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Interference in between the acts of pre- and postselection

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Abstract. **As an alternative approach for measuring the weak effects associated with the artificial preparation of rare events in quantum metrology, we propose the study of the interference pattern generated by acts of pre- and postselection of a quantum system. An example of two Mach–Zehnder interferometers connected by a cross-Kerr nonlinearity is considered. Postselection of photon states at the output of one of the interferometers and the application of a controlled phase shift in one of its arms induces interference phenomena in the photodetection statistics at the output of the second interferometer. The nonlinearity parameter determines the shift and width of the structures in the interference pattern. The main features of this pattern are studied depending on the magnitude of the Kerr nonlinearity and the number of photons at the input of the interferometers.**

Keywords: *quantum metrology, quantum interference, pre- and postselection, weak values.*

1. Introduction

The search for rare events is an important area in modern quantum physics. Some laser cooling mechanisms which rely on random walk [1], proton transfer between water molecules in chemical reactions [2], alpha decay and the creation of Higgs bosons in reactions at the Large Hadron Collider [3] are examples of processes in which the rare events occur with a probability of 10^{-4}–10^{-17}.

In quantum metrology, useful rare events are selected from the data array of the standard repeated procedure of measuring the system under study. The ideological basis of this approach stems from the formulation of quantum mechanics, known as time symmetric quantum mechanics (TSQM) [4]. The symmetry of both directions of time is explicitly introduced into it by selecting events of the successful preparation of a definite initial state of the system (preselection) and detecting the system in a definite final state (postselection). The probability of successful postselection (provided the initial state was successfully prepared) is purposefully made sufficiently small. This circumstance is compensated by the large value of the so-called weak value of a certain observable [5]. It is associated with the degree of freedom of the system playing the role of the meter, and the weak effect corresponds to the interaction of the meter and the rest of the system. The small probability amplitude of successful postselection appears in the denominator of the expression for the weak value, providing what is known in the literature as weak value amplification (WVA). In this form, the WVA method has proven effective in observing the previously predicted optical spin Hall effect [6]. Then this method was applied to measuring spatial and angular displacements, phase displacement, frequency, and other parameters [7].

The purpose of this work is to determine the parameters of the physical interaction between photons in a medium. The photon-photon interaction changes the relative phases of quantum alternatives, modifying the interference pattern. Under typical conditions in nonlinear materials, the phase shift due to the photon–photon interaction χ is $\sim 10^{-18}$ [8]. This value increases with increasing medium length or when using materials with greater nonlinearity. A phase shift of $\chi = 10^{-7}$ per photon was achieved in optical fibre in Ref. [9], and in Ref. [10] this shift was found to be 1.3×10^{-6}. In a resonator, quantum electrodynamics makes it possible to obtain a phase shift of 0.13 [11] for the single-photon regime and in quantum dots a shift of $\pi/4$ was obtained [12]. The interaction of photons via a system of superconductors leads to a phase shift of 0.01 [13]. In the case of small phase shifts, problems arise in obtaining information about the interaction parameters by methods of traditional optical interferometry. Therefore, in nonlinear optics, the approach with the generation of informative rare events is also relevant.

In the experimental work [14] using the effect of electromagnetically induced transparency, the possibility of producing a noticeable phase shift in a coherent state through postselection was shown. For a pulse containing thousands of photons, the shift was 10^{-5} per single photon. In the context of the present paper, Ref. [15] is especially important for us. In Ref [15], the WVA method is implemented in a setup of two Mach–Zehnder interferometers to determine the Kerr nonlinearity parameter of a medium through which interferometers are coupled. To the input of the interferometer, which plays the role of a 'system', a single-photon state is applied, and then postselection is carried out in one of the output arms. The second interferometer is a meter with a coherent state at the input. An effective increase in the number of photons in the system and, accordingly, a phase shift due to Kerr nonlinearity is ensured by the imbalance of the beam splitter in the interferometer. In a later paper [16] by the same authors, the achieved weak value of the number of quanta was eight.

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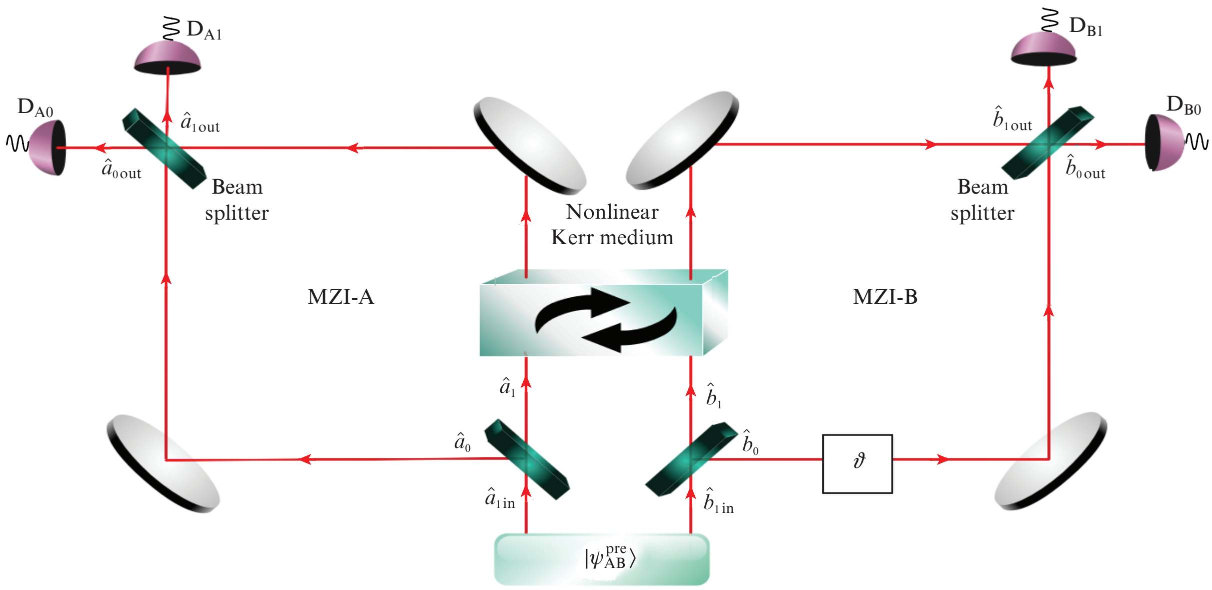

Figure 1. Interference experiment with a multiphoton state of light; postselection consists of accounting for realisations with certain fractions of the total number of photons recorded at the output of the MZI-B interferometer (D are detectors).

Similar in appearance, but fundamentally different scheme is the subject of this work. It is based on the concept of interference, generated by acts of pre- and postselection of quantum states, proposed in [17]. Within the framework of this concept, a specific geometric phase appears as the development of an operational approach to this concept [18].

The concept of interference mentioned above assumes the existence of a Mach–Zehnder interferometer with selection of radiation states at the input and output, in one arm of which a controlled phase shift ϑ is introduced, and the radiation in the other arm interacts with a standard element of environment. Next, a separate interference experiment is carried out with this element, the results of which are taken into account if the pre- and postselection in the Mach–Zehnder interferometer is successful. The shift of the interference pattern can be controlled by changing ϑ. In the additional shift, which is the geometric phase*, the interaction parameters are reflected.

The scheme considered below, as well as the scheme from Ref. [15], consists of two Mach–Zehnder interferometers coupled via a nonlinear Kerr medium. One of them is subject to pre- and postselection and provides a controlled phase shift, while the radiation in the second interferometer serves as an element of the environment. At the output of the second interferometer, the interference process is recorded. We demonstrate the possibility of detecting a small Kerr phase shift in this process. In Section 2, we consider a scheme in which interference arises within the framework of TSQM. Its application in the case of single-photon states at the input is considered in Section 3. Next, we study the curious parallels between the postselection regime used and the 'quantum eraser' scenario. This allows us to trace the nature of correlations between two different rare events, which opens up new prospects in metrology. In Section 4, we return to discussing the differences between the scheme [15] based on the WVA process and the scheme of the present work.

* In Ref. [18] the existence of two geometric phases is demonstrated in the general case under the conditions of pre- and postselection and their gauge invariance is proved.

2. TSQM interference in a system of two interferometers

Suppose that a two-mode state $|\psi\rangle_{\rm AB}^{\rm pre}$ with n photons in each mode is prepared at the inputs of two Mach–Zehnder interferometers (MZI) (Fig. 1):

$$|\psi\rangle_{\rm AB}^{\rm pre} = |n\rangle_{a_1 \rm in} \otimes |n\rangle_{b_1 \rm in}\,. \tag{1}$$

Our goal is to analyse the relationship between the output state of mode A (interference pattern) and the state of mode B after their interaction in the Kerr medium. For this purpose, a controlled phase shift ϑ is provided in one of the arms of the MZI-B; χ is the parameter of Kerr interaction (phase shift in the interaction of a pair of photons from modes a_1 and b_1). We assume that the transformations on the beam splitters have the form:

$$\begin{gathered}\hat{a}_{0\,\rm out} = \frac{1}{\sqrt{2}}(\hat{a}_0 + \hat{a}_1), \quad \hat{a}_{1\,\rm out} = \frac{1}{\sqrt{2}}(\hat{a}_0 - \hat{a}_1), \\ \hat{a}_{0\,\rm in} = \frac{1}{\sqrt{2}}(-\hat{a}_0 + \hat{a}_1), \quad \hat{a}_{1\,\rm in} = \frac{1}{\sqrt{2}}(\hat{a}_0 + \hat{a}_1).\end{gathered} \tag{2}$$

Similar relations hold for mode B. In terms of the creation operators, according to relations (2), the initial state is

$$|\psi\rangle_{\rm AB}^{\rm pre} = |n\rangle_{a1\,\rm in} \otimes |n\rangle_{b1\,\rm in} = \frac{(\hat{a}_0^\dagger + \hat{a}_1^\dagger)^n (\hat{b}_0^\dagger + \hat{b}_1^\dagger)^n}{2^n n!} |{\rm vac}\rangle_{\rm AB}, \tag{3}$$

where $|{\rm vac}\rangle_{\rm AB}$ is the vacuum state. In the model under consideration, the nonlinear interaction of modes is described by the operator $\exp({\rm i}\chi \hat{a}_1^\dagger \hat{a}_1 \hat{b}_1^\dagger \hat{b}_1)$, where $\hat{b}_1^\dagger \hat{b}_1$ and $\hat{a}_1^\dagger \hat{a}_1$ are the photon numbers in the vertical inner arms of the interferometers. Postselection is carried out with a focus on the statistics of photon detection in the vertical and horizontal arms at the MZI-B output:

$$|\psi\rangle_B^{\text{post}} \propto (q_0\hat{b}_{0\ \text{out}}^{\dagger} + q_1\hat{b}_{1\ \text{out}}^{\dagger})^n |\text{vac}\rangle_B.$$

Below q_0 and q_1 take the values 0 or 1, which corresponds to postselection by registration of all photons in the horizontal or vertical output arm. In a different situation, as a postselection state it is possible to use, e.g., $|\psi\rangle_B^{\text{post}} \propto |n-n_1, n_1\rangle_B = b_{0\ \text{out}}^{\dagger\ n-n_1} b_{0\ \text{out}}^{\dagger\ n_1}|\text{vac}\rangle$, where n is the total number of photons and n_1 is their number in the vertical output arm of the MZI-B.

In terms $\hat{b}_0$ and $\hat{b}_1$, the postselection state is

$$|\psi\rangle_B^{\text{post}} \propto [(q_0+q_1)\hat{b}_0^{\dagger} + (q_0-q_1)\hat{b}_1^{\dagger}]^n |\text{vac}\rangle_B. \tag{4}$$

At the output of the MZI-A, the unnormalised conditional state after preparation $|\psi\rangle_{AB}^{\text{pre}}$ and postselection in the state $|\psi\rangle_B^{\text{post}}$ is

$$|\tilde{\psi}\rangle_A \propto {}_B^{\text{post}}\langle\psi|\hat{U}|\psi\rangle_{AB}^{\text{pre}}, \tag{5}$$

where $\hat{U} = \exp(\mathrm{i}\chi\hat{a}_1^{\dagger}\hat{a}_1\hat{b}_1^{\dagger}\hat{b}_1 + \mathrm{i}\vartheta\hat{b}_0^{\dagger}\hat{b}_0)$. Let us explain the meaning of the right-hand side of the above expression. The state $|\psi\rangle_{AB}^{\text{pre}}$ belongs to the tensor product $\mathcal{H}_A \otimes \mathcal{H}_B$ of the Hilbert spaces of the photon modes in the interferometers A and B and can be factorised, i.e., is separable. The operator $\hat{U}$ acts on this tensor product, as a result of which, in the general case, an entangled state arises. Multiplication by a conjugate vector ${}_B^{\text{post}}\langle\psi|$ from $\mathcal{H}_B^*$ affects the second factors in the components of this entangled state and turns it into a vector from $\mathcal{H}_A$.

The calculation of the right-hand side of relation (5) using equations (3) and (4) yields an expression for conditional unnormalised state

$$|\tilde{\psi}\rangle_A \propto \sum_{k=0}^{n} \Xi(k,n,\vartheta,\chi)|n-k\rangle_{a_0} \otimes |k\rangle_{a_1}. \tag{6}$$

Here

$$\Xi(k,n,\vartheta,\chi) = \sqrt{\frac{n!}{k!(n-k)!}}$$
$$\times[(q_0+q_1)\exp(\mathrm{i}\vartheta) + (q_0-q_1)\exp(\mathrm{i}k\chi)]^n.$$

The conditional statistical operator normalised to unit trace at the output of MZI-A after a successful postselection of mode B has the form:

$$\rho_A = \frac{|\tilde{\psi}\rangle_A\langle\tilde{\psi}|}{{}_A\langle\tilde{\psi}|\tilde{\psi}\rangle_A}. \tag{7}$$

Here the denominator represents the probability of successful postselection. The interference pattern at the MZI-A output is given by the difference

$$I_A = \langle(a_0^{\dagger}a_0)_{\text{out}} - (a_1^{\dagger}a_1)_{\text{out}}\rangle, \tag{8}$$

where averaging is performed over state (7) taking relations (2) into account. Dependence of the probability of postselection on ϑ

$$P = {}_A\langle\tilde{\psi}|\tilde{\psi}\rangle_A \propto \sum_{k=0}^{n} \frac{n!}{k!(n-k)!}\cos^{2n}\left(\frac{\vartheta-k\chi}{2}\right)$$

can be used to generate rare events.

3. Results for single-photon and multiphoton states

In the case of single-photon states at the input, $|1\rangle_A|1\rangle_B$, the calculation of (8) yields

$$I_A = \frac{\nu + \sqrt{2\nu}\cos(\vartheta-\theta_g)}{2+\sqrt{2\nu}\cos(\vartheta-\theta_g)}, \tag{9}$$

where $\nu = 1 + \cos\chi$; $\theta_g = \chi/2$ is the operational geometric phase in systems with pre- and postselection [17, 18]. This expression describes the phenomenon of interference due to postselection of the state of the photon of mode B (B-photon). The interference pattern is controlled by phase ϑ.

Thus, the process of pre- and postselection with an arbitrarily weak (but not zero) Kerr interaction between the modes and the possibility of control of the phase ϑ allows transformation of any value of the phase from postselected mode B to mode A. [Note that, as follows from (9), the dependence on ϑ disappears only at $\chi = 0$, i.e., the interference structure becomes infinitely narrow.] The total phase $\vartheta - \theta_g = \pi$ can be transferred to the MZI-A at $\vartheta = \pi + \chi/2$. In this case, a contrast switching is implemented [between $I_A = \sqrt{\nu/2}$ for $\vartheta - \theta_g = m\pi$, $m = 0, 2, 4, \ldots$ and $I_A = -\sqrt{\nu/2}$ for $\vartheta - \theta_g = (2m+1)\pi + \chi/2$, $m = 0, 1, 2, \ldots$] for small values of χ, which is demonstrated by the solid curve in Fig. 2.

The visibility of the interference pattern decreases as χ approaches π. The reason for this is as follows: in the vertical inner arm of the MZI-A, a phase shift $\chi = \pi$ appears on average in every second realisation of the experiment, when the photon in the MZI-B moves along the vertical arm after the input beam splitter. Due to the equal probability of the phase shift appearance or its absence, the photon of mode A is equally likely to be detected in horizontal or vertical exit from the MZI-A. These probabilities are independent of ϑ and the visibility is zero.

It is interesting to trace the similarities and differences between the phenomenon under consideration and the 'quantum eraser' effect [19]. The full two-mode state at the exit from interferometers for single-photon states of modes at the input $|1\rangle_A|1\rangle_B$ has the form:

$$|\psi\rangle^{\text{out}} = \frac{1}{4}[\alpha|0\rangle_A|0\rangle_B + \beta|0\rangle_A|1\rangle_B$$
$$+\gamma(|1\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B)]. \tag{10}$$

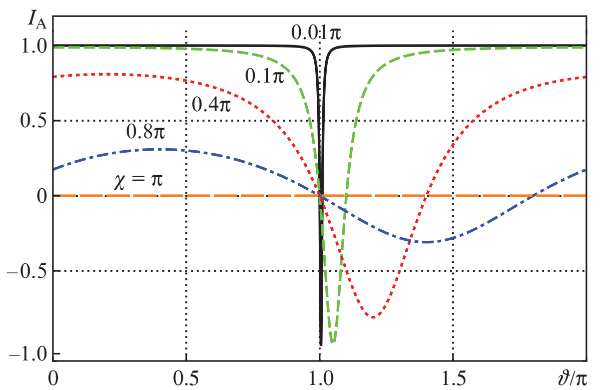

Figure 2. Differences in probabilities of detecting an A-photon [expression (9)] as functions of the controlled phase shift along the path of a B-photon for various values of χ.

Here $\alpha = 2e^{i\vartheta} + e^{i\chi} + 1$; $\beta = -2e^{i\vartheta} + e^{i\chi} + 1$; $\gamma = e^{i\chi} - 1$. Calculating a trace over the basis of states of mode A, we obtain

$$\rho_B^{out} = \mathrm{Tr}_A(|\psi\rangle^{out}\langle\psi|) = \frac{1}{16}\begin{pmatrix}\alpha\alpha^* + \gamma\gamma^* & \alpha\beta^* + \gamma\gamma^* \\ \beta\alpha^* + \gamma\gamma^* & \beta\beta^* + \gamma\gamma^*\end{pmatrix}. \quad (11)$$

The interference pattern at the MZI-B output (difference of photon detection probabilities in the vertical and horizontal arms),

$$I_B \propto \alpha\alpha^* - \beta\beta^* = \cos\left(\frac{\chi}{2}\right)\cos\left(\vartheta - \frac{\chi}{2}\right), \quad (12)$$

depends on ϑ and parameter χ. Similarly, for the state at the MZI-A output, we have

$$\rho_A^{out} = \frac{1}{16}\begin{pmatrix}\alpha\alpha^* + \beta\beta^* & \alpha\gamma^* + \beta\gamma^* \\ \gamma\alpha^* + \gamma\beta^* & 2\gamma\gamma^*\end{pmatrix}$$

and the corresponding interference signal

$$I_A \propto \alpha\alpha^* + \beta\beta^* - 2\gamma\gamma^* = \frac{1}{2}(1 + \cos\chi). \quad (13)$$

In a different problem statement, similar to the observation scheme in the implementation of the 'quantum eraser,' the information about the path of the mode-B photon appears. For this purpose, it is enough to remove the output beam splitter in the MZI-B. The state of both modes in this case is

$$|\psi\rangle^{out} = \frac{1}{2\sqrt{2}}[(e^{i\chi} - 1)|0\rangle_A + (e^{i\chi} + 1)|1\rangle_A] \otimes |0\rangle_B + \frac{1}{\sqrt{2}}e^{i\vartheta}|0\rangle_A \otimes |1\rangle_B. \quad (14)$$

As expected, the expression for I_A remains the same [see Eqn (13)] and depends only on χ.

The joint consideration of expressions (13), (14), and (9) allows comparing the interference processes involving the photon of mode A in situations with different types of 'processing' information at the output of the MZI-B. The dependence of the interference signal on the controlled phase ϑ appears during the transition from the first two scenarios to the postselection scenario. This allows a comparison with the 'quantum eraser' effect. The postselection included in the scenario of the latter [19] is similar to the postselection necessary for the appearance of the dependence upon ϑ. In 'quantum eraser', however, with successful postselection, information about the particle paths disappears, while in our scheme, pre- and postselection by no means affect the possibility of obtaining such information.

Consider briefly the issue of the probability of successful postselection. Such probability increases with the value of the interaction parameter. This decreases the visibility of the interference pattern. At $\chi \approx \pi$, postselection becomes practically unnecessary, because $P_{B0} \approx 1$; at the same time, the interference pattern becomes uninformative due to zero visibility. And vice versa, rare postselection events concentrate valuable information about the magnitude of the interaction parameter, reflected in the width and shift of the interference fringes (see Fig. 2).

In our scheme, at $\chi \lessapprox 0.1\pi$, switching between the detectors at the MZI-A output occurs in a narrow range of phase ϑ

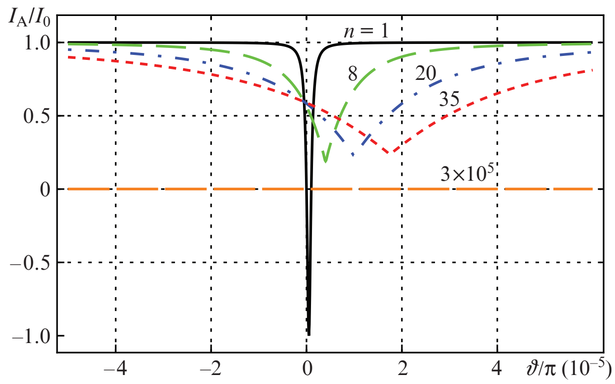

Figure 3. Normalised interference signal as a function of the controlled phase at $\chi = 10^{-5}$ for various numbers of photons n in the input mode states.

values in the MZI-B. In this case, the probability of successful postselection (11) at $\chi = 0.1$ is

$$P = \sin^2\left(\frac{\chi}{4}\right) \approx 6.25\times10^{-4}. \quad (15)$$

Thus, on average, one realisation out of 1.6×10^3 is an informative event.

In an ideal experiment, for the considered value of χ, with the success of postselection, the probability of detecting the photon mode A in the vertical output arm at $\vartheta = \pi + \chi/2$ is $P_{A1} = 0.9994$. This means that, on average, only six successful post-selections out of 10^4 do not show the switching phenomenon, i.e., almost every success is accompanied by pair triggering of D_{A1} and D_{B0} detectors. This is the advantage of the proposed scheme as a tool of quantum metrology.

The results of numerical calculations for states with a large number of photons at $\Delta\vartheta = 10^{-7}$ (phase step in the numerical calculation), $\chi = 10^{-5}$, $q_0 = 0$, $q_1 = 1$ are presented in Fig. 3. It is seen that with increasing number of photons, the interference structure broadens, shifts, and its visibility decreases. The phase shift can be determined from the equation $\theta_g|_{\partial I/\partial\vartheta = 0}$, which yields $\theta_g = n\chi/2$ for the geometric phase. The signal I_A is now normalized to $I_0 = \langle(\hat{a}_0^\dagger\hat{a}_0)_{out} + (\hat{a}_1^\dagger\hat{a}_1)_{out}\rangle = n$ (n being the number of photons in one mode). At $\theta_g = n\chi/2 = \pi$, the visibility of the interference pattern vanishes. The specific beak-like shape of the minimum of the curves in the interference patterns makes them convenient for accurate determination of χ. Naturally, one has to pay for this with a low probability of successful post-selection for large n and small χ. Moreover, the existence of a certain optimum in the number of photons in the prepared states of the modes is obvious, because with its growth, the visibility of the interference pattern decreases.

4. Discussion and conclusions

It makes sense to return to the already mentioned difference between the WVA scheme from [15] and the scheme discussed here, since it may be suspected that the latter is also some modification of WVA. However, this is not the case. The weak imbalance of the beam splitter used in [15] leads to a real weak number of photons passing through the Kerr medium in a postselected interferometer, i.e. effectively increases their number. This is a prerequisite of the 'classical' version of the WVA scheme. In the above scheme, the probability of postselection success is controlled by phase ϑ rather than the imbalanced beam splitter. The weak value of the number of photons turns out to be almost pure imaginary in the limit of small χ. The physical meaning of the real and imaginary parts of the weak value is fundamentally different [20] (in particular, because of the presence of the imaginary part of the weak value, the probability of the postselection success depends on χ). We will not discuss this difference here, since in the context of the present work weak values did not appear.

To summarise, it can be argued that in this paper, using the Kerr effect as an example, we propose a new method for determining the parameters of weak nonlinear effects in optics. We have shown that in the case of weak coupling between two separated systems, namely, Mach–Zehnder interferometers, pre- and postselection of quantum states and the resulting interference process can be useful in situations where the standard quantum-optical approach (only with state preparation) does not provide sufficient information about the interaction. Postselection of the state at the output of one interferometer creates an interference pattern in the output signal of another interferometer, prescribed by the controlled phase delay inside the postselected interferometer. The proposed approach does not reduce to the well-known WVA approach and focuses on a specific geometric phase, which manifests itself as a shift in the interference pattern.

The most important results are shown in Fig. 3. One can see the possibility of obtaining a structure not as narrow as in the case of single-photon states at the input of interferometers, with a good visibility. A relation is shown between the probability of successful postselection (localisation of photons in a specific output arm of the MZI-B) and the visibility of the interference pattern, which reflects a kind of competition between the wave and particle properties of light quanta. The rarity of postselection success events is not a fundamental obstacle to modern quantum technology. In a well-known recent experiment on testing the Bell inequalities [21], the frequency of occurrence of informative events did not exceed one event per hour.

In this paper, the problems of the finite efficiency of the detectors and the possibility of photon loss were not considered. These issues are supposed to be the subject of a separate publication.

In our consideration, we used a quantum optical system. However, we expect the manifestation of the effect of interference generated by pre- and postselection of states in other quantum systems. In addition, the model is interesting in terms of studying the fundamental problems of modern quantum physics, such as quantum discord, amplification of weak values, quantum nondemolition measurements, and the development of new methods for quantum metrology of weak physical effects.

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