papers [1 – 13]. These studies showed that in the presence of a periodic alternating bias in a GRL, there are dynamic zones for locking-in frequencies of counterpropagating waves, which turn out to be the widest when the measured frequency nonreciprocity, determined by the rotation velocity, approaches the amplitude of the bias. In the region of a rotation velocity close to the amplitude of the frequency bias, there occur the largest deviations of the GRL frequency response from the ideal one [4, 11 – 13].

In this paper, we theoretically study the frequency response of a GRL when use is made of a periodic rectangular frequency bias with slow modulation of its amplitude. Rectangular biases of two types are considered: with constant amplitude and with periodic amplitude modulation. For biases with constant amplitude, the obtained results are consistent with the already known data. In this case, the removal of the constraints previously used in the analysis [see inequality (6) below] has made it possible to find a new feature that characterises the behaviour of the widths of the dynamic lock-in zones with a change in the bias period. For biases with low-frequency periodic amplitude modulation, a new possibility has been revealed for reducing nonlinear distortions of the frequency response.

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2. Rectangular frequency bias

We will study the frequency response of the GRL by using the differential equation for the phase difference $\varPhi$ of the counterpropagating waves:

$$\varPhi = \varOmega + \varOmega_{\rm B}(t) + \varOmega_{\rm L}\sin\varPhi, \quad (1)$$

where $\varOmega = K\dot{\vartheta}$ is the frequency nonreciprocity of the resonator, proportional to the angular velocity of the GRL rotation $\dot{\vartheta}$; K is the scale factor; $\varOmega_{\rm B}(t)$ is the alternating frequency bias; and $\varOmega_{\rm L}$ is the half-width of the static lock-in zone.

First, consider a rectangular periodic alternating bias:

$$\varOmega_{\rm B}(t) = \begin{cases} \varOmega_{\rm p} \ \text{at} \ 0 < t < T_{\rm p}/2, \\ -\varOmega_{\rm p} \ \text{at} \ T_{\rm p}/2 < t < T_{\rm p}, \end{cases} \quad (2)$$

where $\varOmega_{\rm p}$ and $T_{\rm p}$ are the amplitude and period of the bias.

For periods of time when the frequency bias is constant, equation (1) is solved analytically, and the phase difference $\varPhi$ of the counterpropagating waves can be calculated for each such interval using the formulae that determine this solution. Using an analytical solution has significantly reduced the time spent on calculations. In this work, we studied the frequency response of the GRL (the dependence of the average beat frequency $f_{\rm b}$ of counterpropagating waves on the measured frequency nonreciprocity $\varOmega/2\pi$). In the presence of a frequency bias, the beat frequency contains an alternating part oscillating with a frequency of $1/T_{\rm p}$. To eliminate these pulsations, the beat frequency was averaged. The change in the phase difference $\Delta\varPhi$ of the counterpropagating waves was calculated over a time equal to the integer $n_{\rm p}$ of the bias periods $T_{\rm p}$. The average value of the beat frequency $f_{\rm b}$ was determined by the expression

$$f_{\rm b} = \frac{\langle\dot{\varPhi}\rangle}{2\pi} = \frac{\Delta\varPhi}{2\pi n_{\rm p} T_{\rm p}}. \quad (3)$$

In the calculations below, the average value of the beat frequency was calculated at $n_p = 3000$.

In a GRL with a periodic alternating bias, there arise dynamic lock-in zones of the frequencies of counterpropagating waves. Inside the dynamic lock-in zones, the average circular beat frequencies are constant and are determined by the formula

$$\langle \dot{\Phi} \rangle = 2\pi n / T_p, \qquad (4)$$

where $n = 0, 1, 2, 3, \ldots$ is the serial number of the zone.

The widths of the dynamic zones in the case of a rectangular bias are determined by the approximate expressions (see, for example, [5]):

$$\Gamma_n = 2\Omega_L \frac{\gamma}{\gamma + n} \frac{\sin[(\pi/2)(\gamma - n)]}{(\pi/2)(\gamma - n)}, \qquad (5)$$

where $\gamma = T_p \sqrt{\Omega_p^2 - \Omega_L^2}/2\pi$.

Figure 1 shows the normalised frequency response $f_b T_p$ of a GRL with a rectangular frequency bias, calculated at $\Omega_L/2\pi = 300$ Hz, a bias amplitude $\Omega_p/2\pi = 56888$ Hz, and a bias period $T_p = 0.004$ s. The straight line corresponds to an ideal frequency response $\langle \dot{\Phi} \rangle = \Omega$ (in the absence of nonlinear distortions).

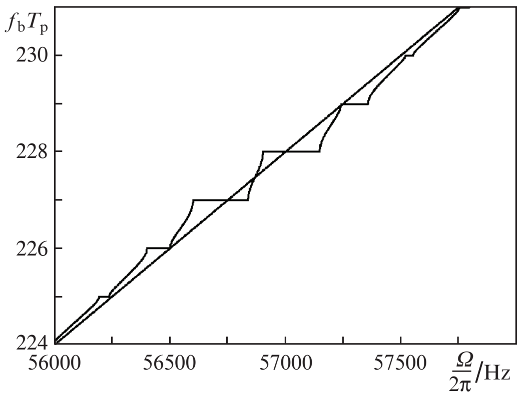

Figure 1. Normalised frequency response (dependence of $f_b T_p$ on the measured frequency nonreciprocity $\Omega/2\pi$) for a rectangular frequency bias with a period $T_p = 0.004$ s. The straight line corresponds to an ideal frequency response $\langle \dot{\Phi} \rangle = \Omega$.

The frequency response is shown in a small range of Ω values close to the amplitude of the bias. In this range, there are wide dynamic lock-in zones, following each other with an interval of 250 Hz (with a magnetic field switching frequency $1/T_p$). The largest dynamic zones have a width close to that of the static lock-in zone Ω_L.

Figure 2 shows the normalised frequency response $f_b T_p$ of a GRL with a rectangular frequency bias, which was calculated for the same parameters as in Fig. 1, with the exception of the bias period T_p, which in this case is 0.04 s. The straight line in Fig. 2 corresponds to the frequency response in the absence of nonlinear distortions. As in Fig. 1, the widest dynamic zones are observed in the range of Ω values close to the amplitude of the bias; however, the widths of these zones are much smaller than those in Fig. 1.

The widths of the dynamic lock-in zones in Fig. 1 are in good agreement with the results of calculations by formula

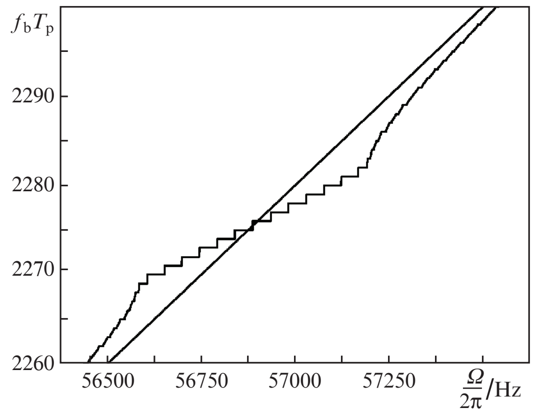

Figure 2. Normalised frequency response for a rectangular frequency bias with a period $T_p = 0.04$ s.

(5), while in Fig. 2 they are significantly smaller than those predicted by this formula. This is due to the fact that the theoretical dependence defined by formula (5) is valid only when the inequality

$$\Omega_L/2\pi < 1/T_p \qquad (6)$$

is met.

This inequality holds for the case of Fig. 1 and does not hold for Fig. 2. One can see from Fig. 2 that despite the small widths of the dynamic lock-in zones, nonlinear distortions of the frequency response turn out to be significant. Figure 3 shows the deviations of the frequency response from the ideal one, $\delta f = f_b - \Omega/2\pi$, in the range of Ω values close to the amplitude of the bias. Here, curve (*1*) corresponds to Fig. 1, and curve (*2*) corresponds to Fig. 2. As can be seen from Fig. 3, the maximum deviation δf is independent of the bias period T_p.

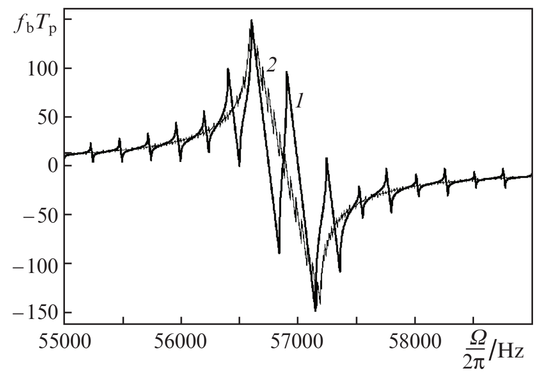

Figure 3. Nonlinear distortions of the frequency response for biases with a period T_p = (*1*) 0.004 and (*2*) 0.04 s.

3. Rectangular bias with periodic amplitude modulation

In this paper, we have studied the possibility of reducing nonlinear distortions of the frequency response of a GRL using a periodic rectangular bias with slow amplitude modulation. Let us consider the frequency bias shown in Fig. 4. The bias period is T_p = 40 ms; each period has ten rectangular links with a duration of 4 ms. For the first five links, the amplitude of the alternating bias in each subsequent link increases in 6 kHz steps from the initial value of 30 kHz to the final value of 60 kHz; for the next five links, the amplitude decreases in 6 kHz steps from the initial value of 60 kHz to the final value of 30 kHz.

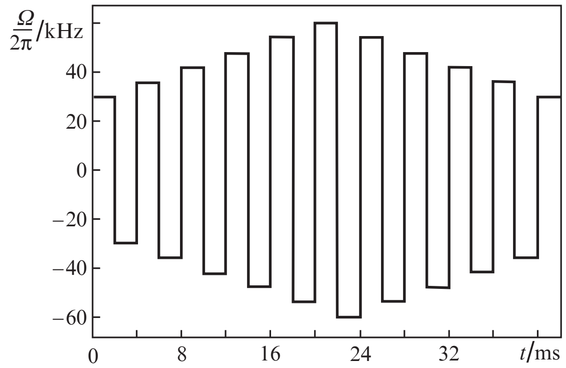

Figure 4. Shape of a rectangular bias with slow amplitude modulation.

At time intervals of 2 ms duration, the frequency bias is constant, and equation (1) is solved analytically. The phase difference Φ of the counterpropagating waves was calculated at each of these intervals using formulae that determine the analytical solution of equation (1). Figure 5 shows the deviations of the frequency response from the ideal one, $\delta f = f_b - \Omega/2\pi$, in the range of Ω values corresponding to the largest distortions of the frequency response. As can be seen from Fig. 5, the maximum deviation δf for the frequency bias in question decreased fivefold as compared to the case of a periodic rectangular bias.

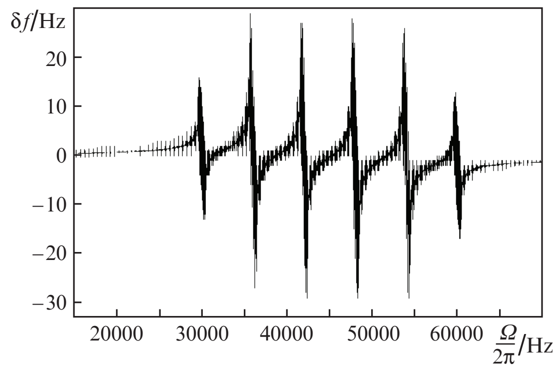

Figure 5. Nonlinear distortions of the frequency response $\delta f = f_b - \Omega/2\pi$ in the case of the bias shown in Fig. 4.

Consider another similar bias with slow amplitude modulation. The bias period is T_p = 80 ms; each period has twenty rectangular links with a duration of 4 ms. For the first ten links, the amplitude of the alternating bias increases in 6 kHz steps from the initial value of 30 kHz to the final value of 90 kHz, for the next ten links, the amplitude decreases in 6 Hz steps from the initial value of 90 kHz to the final value of 30 kHz.

Figure 6 shows the deviations of the frequency response from the ideal one, $\delta f = f_b - \Omega/2\pi$, in a wide range of Ω values. One can see that the maximum deviation δf for the frequency bias in question decreased tenfold as compared to the case of a periodic rectangular bias with constant amplitude.

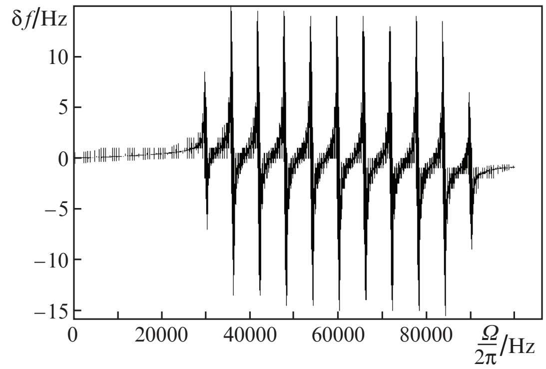

Figure 6. Nonlinear distortions of the frequency response $\delta f = f_b - \Omega/2\pi$ in the case of a bias with a period T_p = 0.08 s (other values of the bias parameters are given in the text of the paper).

Thus, the performed studies of the frequency response of GRLs with a periodic rectangular bias have shown that the largest dynamic lock-in zones have a width close to that of the static lock-in zone Ω_L when the inequality $\Omega_L/2\pi < 1/T_p$ is fulfilled. At $\Omega_L/2\pi \gg 1/T_p$, the widths of the dynamic lock-in zones are much smaller; however, despite this fact, the nonlinear distortions of the frequency response turn out to be significant. We have found that the use of a rectangular frequency bias with slow amplitude modulation can markedly reduce the nonlinear distortions of the frequency response.

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