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Laser cooling of atoms at narrow optical transitions in fields with a polarisation gradient

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Abstract. **The efficiency of the polarisation mechanisms of laser cooling of atoms is analysed using narrow optical transitions for which the recoil energy is greater than or comparable to the natural line width. By the example of atoms with the $J_g = 1 \to J_e = 2$ optical transition (J_g and J_e are the total angular momenta of the ground and excited states) in the resonant light fields of $\sigma_+ - \sigma_-$ and lin⊥lin configurations, the analysis of the minimum achievable energies of laser-cooled atoms is performed. It is shown that the polarisation mechanisms of laser cooling under conditions of a significant influence of recoil effects become less efficient and do not lead to average kinetic energies of an ensemble of atoms below the Doppler limit.**

Keywords: *laser cooling, clock optical transitions, recoil effect, optical lattices.*

1. Introduction

A laser is a powerful and perfect tool for an efficient control of the translational degrees of freedom of atoms. Currently, laser cooling has become a field of science at the junction of laser physics and atomic optics (see, for example, [1–3]), which has many prospects and applications. The developed methods of laser cooling and cold atoms are widely used, in particular, for the production of Bose–Einstein condensate of neutral atoms [4, 5], as well as in the fields of quantum informatics [6], atomic nanolithography [7] and interferometry [8]. The combination of laser cooling and modern methods of precision spectroscopy allows one to create frequency and time standards, the relative stability and accuracy of which reaches values on the order of 10^{-18} [9–11]. To date, various methods have been developed for the localisation and cooling of atoms (magneto-optical and dipole traps, optical lattices, etc.), which have become an integral part of modern fundamental and applied science. Such progress would not have been possible without a theoretical analysis and consideration of the processes accompanying the atom–electromagnetic field interaction. A theoretical description, taking into account the multi-level atomic structure, level degeneracy, spontaneous decay, recoil effect and field polarisation effect, is an extremely difficult task. The beginning of its solution was laid in the 1970s–1980s by studying the simplest system: a two-level atom in a resonant light field [1, 2, 12–14]. An analytical study of this model using the semiclassical approach (see, for example, [1, 2, 15–20]), which considers cooling in terms of light-induced forces and their fluctuations (diffusion in the momentum space), made it possible to clearly understand the main cooling mechanisms. However, the semiclassical approach describes only those cases when the single-photon recoil frequency $\omega_r = \hbar \boldsymbol{k}^2/(2M)$ ($2M$ is the mass of the atom, and $\boldsymbol{k}$ is the wave vector) is small compared to the natural width of the line of the cooling transition γ (i.e., the recoil parameter $\varepsilon_r = \omega_r/\gamma \ll 1$), and is not suitable for describing the kinetics of laser cooling of atoms using narrow transitions, such as intercombination transitions of ytterbium, magnesium, calcium, and strontium [21–24].

Earlier, we proposed an effective method [25–27], which allows one to find a numerical stationary solution of the quantum kinetic equation for the atomic density matrix in a resonant light field taking into account the recoil effects in the atom–field photon interaction, containing complete information about both internal and translational degrees of freedom of atoms. For example, this solution makes it possible to find stationary distributions over the momenta and coordinates of atoms cooled by the laser field of two counterpropagating waves, as well as significantly expand the field of theoretical analysis of laser cooling and, in particular, obtain information about the dynamics [28] of laser cooling of atoms using light waves resonant to narrow optical transitions. Thus, we have recently shown in [24] that the kinetics of atoms with a ground state nondegenerate over the projection of the angular momentum, characterised by a large recoil parameter $\varepsilon_r = \omega_r/\gamma \gtrsim 1$ [for example, intercombination transitions $^1S_0 \to {}^3P_1$ of ^{88}Sr ($\varepsilon_r = 0.635$), ^{40}Ca ($\varepsilon_r = 32.3$) and ^{24}Mg ($\varepsilon_r = 1100$) atoms] can be described in a unified manner, which was formulated as a "scaling law" for laser cooling of atoms using narrow optical transitions. It has been also shown in this paper that the optimal detuning value for effective cooling at narrow optical transitions differs from the optimal detuning during Doppler cooling and is universally expressed in units of the recoil frequency.

It is well known (see, for example, [15, 19]) that the presence of additional contributions to the dissipative friction force arising from the existence of polarisation gradients of the light field (the orientation gradient of the polarisation vector in the $\sigma_+ - \sigma_-$-configuration field formed by counterpropagating waves with opposite circular polarisations, or the ellipticity gradient of the polarisation vector in the lin⊥lin-configuration field formed by counterpropagating waves with

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orthogonal linear polarisations) leads to the possibility of laser cooling of atoms below the Doppler limit, determined by the temperature $T_D \approx \hbar\gamma/(2k_B)$. It should be noted that the results obtained correspond to a semiclassical description of the kinetics of atoms and are applicable for laser cooling of atoms using light fields that are resonant to closed optical transitions characterised by an extremely small recoil parameter, $\varepsilon_r \ll 1$. The possibility of efficient laser cooling of atoms in a monochromatic field using narrow optical transitions, i.e., transitions with a smaller value of the natural width γ ($\varepsilon_r > 1$), was doubtful, because single absorption of a photon lead to the fact that the atom came out of resonance and ceased to interact effectively with the field. It was shown in [24] that this problem can be solved by a moderate increase in the light field intensity, which results in possible efficient cooling of atoms under field broadening conditions. Nevertheless, the effect of the polarisation mechanisms of laser cooling at $\varepsilon_r > 1$ is still to be evaluated.

In the present work, we study the possibilities of laser cooling in light fields with a polarisation gradient resonant to a narrow optical transition, i.e., under the conditions of a significant influence of the recoil effects on the resonant nature of the interaction of light with atoms. As an example, we consider atoms with the optical transition $J_g = 1 \to J_e = 2$ (J_g and J_e are the total angular momenta of the ground and excited states), which allow the appearance of sub-Doppler laser cooling mechanisms both in the lin⊥lin- and $\sigma_+-\sigma_-$-configuration fields. The data of polarisation configurations of light fields for deep laser cooling is analysed and compared with the results obtained for a two-level atomic model.

2. Statement of the problem

Let us consider a one-dimensional motion of atoms (along the z axis) with a closed optical transition $J_g = 1 \to J_e = 2$ in a resonant monochromatic field formed by counterpropagating light waves of equal intensity,

$$\boldsymbol{E}(z,t) = E_0[\boldsymbol{e}_1\exp(ikz) + \boldsymbol{e}_2\exp(-ikz)]\exp(-i\omega t) + \text{c.c.}, \quad (1)$$

where E_0 is the complex amplitude of the light waves; ω is the frequency; and $k = \omega/c$. The polarisations of the counterpropagating waves $\boldsymbol{e}_1$ and $\boldsymbol{e}_2$ can be represented as

$$\boldsymbol{e}_n = \sum_{q=0,\pm1} e_n^{(q)}\boldsymbol{e}_q, \quad n = 1,2, \quad (2)$$

where $\boldsymbol{e}_\pm = \mp(\boldsymbol{e}_x \pm i\boldsymbol{e}_y)/\sqrt{2}$ and $\boldsymbol{e}_0 = \boldsymbol{e}_z$ are the unit vectors in the cyclic basis. Note that the components $e_n^{(0)} = 0$ due to the orthogonality of the vectors $\boldsymbol{e}_n$ and $\boldsymbol{k}$. In particular, counterpropagating waves with orthogonal polarisations form well-known configurations of light fields:

1) lin⊥lin, a configuration of the light field formed by counterpropagating waves with orthogonal linear polarisations (for example, $\boldsymbol{e}_1 = \boldsymbol{e}_x$, $\boldsymbol{e}_2 = \boldsymbol{e}_y$); and

2) $\sigma_+-\sigma_-$, a configuration of the light field formed by counterpropagating waves with orthogonal circular polarisations $\boldsymbol{e}_1 = \boldsymbol{e}_+$, $\boldsymbol{e}_2 = \boldsymbol{e}_-$.

The evolution of an ensemble of atoms with a low concentration, when interatomic interaction can be neglected, is determined by the quantum kinetic equation for the atomic density matrix

$$\frac{d\hat\rho}{dt} = -\frac{i}{\hbar}[\hat H,\hat\rho] + \hat\Gamma\{\hat\rho\}, \quad (3)$$

where $\hat H$ is the Hamiltonian, and $\hat\Gamma\{\hat\rho\}$ describes the spontaneous relaxation of atomic levels.

The Hamiltonian $\hat H$ of the atom can be represented as the sum of the contributions:

$$\hat H = \frac{\hat p^2}{2M} + \hat H_0 + \hat V, \quad (4)$$

where the first term is the kinetic energy operator; the second term $\hat H_0 = -\hbar\delta\hat P_e$ is the Hamiltonian of a free atom in the basis of a rotating wave; $\delta = \omega - \omega_0$ is the detuning of the optical field frequency ω from the atomic transition frequency ω_0;

$$\hat P_e = \sum_\mu |J_e,\mu\rangle\langle J_e,\mu| \quad (5)$$

is the operator of projections onto the levels of the excited state $|J_e,\mu\rangle$, characterised by the total angular momentum J_e and the projection of the angular momentum μ onto the quantisation axis; and the last term $\hat V$ describes the interaction of an atom with field (1). In the electric dipole approximation and in the rotating wave approximation, the interaction operator takes the form

$$\hat V = -[\langle e\|d\|g\rangle(\hat{\boldsymbol{D}}\hat{\boldsymbol{E}}) + \text{h.c.}], \quad (6)$$

where $\langle e\|d\|g\rangle$ is the reduced dipole moment of the optical transition, and the cyclic components of the operator $\hat{\boldsymbol{D}}$ are expressed in terms of the Clebsch–Gordan coefficients:

$$\hat D_q = \sum_{\mu,m} C^{J_e,\mu}_{J_g,m;1,q}|J_e,\mu\rangle\langle J_g,m|, \quad q = 0,\pm1, \quad (7)$$

and determine the matrix elements of the interaction operator of the magnetic sublevels. Accordingly, the interaction operator (6) can be written as

$$\hat V = \Omega_0\sum_{q=0,\pm1}\hat D_q e_1^{(q)}\exp(ikz) + \Omega_0\sum_{q=0,\pm1}\hat D_q e_2^{(q)}\exp(-ikz) + \text{h.c.}, \quad (8)$$

where $\Omega_0 = -\langle e\|d\|g\rangle E_0/\hbar$ is the Rabi frequency for each of the counterpropagating waves. The scheme of spontaneous and stimulated transitions for atoms with a closed optical transition is shown in Fig. 1.

The non-Hamiltonian addition $\hat\Gamma\{\hat\rho\}$ in the equation for the density matrix describes the relaxation of the excited state degenerate from the projection of the angular momentum. In general terms, taking into account the recoil effects, the relaxation operator as a result of spontaneous emission of photons has the form

$$\hat\Gamma\{\hat\rho\} = -\frac{\gamma}{2}(\hat P_e\hat\rho + \hat\rho\hat P_e) + \hat\gamma\{\hat\rho\},$$

$$\hat\gamma\{\hat\rho\} = \frac{3}{2}\gamma\Big\langle\sum_{\zeta=1,2}(\hat{\boldsymbol{D}}\boldsymbol{e}'_\zeta(\boldsymbol{k}))^+\exp(-i\boldsymbol{k}\hat{\boldsymbol{r}})\hat\rho\exp(i\boldsymbol{k}\hat{\boldsymbol{r}}) \times (\hat{\boldsymbol{D}}\boldsymbol{e}'_\zeta(\boldsymbol{k}))\Big\rangle_{\Omega_k}. \quad (9)$$

The symbol $\langle\ldots\rangle_{\Omega_k}$ means averaging over all directions Ω_k of emission of spontaneous photons, and $\boldsymbol{e}'_\zeta$ is the polarisation vector of spontaneous photons ($\zeta = 1,2$).

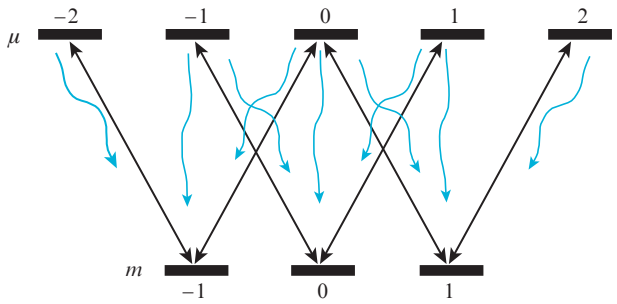

Figure 1. Scheme of spontaneous (wavy arrows) and stimulated (straight arrows) transitions between the magnetic sublevels of the atom μ (excited state) and m (ground state) for atoms with a closed optical $J_g = 1 \to J_e = 2$ transition interacting with the electromagnetic wave field (1).

The solution of quantum kinetic equation (3) for an atomic density matrix is a rather complicated and laborious task requiring the use of various approximations. In our papers [25–27] we proposed an efficient method that allows one to find a numerical stationary solution of the equation for the atomic density matrix in a resonant light field, taking into account recoil effects in the interaction with field photons that contains complete information about both internal and translational degrees of freedom atoms. The method consists in the fact that the equation for the atomic density matrix is divided into a system of equations for its spatial harmonics. After separation of the recurrence relation between spatial harmonics, the system of equations can be solved using the method of continued fractions. We will use this method to analase the minimum attainable kinetic energies of an atomic ensemble under conditions of laser cooling in the fields of $\sigma_+ - \sigma_-$ and lin⊥lin- configurations.

3. Cooling of atoms in the fields of lin⊥lin and $\sigma_+ - \sigma_-$ configurations

The stationary solution of the quantum kinetic equation (3) for the atomic density matrix is defined by the following main parameters of the problem: the detuning of the light field from the atomic resonance δ; the amplitude or intensity of the light field (Rabi frequency Ω_0); the recoil parameter ε_r; the polarisation configuration of the light field and the type of

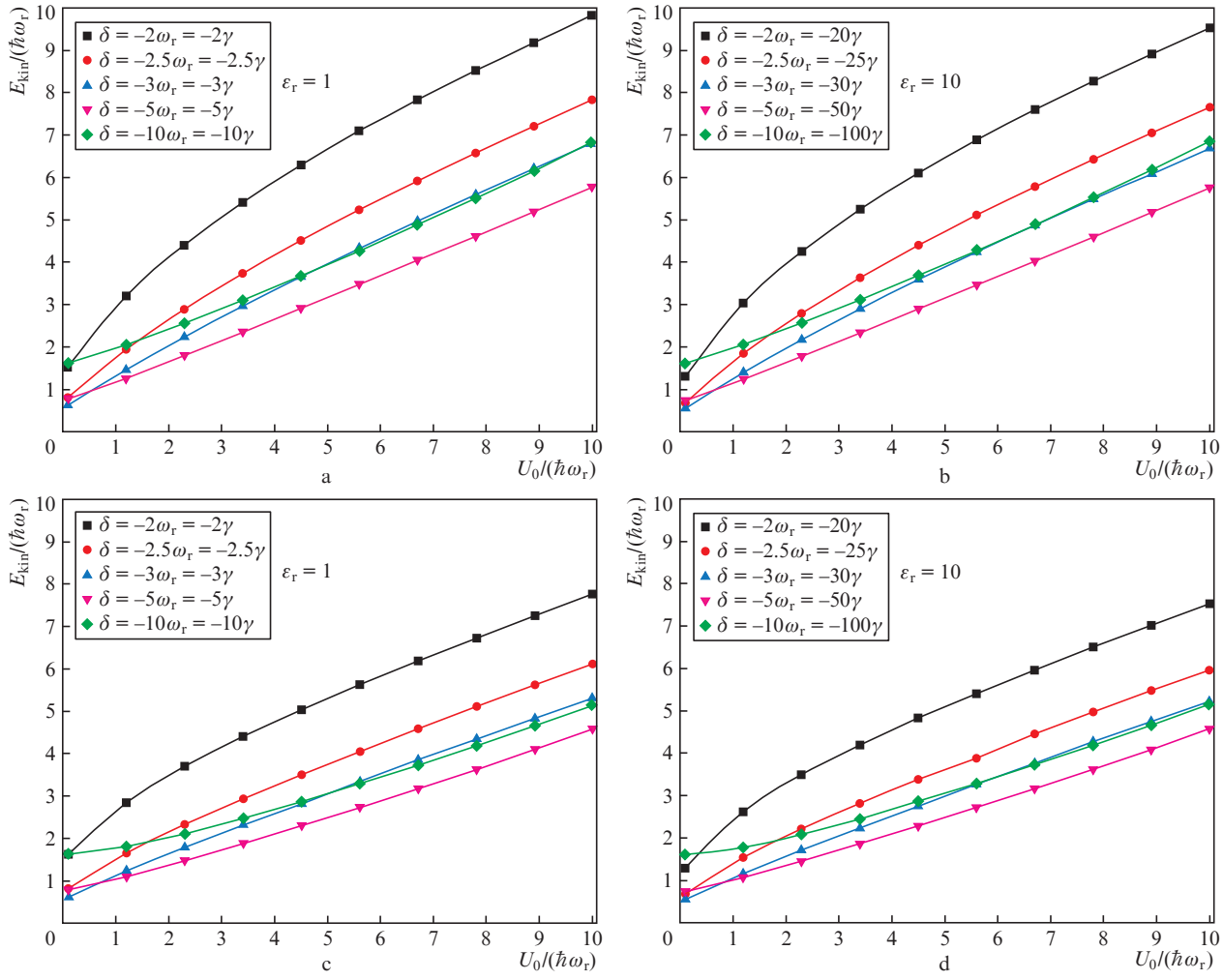

Figure 2. Average kinetic energy E_{kin} of the cold atoms in the lin⊥lin-configuration field as a function of the field shift parameter U_0 at various light field detunings δ for (a, b) $J_g = 0 \to J_e = 1$ transitions with $\varepsilon_r =$ (a) 1 and (b) 10, as well as (c, d) $J_g = 1 \to J_e = 2$ transitions with $\varepsilon_r =$ (c) 1 and (d) 10.

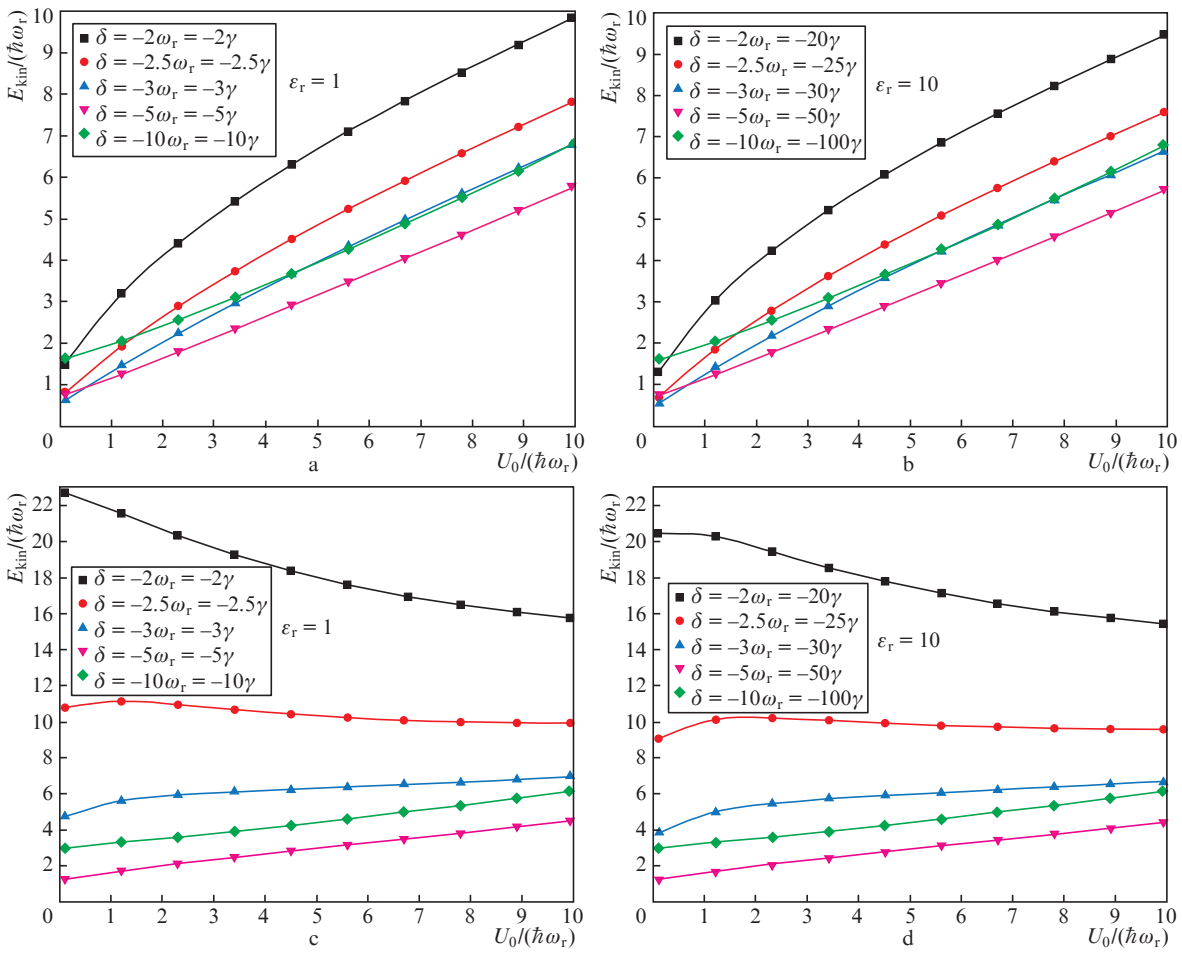

Figure 3. Average kinetic energy E_{kin} of the cold atoms in the $\sigma_+-\sigma_-$-configuration field as a function of the field shift parameter U_0 at various light field detunings δ for (a, b) $J_g = 0 \to J_e = 1$ transitions with $\varepsilon_r =$ (a) 1 and (b) 10, as well as (c, d) $J_g = 1 \to J_e = 2$ transitions with $\varepsilon_r =$ (c) 1 and (d) 10.

optical transition $J_g \to J_e$ (angular momenta of the ground and excited states).

As noted above, to analyse the polarisation effects of laser cooling, we consider an atom with an optical $J_g = 1 \to J_e = 2$ transition, which makes sub-Doppler mechanisms of laser cooling possible in both the lin⊥lin- field and in the $\sigma_+-\sigma_-$ configuration field that will allow for a comparative analysis. Also, instead of the intensity parameter, we will use the dimensionless parameter $U_0 = 4\Omega_0^2|\delta|/[3\omega_r(\delta^2 + 1/4)]$, which characterises the optical level shift in the light field. The choice of this parameter is due to the fact that in the limit of small recoil parameters ($\varepsilon_r \ll 1$) and for sufficiently large detunings, it is universal for determining the stationary state of an atomic ensemble in a light field [27, 29].

Figures 2 and 3 show the dependences of the average kinetic energy of cold atoms in the fields of lin⊥lin- and $\sigma_+-\sigma_-$ configurations, respectively, on the parameter U_0 for different recoil parameters. In addition to the results for atoms with an optical $J_g = 1 \to J_e = 2$ transition, we present results for atoms with a nondegenerate ground state, with an optical $J_g = 0 \to J_e = 1$ transition, for which sub-Doppler laser cooling mechanisms do not arise.

First, we note that due to the essentially nonequilibrium distribution of atoms over momenta in a light field [30, 31], the state of an atomic ensemble cannot be determined in terms of temperature. For a qualitative assessment of laser cooling of atoms, we use the average kinetic energy of the atomic ensemble. We also note the identity of the dependences of the kinetic energy of cold atoms on the parameter U_0 with the optical $J_g = 0 \to J_e = 1$ transition for different recoil parameters in Figs 2a and 2b in the lin⊥lin-configuration field and in Figs 3a and 3b in the $\sigma_+-\sigma_-$-configuration field. For atoms with an optical $J_g = 0 \to J_e = 1$ transition, the ground state is nondegenerate, which leads to results identical to those for a two-level atomic model [32]. The equivalence of the results for various recoil parameters is a consequence of the scaling law for laser cooling of atoms using narrow optical transitions [24]. Here, the minimum average kinetic energy E_{kin} of the ensemble of atoms is achieved in weak optical fields with the detuning $\delta = -3\omega_r$ and amounts to $\sim 0.5\hbar\omega$. In stronger fields, the energy grows, and the optimal detuning shifts to the red.

The degeneracy of the ground state, which leads to the appearance of sub-Doppler friction mechanisms in the quasiclassical limit $\varepsilon_r \ll 1$, does not lead to noticeable differences from the results for the two-level model at $\varepsilon_r \geqslant 1$, i.e., under conditions of a significant influence of recoil effects (cf. the dependences for atoms with optical transitions $J_g = 0 \to J_e = 1$ and $J_g = 1 \to J_e = 2$ in Figs 2a and 2c, and 2b and 2d). The difference in the dependences in Figs 2a and 2c, as well as in Figs 2b and 2d, reduces only to the renormalisation of the parameter U_0 by a factor depending on the angular momenta of the energy levels of the ground J_g and excited J_e states. It

also confirms the fulfilment of the scaling law for atoms with levels degenerated by the projection of the angular momentum in the lin⊥lin-configuration field.

Figure 3 shows the dependences of the average kinetic energy of cold atoms on the parameter U_0 under conditions of laser cooling in the field of the $\sigma_+-\sigma_-$ configuration. Figures 3a and 3b show the dependences for atoms with the optical $J_g = 0 \to J_e = 1$ transition, which correspond to the results for the two-level model and coincide with the dependences in Figs 2a and 2b. The influence of the polarisation effects of laser cooling in the $\sigma_+-\sigma_-$-configuration field is shown in Figs 3c and 3d. In the limit of a low field intensity, laser cooling of atoms with a ground state degenerated by the projection of the angular momentum in the field of the $\sigma_+-\sigma_-$ configuration leads to slightly larger values of the kinetic energy $E_{kin} \approx \hbar\omega_r$ (the minimum values are achieved with the detuning $\delta = -5\omega_r$ than those for the two-level model ($E_{kin} \approx 0.5\hbar\omega_r$, $\delta = -3\omega_r$). For smaller detunings, the polarisation mechanisms of laser cooling in the $\sigma_+-\sigma_-$ field lead to significantly higher kinetic energies of the atoms. We also note the equivalence of the results for atoms with different recoil parameters $\varepsilon_r \geqslant 1$ at detunings $\delta < -2\omega_r$.

4. Conclusions

We have performed an analysis of laser cooling of neutral atoms using narrow optical transitions ($\varepsilon_r \geqslant 1$) with levels degenerate over the projection of the angular momentum in the fields of lin⊥lin- and $\sigma_+-\sigma_-$ configurations with nonuniform polarisations. It is shown that polarisation mechanisms that allow cooling below the Doppler limit when using narrow optical transitions become less effective. Moreover, the use of the $\sigma_+-\sigma_-$-configuration field leads to large values of the kinetic energy of cold atoms in the limit of low light field intensities. The results obtained for the average kinetic energy of cold atoms in the field of the lin⊥lin configuration are equivalent to those obtained in the framework of the two-level model, i.e., without taking into account the polarisation mechanisms of laser cooling. Thus we can draw a conclusion that for the laser cooling of atoms with optical $J_g \to J_e$ transitions (with transitions where polarisation mechanisms of sub-Doppler laser cooling are possible in the semiclassical limit) to be most efficient, in using narrow transitions to achieve the lowest kinetic energies of the atomic ensemble use should be made of the llin⊥lin-configuration field or a uniform polarisation field configuration in which the problem becomes equivalent to a two-level atom model.

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