Quantum Electronics **50** (6) 537 – 542 (2020)

https://doi.org/10.1070/QEL17334

Control of atomic Bose – Einstein condensate with interferometric feedback probing

V.A. Tomilin, L.V. Il'ichov

Abstract. **We consider the problem of decoherence in a Bose – Einstein condensate (BEC) of noninteracting atoms during interferometric probing with a classical monochromatic external field. The condensate is located in one of the arms of the Mach – Zehnder interferometer, while part of the radiation from the interferometer output is fed back to the input, thereby closing the coherent feedback loop. A more general setting of the problem is also considered, in which the condensate is located in a system of interferometers 'nested' into each other, while part of the output radiation of each of them is also fed back to its input, closing the so-called multiloop feedback. The possibility of effective control of decoherence rates of various matrix elements is shown. The application of feedback of the proposed type to BEC in a double-well potential is investigated. It is found that this feedback allows efficient control of the distribution of atoms between the wells.**

Keywords: *Bose – Einstein atomic condensate, interferometric probing, feedback, decoherence rate.*

1. Introduction

Atomic Bose – Einstein condensate (BEC) is a nonclassical state of matter. It is characterised by the majority of atoms being in a single quantum state. This state determines the average spatial density of atoms and certain phase relations between the atomic arrangements. In other words, BEC posesses spatial coherence. It can be detected using interference experiments with beams of optical radiation probing various regions of the volume occupied by the condensate. However, probing with even a strongly nonresonant field makes the condensate an open quantum system, which leads to the loss of macroscopic coherence. It is of interest to study this process and search for methods to control it.

Quantum control theory appeared in the 80s of the last century [1] and in recent years has been significantly developed due to the progress of experimental techniques that allow experiments with single quantum objects. Feedback is one of the most effective methods of control. The main applications of this theory include controlling the evolution of quantum systems and engineering of quantum states such as squeezed [2], entangled [3], and superposition states [4]. As applied to atoms localised in an optical trap, the feedback is mainly aimed at increasing the efficiency of their optical cooling [5, 6]. The main method in this case is the phase-contrast imaging of the condensate with the subsequent tuning of the trapping potential [7, 8].

In the present work, we propose a new feedback control scheme for the atomic BEC, based on interferometric probing of the condensate, with part of interferometer output radiation being redirected back to its input. Describing BEC with a simple model, we have shown that this type of feedback allows efficient suppression of the BEC coherence loss. In addition, a new method for controlling the spatial distribution of atoms in a BEC is proposed. Its advantage over standard approaches is that it does not involve adjustments of the trapping potential. Instead, the phase shifts situated in the interferometers undergo rapid switchings.

Section 2 describes the model of BEC decoherence. Section 3 discusses the use of interferometric feedback for controlling decoherence within the framework of the chosen model. In Section 4, we study a modified feedback option with the addition of phase shift switching and its use for controlling distribution of atoms in a BEC trapped in a double-well potential. The results of the work and their discussion are presented in the Conclusions.

2. Decoherence of atomic condensate

The present work is devoted to the study of a new scheme for BEC control; therefore, to describe the condensate itself, we will use a simplified model that does not take into account the effects of interatomic interactions. The condensate is represented by a single boson mode. We also consider the probe radiation to be classical and nonresonant to atomic transitions, and the radiation wavefront to be wide enough so that the field is uniform over the characteristic scale of BEC localisation. In this case, the interaction of the condensate atoms with the radiation is dispersive. As a result of this interaction, the complex amplitude $\mathcal{E}$ of the probe radiation acquires a phase shift proportional to the number of atoms in the condensate: $\mathcal{E} \to \mathcal{E}e^{i\phi\hat{n}}$, where the parameter ϕ is the phase shift

V.A. Tomilin, L.V. Il'ichov Institute of Automation and Electrometry, Siberian Branch, Russian Academy of Sciences, prosp. Akad. Koptyuga 1, 630090 Novosibirsk, Russia; e-mail: 8342tomilin@mail.ru

Received 11 March 2020; revision received 30 March 2020
Kvantovaya Elektronika **50** (6) 537 – 542 (2020)
Translated by V.L. Derbov

introduced by a single atom, and $\hat{n}$ is the operator of the number of atoms in the condensate mode*.

In the master equation describing the evolution of the condensate state, the interaction of radiation with the condensate is described by the dissipative part, which has the form of a Lindblad structure. The process of decoherence (dephasing) is associated with the irreversible departure of radiation carrying information about the condensate from the interferometer. The corresponding Lindblad operator in the case of a quantized field is a photon annihilation operator. Obviously, in our case it must be proportional to the amplitude of the radiation that has left the resonator [11], which, after acquiring a phase shift, becomes an operator. Thus, the equation for the density matrix $\hat{\rho}$ has the form:

$$\left(\frac{\mathrm{d}}{\mathrm{d}t}\hat{\rho}\right)_{\mathrm{deph}} = \nu \mathrm{e}^{\mathrm{i}\phi\hat{n}}\hat{\rho}\mathrm{e}^{-\mathrm{i}\phi\hat{n}} - \nu\hat{\rho}\,. \tag{1}$$

The parameter ν is proportional to the beam intensity and determines the decoherence rate.

To begin with, we consider the decoherence process on its own, without taking into account the dynamics of the condensate itself. It is easy to proceed from Eqn (1) to a set of independent equations for the matrix elements of the statistical operator $\hat{\rho}$ in the Fock basis $\{|n\rangle;\ n = 0, \ldots, N_{\mathrm{at}}\}$ of the atomic mode (N_{at} being the total number of atoms in the condensate mode):

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle m|\hat{\rho}|n\rangle = (-\gamma_{mn} + \mathrm{i}\omega_{mn})\langle m|\hat{\rho}|n\rangle,$$

$$\omega_{mn} = \nu\sin[(m-n)\phi], \tag{2}$$

$$\gamma_{mn} = \nu\{1-\cos[(m-n)\varphi]\}.$$

3. Interferometric probing scheme with coherent feedback

The proposed feedback scheme is organised as follows. The condensate is located in the Mach–Zehnder interferometer, to the input of which classical coherent radiation is supplied, as described in the previous section. The radiation from one of the outputs of the interferometer is redirected back to the input (balanced) beam splitter (Fig. 1a). Thus, feedback arises in the system, similar to that found in electronic circuits: part of the system output signal is fed back to the input. According to the classification adopted in the theory of quantum feedback, feedback of this type is referred to as coherent [12], because it implies no explicit measurement in the controlled system.

Let us find the relationship between input and output signals. According to Section 2, as a result of the interaction, the amplitude of the radiation transmitted through the condensate becomes an operator acting in the space of condensate states. In the presence of feedback, this applies to the amplitudes of all fields except the input one. As usual, we will denote these operators by the symbol ^ over the corresponding variable. We emphasise that the probing field acquires the operator nature exclusively as a result of interaction with the condensate. For the amplitudes at the input beam splitter, we have the expressions (see the notation in Fig. 1a):

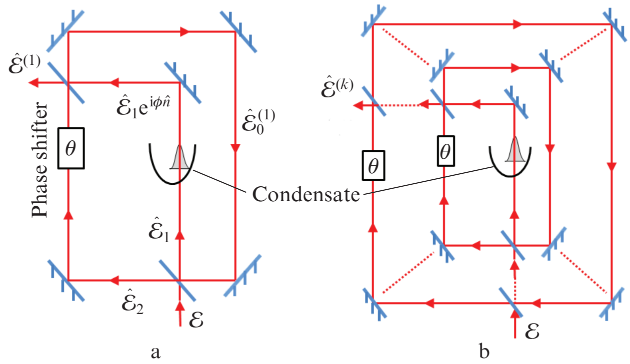

Figure 1. Scheme of interferometric probing with (a) one coherent feedback loop and (b) several feedback loops.

$$\hat{\mathcal{E}}_1 = \frac{1}{\sqrt{2}}[\mathcal{E} + \hat{\mathcal{E}}_0^{(1)}],$$
$$\hat{\mathcal{E}}_2 = \frac{1}{\sqrt{2}}[-\mathcal{E} + \hat{\mathcal{E}}_0^{(1)}]. \tag{3}$$

A similar relation can be written for the output signal:

$$\hat{\mathcal{E}}_0^{(1)} = \frac{1}{\sqrt{2}}(\hat{\mathcal{E}}_2\mathrm{e}^{\mathrm{i}\theta} + \hat{\mathcal{E}}_1\mathrm{e}^{\mathrm{i}\phi\hat{n}}),$$
$$\hat{\mathcal{E}}^{(1)} = \frac{1}{\sqrt{2}}(-\hat{\mathcal{E}}_2\mathrm{e}^{\mathrm{i}\theta} + \hat{\mathcal{E}}_1\mathrm{e}^{\mathrm{i}\phi\hat{n}}). \tag{4}$$

It is assumed here that the delay introduced by the feedback circuit is negligible. By virtue of relations (3), (4), we obtain the expression

$$\hat{\mathcal{E}}^{(1)} = \frac{\mathrm{e}^{\mathrm{i}\theta} + (1-2\mathrm{e}^{\mathrm{i}\theta})\mathrm{e}^{\mathrm{i}\phi\hat{n}}}{2-\mathrm{e}^{\mathrm{i}\theta}-\mathrm{e}^{\mathrm{i}\phi\hat{n}}}\mathcal{E}\,. \tag{5}$$

Both the numerator and the denominator in (5) are operator expressions. Division in this case means multiplication by the operator, inverse of the denominator. It is easy to show that the operator on the right-hand side is unitary, and therefore we can write the expression $\hat{\mathcal{E}}^{(1)} = \mathrm{e}^{\mathrm{i}\chi_1(\hat{n})}\mathcal{E}$.

This scheme can be generalised by placing the resulting system with a feedback circuit in the arm of the new Mach–Zehnder interferometer and connecting again one of its outputs to the input. This procedure can be repeated, resulting in a sequence of 'nested' interferometers (Fig. 1b), each containing a feedback loop. As far as the authors know, the concept of multi-loop feedback has not been described in the earlier literature. If we number interferometers from 1 to N, then for the output amplitudes of each of them, we can compose the following relation (taking into account the fact that each interferometer generally contributes its own phase shift θ_k):

*A more consistent theory would require a quantum treatment of both the condensate and the probe field. Their interaction would be reflected in the Hamiltonian by the term $\propto \hat{n}\hat{n}_{\mathrm{ph}}$, where $\hat{n}_{\mathrm{ph}}$ is the number of photons in the probe beam. However, in the case of small photon fluctuations, the last operator can be replaced by the average number of photons in the beam $\bar{n}_{\mathrm{ph}}$. This is valid, e.g., for Glauber coherent states $|\alpha\rangle$ with $|\alpha| \gg 1$. One of the possible approaches to accounting for fluctuations of the probe field is discussed in Refs [9, 10].

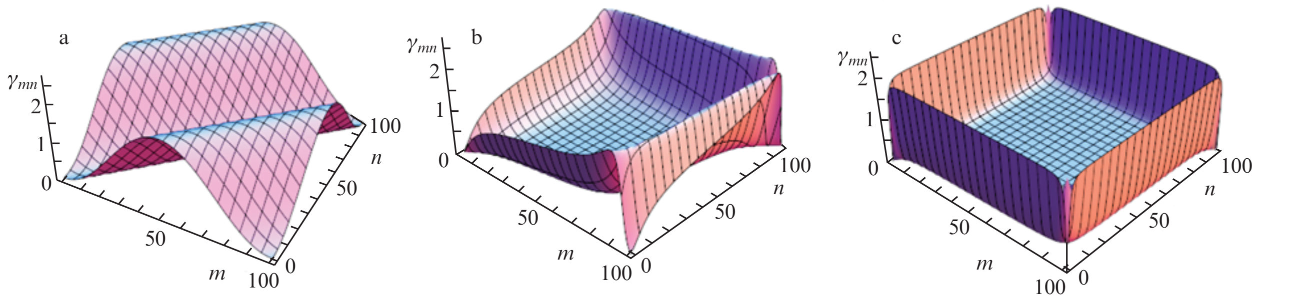

Figure 2. Decoherence rates γ_{mn} of matrix elements $\langle m|\hat{\rho}|n\rangle$ (a) without feedback, (b) with one feedback loop and (c) with five feedback loops for $\theta = \pi/4$ and $\phi = \pi/50$.

$$\hat{\mathcal{E}}^{(k)} = \mathrm{e}^{\mathrm{i}\chi_k(\hat{n})}\mathcal{E},$$
$$\mathrm{e}^{\mathrm{i}\chi_k(\hat{n})} = \frac{\mathrm{e}^{\mathrm{i}\theta_k} + (1 - 2\mathrm{e}^{\mathrm{i}\theta_k})\mathrm{e}^{\mathrm{i}\chi_{k-1}(\hat{n})}}{2 - \mathrm{e}^{\mathrm{i}\theta_k} - \mathrm{e}^{\chi_{k-1}(\hat{n})}}. \tag{6}$$

In the case when all phase shifts are similar and equal to θ, this relation becomes recurrent and has a solution in the closed form:

$$\mathrm{e}^{\mathrm{i}\chi_k(\hat{n})} = \frac{\mathrm{e}^{\mathrm{i}\chi_1(\hat{n})}(2^{k-1}\mathrm{e}^{\mathrm{i}\theta} - 1) + \mathrm{e}^{\mathrm{i}\theta}(1 - 2^{k-1})}{\mathrm{e}^{\mathrm{i}\chi_1(\hat{n})}(2^{k-1} - 1) + \mathrm{e}^{\mathrm{i}\theta} - 2^{k-1}}. \tag{7}$$

It is easy to verify that the modulus of the right-hand side of Eqn (7) is also equal to unity, i.e., the amplitude of the output radiation remains unchanged. The parameter θ is easy to control, thereby realising a simple method for changing feedback parameters. In the remainder of the paper, we will focus on the case of identical phase shifts.

Taking Eqn (7) into account, the equation of evolution for a condensate placed in a chain of N nested interferometers with coherent feedback has the form [by analogy with Eqn (1)]:

$$\left(\frac{\mathrm{d}}{\mathrm{d}t}\hat{\rho}\right)_{\mathrm{deph}} = \nu\mathrm{e}^{\mathrm{i}\chi_N(\hat{n})}\hat{\rho}\mathrm{e}^{-\mathrm{i}\chi_N(\hat{n})} - \nu\hat{\rho}, \tag{8}$$

and in terms of matrix elements we obtain the expression

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle m|\hat{\rho}|n\rangle = \nu[\mathrm{e}^{\mathrm{i}\chi_N(m)}\mathrm{e}^{-\mathrm{i}\chi_N(n)} - 1]\langle m|\hat{\rho}|n\rangle. \tag{9}$$

Consider how the introduction of coherent feedback affects the process of dephasing in the probed BEC. Figure 2 shows the decoherence rates of the matrix elements of the condensate statistical operator. It can be seen that even when using a single interferometer, the feedback efficiently suppresses decoherence of most matrix elements. With an increase in the number of interferometers, the effect becomes even more pronounced and the decoherence rate remains unchanged only for matrix elements with numbers n for which $\phi n = 2\pi$. This is explained by the fact that the matrix elements of expression (5) in states with such n tend to $\mathcal{E}$, thus reducing the feedback effect on the decoherence of the corresponding density matrix elements.

4. Coherent feedback with phase switching

In experimental applications, the problem of preparing condensate with a given spatial distribution often arises. In particular, this is important when studying various aspects of the tunnelling process between multiple localisations of the BEC [13, 14]. In the scheme considered above, the spatial structure of the condensate did not arise. In the general case, the situation when the probe beam illuminates only part of the space occupied by the condensate is of interest. In this case, the trap in which the condensate is localised is assumed created independently (not using probe radiation). Obviously, this leads to the separation of condensate into the illuminated and nonilluminated parts and destroys the coherence between them. In the present section, a model of condensate in a double-well trap will be used to describe this process. The intrinsic dynamics of such condensate is described by the Hamiltonian of two bosonic modes:

$$\hat{H} = \omega_{\mathrm{A}}\hat{a}^{\dagger}\hat{a} + \omega_{\mathrm{B}}\hat{b}^{\dagger}\hat{b} + \lambda(\hat{a}^{\dagger}\hat{b} + \hat{b}^{\dagger}\hat{a}). \tag{10}$$

Here the operators $\hat{a}$, $\hat{a}^{\dagger}$ and $\hat{b}$, $\hat{b}^{\dagger}$ correspond to the creation and annihilation of photons in wells A and B, and the constant λ quantifies the process of atomic tunnelling from one well to another.

Suppose now that atoms in only one well (let it be well A for definiteness) are subject to optical probing. From Eqn (1) it can be seen that the diagonal elements of the density matrix do not change in the course of decoherence. They also do not change under the influence of probing with feedback described by Eqn (8), since the Lindblad operators, although they acquire a more complex form, remain unitary and diagonal in the number of atoms in the illuminated well, which demonstrates a certain limitation of the scheme considered in Section 3.

In Ref. [15], a different method for probing a BEC localised in a double-well trap was considered. The condensate was also placed in a Mach–Zehnder interferometer, and only one potential well was illuminated. The control was implemented by fast switching of the trap potential, triggered by the detection of photons in the output channels of the interferometer. This type of feedback is called measurement-based feedback [12]. It was shown that using it allows creating nontrivial distributions of atoms in the wells. However, for this purpose multiple switching of the trap potential is required, which is not very convenient. In addition, a rather strict constraint on the switching time arises. On the one hand, it must be fast enough to neglect the delay in the feedback loop. On the other hand, fast (in comparison with the atomic evolution time inside the well) switching of potential leads to the excitation of higher vibrational levels in the well and to a violation of the two-mode approximation. As will be shown below, the addition of coherent feedback to this scheme makes it possible to achieve a similar effect without the need to change the trap potential itself.

In Section 3, we considered a system of nested feedback interferometers with the same phase shifts in the free arms. In contrast to the potential of the trap in which the condensate is localised, a collective change in these phase shifts is easily feasible. In the scheme shown in Fig. 3, the detection of photons in a particular output channel of the external interferometer causes phase shifts in a coherent feedback system similar to that shown in Fig. 1. To describe such feedback based on phase switching, there is a very convenient formalism of the so-called hybrid statistical operators [16]. The introduction of feedback requires considering the combination of a quantum subsystem (condensate) and a classical subsystem (a device that regulates the state of the controlled phase shift).

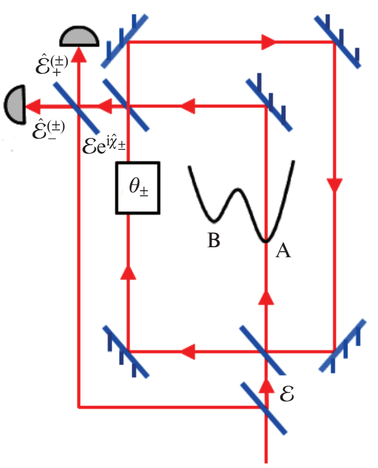

Figure 3. Scheme of interferometric probing of a double-well BEC with coherent feedback and phase switching.

We successfully applied this approach in a series of works devoted to the study of feedback based on phase switching in systems of single emitters [17]. Its essence is that the ordinary statistical operator of a quantum system is replaced by a set of hybrid operators, numbered by the classical index. Each of these operators is a conditional state of the system corresponding to a particular state of the feedback circuit. In our case, there are only two such states corresponding to one or another output of the external Mach–Zehnder interferometer. We will denote them by superscripts '+' and '–'. Another difference from Eqns (1) and (8) considered above is the existence of two output channels instead of one; the corresponding Lindblad operators are labelled by the subscripts '+' and '–'. The Lindblad operators also depend on the current value of the phase shifts θ; therefore, they additionally acquire the classical index '±'. Keeping this in mind, we can write the system of quantum master equations for new hybrid operators:

$$\frac{\mathrm{d}}{\mathrm{d}t}\hat{\rho}^{(+)} = -\mathrm{i}[\hat{H},\hat{\rho}^{(+)}] + \nu\hat{\mathcal{E}}_+^{(+)}\hat{\rho}^{(+)}\hat{\mathcal{E}}_+^{(+)\dagger} + \nu\hat{\mathcal{E}}_+^{(-)}\hat{\rho}^{(-)}\hat{\mathcal{E}}_+^{(-)\dagger}$$
$$-\frac{\nu}{2}\{\hat{\mathcal{E}}_+^{(+)\dagger}\hat{\mathcal{E}}_+^{(+)} + \hat{\mathcal{E}}_-^{(+)\dagger}\hat{\mathcal{E}}_-^{(+)}, \hat{\rho}^{(+)}\}, \tag{11}$$
$$\frac{\mathrm{d}}{\mathrm{d}t}\hat{\rho}^{(-)} = -\mathrm{i}[\hat{H},\hat{\rho}^{(-)}] + \nu\hat{\mathcal{E}}_-^{(-)}\hat{\rho}^{(-)}\hat{\mathcal{E}}_-^{(-)\dagger} + \nu\hat{\mathcal{E}}_-^{(+)}\hat{\rho}^{(+)}\hat{\mathcal{E}}_-^{(+)\dagger} -$$
$$-\frac{\nu}{2}\{\hat{\mathcal{E}}_-^{(-)\dagger}\hat{\mathcal{E}}_-^{(-)} + \hat{\mathcal{E}}_+^{(-)\dagger}\hat{\mathcal{E}}_+^{(-)}, \hat{\rho}^{(-)}\},$$

where

$$\hat{\mathcal{E}}_+^{(\pm)} = \frac{1}{\sqrt{2}}[\mathrm{e}^{\mathrm{i}\chi_\pm(\hat{n}_\mathrm{A})} - 1];$$
$$\hat{\mathcal{E}}_-^{(\pm)} = \frac{1}{\sqrt{2}}[\mathrm{e}^{\mathrm{i}\chi_\pm(\hat{n}_\mathrm{A})} + 1]; \tag{12}$$

operators $\chi_\pm(\hat{n}_\mathrm{A})$ have the form similar to Eqn (7), but with the replacement $\theta \to \theta_\pm$; curly brackets denote an anticommutator.

It is worth commenting on the structure of dissipative parts in the system (11). The quantum jump itself – a sharp change in the quantum state of the system as a result of detecting a photon at the output of the interferometer – is described by the so-called sandwich term in the Lindblad structure [18]. Depending on the value of the phase shift before this detection, two types of sandwich terms are possible. The hybrid statistical operator contained in each of them has a phase shift index prior to the switching.

It can be seen that even the use of the simplest two-mode model leads to equations that are too complicated for a complete analytical treatment. Due to the focus of the present work, the study of the feedback is of main interest; therefore, it is natural to concentrate on the case of a maximum impact of feedback. Since phase switchings are triggered by photodetections, it is natural to consider the case when they occur frequently, i.e., the case of high intensities of probe radiation. This corresponds to a regime of strong decoherence. In this case, the dissipative parts of Eqns (12) are dominant.

For the dissipative part, we can choose its 'natural' basis, in which this part can be reduced to a balance equation resembling the Pauli equation, i.e., the Fock basis for the illuminated well. If the decoherence rate in the illuminated well is high enough, then we can assume that the coherence in this basis does not extend beyond one term from the diagonal. Then for hybrid statistical operators the following ansatz can be used [19]:

$$\hat{\rho}^{(\pm)} = \sum_n (|n\rangle_\mathrm{A}\langle n| \otimes \hat{\rho}_n^{(\pm)} + |n+1\rangle_\mathrm{A}\langle n| \otimes \hat{\rho}_{+n}^{(\pm)}$$
$$+ |n\rangle_\mathrm{A}\langle n+1| \otimes \hat{\rho}_{-n}^{(\pm)}), \tag{13}$$

where, obviously, $\hat{\rho}_{-n}^{(\pm)} = \hat{\rho}_{+n}^{(\pm)\dagger}$, and $\mathrm{Tr}(\hat{\rho}_n^{(+)} + \hat{\rho}_n^{(-)}) = p_n$ is the probability of observing n atoms in well B. Operators $\hat{\rho}_n^{(\pm)}$, $\hat{\rho}_{+n}^{(\pm)}$, and $\hat{\rho}_{-n}^{(\pm)}$ belong to the nonilluminated well B. However, it turns out that going for the following linear combinations is more convenient:

$$\hat{\rho}_n = \hat{\rho}_n^{(+)} + \hat{\rho}_n^{(-)},$$
$$\hat{r}_n = \hat{\rho}_n^{(+)} - \hat{\rho}_n^{(-)},$$
$$\hat{\rho}_{\pm n} = \hat{\rho}_{\pm n}^{(+)} + \hat{\rho}_{\pm n}^{(-)}, \tag{14}$$
$$\hat{r}_{\pm n} = \hat{\rho}_{\pm n}^{(+)} - \hat{\rho}_{\pm n}^{(-)}.$$

The equations for these linear combinations have the form

$$\frac{d}{dt}\hat{\rho}_n = -i\omega_B[\hat{n}_B,\hat{\rho}_n] - i\lambda\sqrt{n+1}\,\hat{b}^\dagger\hat{\rho}_{+n} + i\lambda\sqrt{n}\,\hat{\rho}_{+(n-1)}\hat{b}^\dagger - $$
$$- i\lambda\sqrt{n}\,\hat{b}\hat{\rho}_{-(n-1)} + i\lambda\sqrt{n+1}\,\hat{\rho}_{-n}\hat{b},$$
$$\frac{d}{dt}\hat{r}_n = -i\omega_B[\hat{n}_B,\hat{r}_n] - i\lambda\sqrt{n+1}\,\hat{b}^\dagger\hat{r}_{+n} + i\lambda\sqrt{n}\,\hat{r}_{+(n-1)}\hat{b}^\dagger$$
$$- i\lambda\sqrt{n}\,\hat{b}\hat{r}_{-(n-1)} + i\lambda\sqrt{n+1}\,\hat{r}_{-n}\hat{b} - \nu[2+\cos\chi_+(n)$$
$$- \cos\chi_-(n)]\hat{r}_n - \nu[\cos\chi_+(n) + \cos\chi_-(n)]\hat{\rho}_n, \qquad (15)$$
$$\frac{d}{dt}\hat{\rho}_{+n} = -i\omega_A\hat{\rho}_{+n} - i\omega_B[\hat{n}_B,\hat{\rho}_{+n}] - i\lambda\sqrt{n+1}\,\hat{b}\hat{\rho}_n$$
$$+ i\lambda\sqrt{n+1}\,\hat{\rho}_{n+1}\hat{b} - \frac{\nu}{2}\{2 - e^{i[\chi_+(n+1)-\chi_+(n)]}$$
$$- e^{i[\chi_-(n+1)-\chi_-(n)]}\}\hat{\rho}_{+n} - \frac{\nu}{2}\{e^{i[\chi_-(n+1)-\chi_-(n)]}$$
$$- e^{i[\chi_+(n+1)-\chi_+(n)]}\}\hat{r}_{+n},$$
$$\frac{d}{dt}\hat{r}_{+n} = -i\omega_B[\hat{n}_B,\hat{r}_{+n}] - i\lambda\sqrt{n+1}\,\hat{b}\hat{r}_n + i\lambda\sqrt{n+1}\,\hat{r}_{n+1}\hat{b}$$
$$- 2\nu\hat{r}_{+n} - \frac{\nu}{2}[e^{i\chi_+(n+1)} + e^{-i\chi_+(n)} + e^{i\chi_-(n+1)} + e^{i\chi_-(n)}]\hat{\rho}_{+n}$$
$$- \frac{\nu}{2}[e^{i\chi_+(n+1)} + e^{-i\chi_+(n)} - e^{i\chi_-(n+1)} - e^{i\chi_-(n)}]\hat{r}_{+n}.$$

Note that the first equation in (15) is not different from the equation in the absence of feedback obtained in [19]. However, the operators $\hat{\rho}_{\pm n}$ evolve already in a completely different way, which leads to different stationary distributions. Let us check under what relations between the parameters of the problem it is legitimate to use ansatz (13). For this purpose, let us analyse the equations for the operators $\hat{\rho}_{+n}$ and $\hat{r}_{+n}$, since it is exactly their matrix elements that determine the coherence in the illuminated well.

In the equation for $\hat{\rho}_{+n}$, the 'source' terms are proportional to $\lambda\hat{b}\hat{\rho}_n$ and $\lambda\hat{\rho}_{n+1}\hat{b}$, and in the equation for $\hat{r}_{+n}$, terms proportional to $\lambda\hat{b}\hat{r}_n$ and $\lambda\hat{r}_{n+1}\hat{b}$. Thus, the coherence creation rate depends only on λ. The terms proportional to ν describe the loss of coherence. Therefore, if the rate of coherence loss significantly exceeds $|\lambda|$, then expression (13) can be considered valid. Figure 4 shows examples of the calculated coherence loss rates of the operators entering the ansatz (13) in comparison with the coherence loss rate of $\hat{\rho}_{+n}$ in the absence of feedback (the latter is equal to $\nu(1-\cos\phi)$ [19]). It can be seen that the strong decoherence condition is rather well satisfied for $\nu(1-\cos\phi) \gg |\lambda|$, and in this range of ν values, the introduction of ansatz (13) is quite justified. The fulfilment of this condition is quite easy to ensure by increasing the intensity of the probe radiation or by increasing the height of the barrier between potential wells; the latter will lead to a decrease in the tunnelling parameter λ.

One more consequence of the introduced approximation is that the evolution of the operators $\hat{r}_{+n}$ and $\hat{\rho}_{+n}$ turns out to be adiabatically subordinate to the slowly evolving operator $\hat{\rho}_n$. Therefore, the time derivatives in the last two equations in (15) can be neglected.

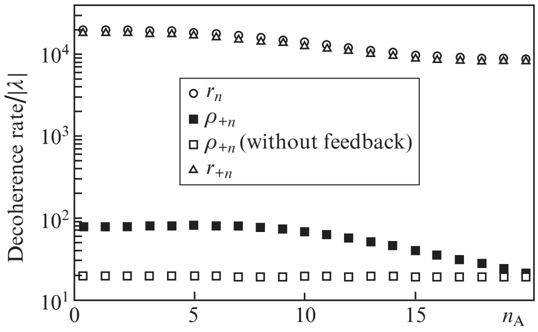

Figure 4. Decoherence rates of the operators describing the state of well B at $N_{at} = 20$, $\omega_A = 5\lambda$, $\omega_B = 7\lambda$, $\phi = \pi/50$, $\nu = 10^4\lambda$, $\theta_\pm = \pm\pi/2$.

Since the purpose of this section is to clarify the possibilities of using feedback to control atomic distributions, we will be primarily interested in the diagonal matrix elements $\langle k|\hat{\rho}_n|k\rangle = \rho_n(k)$. Simple algebraic transformations make it possible to obtain for them a system of balance equations

$$\frac{d}{dt}\rho_n(k) = F_1(\rho_n(k),\rho_{n+1}(k-1),\rho_{n-1}(k+1),r_n(k),$$
$$r_{n+1}(k-1),r_{n-1}(k+1)), \qquad (16)$$
$$\frac{d}{dt}r_n(k) = F_2(\rho_n(k),\rho_{n+1}(k-1),\rho_{n-1}(k+1),r_n(k),$$
$$r_{n+1}(k-1),r_{n-1}(k+1)),$$

where $F_{1,2}$ are linear functions of their arguments (we do not present explicit expressions for them because they are too cumbersome). It can be seen that this system determines the populations of states with a given total number of atoms $N_{at} = n + k$. Finding its solution is straightforward and can be done numerically.

As a criterion that demonstrates the possibility of creating nontrivial atomic distributions in the wells, we choose the quantity $\rho_0(N_{at})/\rho_{N_{at}}(0)$, i.e., the ratio of the probability that all atoms are in the nonilluminated well B to the probability of the opposite situation when all atoms are in well A. When the switching of the phase shift θ is absent, the stationary distribution of atoms between the wells is uniform and this quantity is equal to unity. Figure 5 shows the dependence of this quantity on phase shifts $\theta_\pm$ between which switching occurs. It is seen that when approaching the line $\theta_+ = \theta_-$, the quantity $\rho_0(N_{at})/\rho_{N_{at}}(0)$ expectedly tends to unity. Overall, a rather complex landscape appears with multiple maxima and minima. This suggests that the correct selection of $\theta_\pm$ can allow efficient 'pumping' of atoms from one potential well to another. The nonequilibrium distribution arises as a result of permanently occurring phase shifts that change the phase relations between atoms in different potential wells.

5. Conclusions

In the paper, a fundamentally new interferometric feedback scheme was investigated and its prospectivity was demonstrated by the example of relatively simple theoretical models. It is based on several coherent feedback loops using

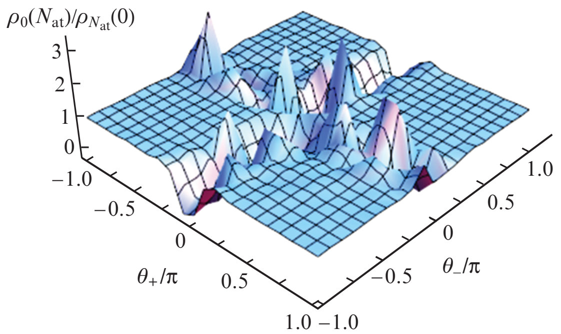

Figure 5. Dependence of the parameter $\rho_0(N_{at})/\rho_{N_{at}}(0)$ on the controlled phase shifts $\theta_\pm/\pi$ for $N_{at} = 10$, $\omega_A = 5\lambda$, $\omega_B = 7\lambda$, $\phi = \pi/50$, $v = 10^4\lambda$.

Mach–Zehnder interferometers. We considered the application of this scheme to the problem of controlling the state of an atomic BEC. It was found that this scheme can efficiently suppress the decoherence process caused by phase contrast probing of the condensate. This effect can find applications in preparation and storage of quantum states.

The proposed scheme also showed its effectiveness for controlling the spatial distribution of atoms in the condensate. In the framework of the two-mode approximation, it was found that multi-loop feedback in combination with measurement-based feedback is capable of creating highly nonequilibrium stationary distributions of condensate atoms localised in a double-well potential.

Although the idea of organising the feedback of the considered type is quite simple, its experimental implementation is associated with certain difficulties. First of all, the main feedback tool, the Mach–Zehnder interferometer, is a highly delicate system to tune and to use. A study of how the features of a real experiment (in particular, the imperfect shape of the atomic cloud) will affect the results obtained will be the subject of a separate consideration. In addition, taking into account interatomic interaction is of undoubted practical interest. Another promising development of the present work is the study of the case of a quantized field. It seems that the probing of the BEC by essentially non-classical radiation (for example, being in a squeezed state) can lead to nontrivial correlations between atoms and the field.

Acknowledgements. One of the authors (V.A. Tomilin) acknowledges the support of the Russian Foundation for Basic Research (Grant No. 19-32-80018). L.V. Il'ichov is grateful for the support provided in the framework of the State Assignement (Project AAAA-A17-117052210003-4, internal FASO number 0319-2016-0002) at the Institute of Automation and Electrometry, Siberian Branch, Russian Academy of Sciences.

References

1. Shapiro J.H., Saplakoglu G., Ho S.-T. *J. Opt. Soc. Am. B*, **4**, 1604 (1987).
2. Iida S., Yukawa M., Yonezawa H., Yamamoto N., Furusawa A. *IEEE Trans. Automat. Contr.*, **57**, 2045 (2012).
3. Li Y., Luo B., Guo H. *Phys. Rev. A*, **84**, 012316 (2011).
4. Negretti A., Poulsen U.V., Mølmer K. *Phys. Rev. Lett.*, **99**, 223601 (2007).
5. Steixner V., Rabl P., Zoller P. *Phys. Rev. A*, **72**, 043826 (2005).
6. Bushev P. et al. *Phys. Rev. Lett.*, **96**, 043003 (2006).
7. Szigeti S.S. et al. *Phys. Rev. A*, **80**, 013614 (2009).
8. Szigeti S.S. et al. *Phys. Rev. A*, **82**, 043632 (2010).
9. Il'ichov L.V. *JETP Lett.*, **106**, 12 (2017) [*Pis'ma Zh. Eksp. Teor. Fiz.*, **106**, 14 (2017)].
10. Il'ichov L.V., Chapovsky P.L. *Quantum Electron.*, **47**, 463 (2017) [*Kvantovaya Elektron.*, **47**, 463 (2017)].
11. Breuer H.-P., Petruccione F. *The Theory of Open Quantum Systems* (Oxford: Oxford University Press, 2002).
12. Wiseman H.M., Milburn G.J. *Quantum Measurement and Control* (Cambridge: Cambridge University Press, 2009).
13. Levy S., Lahoud E., Shomroni I., Steinhauer J. *Nature*, **449**, 579 (2007).
14. Rab M. et al. *Phys. Rev. A*, **77**, 061602(R) (2008).
15. Tomilin V.A., Il'ichov L.V. *Ann. Phys. (Berlin)*, **528**, 619 (2016).
16. Diósi L. arXiv:1401.0476v1 (2014).
17. Tomilin V.A., Il'ichov L.V. *JETP Lett.*, **94**, 676 (2012) [*Pis'ma Zh. Eksp. Teor. Fiz.*, **94**, 734 (2011)]; *JETP*, **116**, 181 (2013) [*Zh. Eksp. Teor. Fiz.*, **143**, 211 (2013)]; *Ann. Phys. (Berlin)*, **526**, 520 (2014).
18. Blanchard Ph., Jadczyk A. *Rep. Math. Phys.*, **36**, 235 (1995).
19. Il'ichov L.V., Chapovsky P.L. *JETP Lett.*, **102**, 14 (2015) [*Pis'ma Zh. Eksp. Teor. Fiz.*, **102**, 18 (2015)].