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Effect of boundary conditions on fluctuations of the Bose condensate of interacting atoms

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Abstract. **For a gas of trapped interacting atoms forming a Bose condensate, we demonstrate a possibility of a significant influence of the boundary conditions on the statistics of the number of particles in the fundamental mode of the system. The analysis is carried out in the Bogoliubov–Popov low-temperature approximation for model homogeneous cubic traps with periodic or combined periodic and zero boundary conditions. It is shown that the effect of the boundary conditions does not weaken even in the region of parameters corresponding to the relatively strong interaction in the Thomas–Fermi asymptotic case, and does not disappear when proceeding to the thermodynamic limit.**

Keywords: *Bose condensation, order parameter fluctuations, Bogoliubov–Popov approximation, Thomas–Fermi asymptotic case, boundary conditions.*

1. Introduction

When describing a Bose gas in a trap with a temperature noticeably below critical, it is traditionally assumed that N atoms of the system are divided into two fractions. The first fraction forms a Bose condensate, i.e. the ground state, macroscopically filled mode of the system with a significant number of particles N_0. The second fraction containing N_{ex} particles forms a residual cloud of noncondensed gas. This division into condensed and noncondensed fractions is random, i.e., the number of particles in the condensate N_0 and outside it N_{ex} are randomly fluctuating quantities (related by the condition $N_0 + N_{ex} = N$ in a canonical ensemble).

Quantum statistics describing this partition becomes an object of laboratory research of Bose systems of various configurations. The analysis is not restricted only to mathematical expectations; e.g., in the experiment [1], the behaviour of the variance of the number of particles in a condensate was studied as a function of the system temperature. The achieved measurement accuracy and the ability to vary the geometry and parameters of the trap allow quantitative comparison of laboratory results with theoretical models. An example is the experiment [2], in which the dependence of the number of noncondensed atoms on the intensity of interparticle interaction was measured (i.e., the effect of the condensate quantum depletion was measured) and the prediction of the Bogoliubov theory about this dependence was confirmed, including the relevant numerical factor.

A universal microscopic description of the Bose condensate fluctuations in an arbitrary trap corresponding to such experiments has not been constructed yet and, in fact, is known only for an ideal gas [3–5]. However, even this particular case already demonstrates nontrivial statistics of the number of particles in the condensate. For a gas without interparticle scattering, it was shown that the studied statistics can be essentially non-Gaussian even in the thermodynamic limit [4–7], which is not typical for the statistical physics of multimode systems. In this case, the variance and higher-order moments turn out to depend on the boundary conditions imposed on the system (and other nonglobal perturbations of the trapping potential). These properties manifest themselves for traps with a sharply increasing confining potential and low energy density of states, which include 3D box traps and other systems that are close to homogeneous. To design and interpret future experiments aimed at studying the statistics of Bose systems, it is important to find out whether its non-Gaussian features also appear in a real situation when interparticle interaction is present. Today, this problem remain an open one.

In this paper, we consider a particular aspect of the above problem, namely, whether the interparticle interaction 'turns off' the effects associated with boundary conditions. It would seem that this can be expected taking into account the following two circumstances. First, in the case of relatively strong interparticle scattering, bringing the system closer to the Thomas–Fermi limit, the condensate effectively shields the external potential, and therefore, the wave functions of the system are able to sense the imposed boundary conditions only in a narrow boundary region of space that is vanishingly small compared to the total volume of the system. Second, in the presence of interparticle scattering, the spectrum of the quasi-particles of the system in the low-energy part (most important for the formation of fluctuations) is modified in accordance with the Bogoliubov transformation from a quadratically increasing function of quantum numbers to a linearly growing one. It is also known [4, 5, 7] that for an ideal Bose gas in a 3D harmonic trap, such a linear increase in energies depending on quantum numbers is completely insufficient to violate the central limit theorem, and, therefore, implies neither non-Gaussian fluctuations nor a significant influence of trapping potential perturbations on the statistics of the system.

The aim of this work is a direct demonstration of the fact that in a wide range of Bose system parameters, the influence of boundary conditions on fluctuations in the number of particles in the condensate, namely, variance and higher-order

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moments, is preserved in the presence of interparticle scattering. This effect is not restricted to the case of a nearly perfect gas; it neither disappears nor even weakens as the system approaches the Thomas–Fermi limit. The analysis is carried out by the example of two model box traps that differ only in superimposed boundary conditions. The choice of such a simple geometry made it possible to obtain the most transparent and almost completely analytical description of the analysed statistical distributions.

The hypothesis about the influence of the boundary conditions on the statistics of the interacting Bose gas was earlier presented in Ref. [8], where it was pointed out why the central limit theorem may appear not applicable for describing its fluctuations. However, the problem of finding real quasi-particles of the system was actually left unsolved, and the calculations were performed for a certain set of hypothetical spectra and structures of excited states, consistent in the Thomas–Fermi limit, but not based on a rigorous solution of the equations for condensate and quasi-particles. Accordingly, the influence of the boundary conditions on the statistics was justified only qualitatively and the detailed mechanism of its implementation remained unclear.

In this paper, we consider specific model traps, for which the quasi-particles of the system and their energies in a wide range of interparticle scattering intensities are found explicitly and consistently with the condensate density profile. This allows, on the one hand, the analytical study of the Bose gas statistics not only in the region of applicability of the Thomas–Fermi approximation, but also in the region of parameters where the wave functions of the condensate and quasi-particles can substantially depend on the interaction strength. On the other hand, a description of the evolution of fluctuations during a continuous transition from the ideal gas limit to the Thomas–Fermi limit clearly shows why the influence of the boundary conditions on statistics is preserved for the Bose gas with significant interaction.

2. Distribution of the number of particles inside and outside the condensate. Role of boundary conditions in the origin of fluctuations

The analysis carried out is based on the Bogoliubov–Popov approximation [9, 10], which is applicable when the Bose gas temperature T is well below the critical temperature, $T \ll T_c$. This approximation assumes splitting of the field operator of a many-particle system into two parts:

$$\hat{\psi}(\boldsymbol{r}) = \sqrt{\langle N_0\rangle}\,\phi(\boldsymbol{r}) + \hat{\psi}_{\text{ex}}(\boldsymbol{r}),$$
$$\hat{\psi}_{\text{ex}}(\boldsymbol{r}) = \sum_j \big(u_j(\boldsymbol{r})\hat{b}_j + v_j^*(\boldsymbol{r})\hat{b}_j^\dagger\big). \tag{1}$$

The first part describes a condensate with a spatial profile $\phi(\boldsymbol{r})$ and the expected large number of particles $\langle N_0\rangle$ in it; the annihilation operator corresponding to this spatial mode is replaced with the classical non-operator numerical value $\sqrt{\langle N_0\rangle}$. The second part, denoted by the subscript ex (excitation), is an operator additive and describes the excitation quasi-particles existing against the background of the condensate, described by the creation/annihilation operators $\hat{b}_j^\dagger$ тв $\hat{b}_j$ and two-component wave functions (u_j, v_j). The condensate wave function (chosen purely real) is described by the Gross–Pitaevsky equation, while the quasi-particles and their energies E_j are governed by the Bogoliubov–de Gennes system of equations:

$$\Big(-\frac{\hbar^2}{2m}\Delta + U - \mu + g\langle N_0\rangle\phi^2 + 2g\langle n_{\text{ex}}\rangle\Big)\phi = 0,$$
$$\Big(-\frac{\hbar^2}{2m}\Delta + U - \mu + 2g\langle N_0\rangle\phi^2 + 2g\langle n_{\text{ex}}\rangle\Big)\begin{bmatrix}u_j\\ v_j\end{bmatrix} + g\langle N_0\rangle\phi^2\begin{bmatrix}v_j\\ u_j\end{bmatrix} = \begin{bmatrix}+E_j u_j\\ -E_j v_j\end{bmatrix}. \tag{2}$$

Both equations include the external trapping potential $U(\boldsymbol{r})$ and depend on the averaged profiles of the condensate density $\langle N_0\rangle\phi^2(\boldsymbol{r})$ and the particle density outside the condensate $\langle n_{\text{ex}}\rangle(\boldsymbol{r})$. The interparticle scattering intensity is characterised by the interaction constant g and μ is the chemical potential. The averaging is performed with the density matrix

$$\hat{\rho} = \exp\Big(-\sum_j E_j\hat{b}_j^\dagger\hat{b}_j/T\Big)\prod_j\big(1 - \text{e}^{-E_j/T}\big),$$

which characterises the equilibrium system and is diagonal in the quasi-particle number representation.

The problem of describing fluctuations implies finding the probability distribution $\rho_0(N_0)$ that describes the random number of particles in the condensate N_0. The same problem can be reformulated as a search for the distribution $\rho_{\text{ex}}(N_{\text{ex}})$ of the total number N_{ex} of particles outside the condensate, because the total number of particles in the entire system is fixed: $N = N_0 + N_{\text{ex}} = \text{const}$. An efficient method of analysing statistics is to use the characteristic function $\Theta(u)$:

$$\Theta(u) \equiv \text{Tr}\big(\text{e}^{\text{i}u\hat{N}_{\text{ex}}}\hat{\rho}\big),$$
$$\rho_{\text{ex}}(N_{\text{ex}}) = \frac{1}{2\pi}\int_{-\pi}^{\pi}\text{e}^{-\text{i}uN_{\text{ex}}}\,\Theta(u)\,\text{d}u,\quad \rho_0(N_0) = \rho_{\text{ex}}(N - N_0). \tag{3}$$

It allows presenting the desired distributions in the form of Fourier integrals, as well as expressing the distribution moments (mathematical expectation, variance σ^2, asymmetry coefficient γ) through generating cumulants:

$$\tilde{\kappa}_m \equiv \frac{\text{d}^m \ln\Theta}{\text{d}(\text{e}^{\text{i}u} - 1)^m}\bigg|_{u=0},\quad \langle N_{\text{ex}}\rangle = \tilde{\kappa}_1,\quad \sigma^2 = \tilde{\kappa}_2 + \tilde{\kappa}_1,$$
$$\gamma = \tilde{\kappa}_3 + 3\tilde{\kappa}_2 + \tilde{\kappa}_1. \tag{4}$$

In such a description, we consider two model traps that hold the gas in a cubic volume $V_{\text{trap}} = [0, L]\times[0, L]\times[0, L]$ and differ only in the boundary conditions imposed on the Gross–Pitaevsky and Bogoliubov–de Gennes equations (2). In the first case, we consider the boundary conditions to be periodic in all directions, which corresponds to a completely homogeneous system. In the second case, we impose periodic boundary conditions along only two directions, while along the third direction the zero boundary conditions are imposed. The wave function of the condensate along this inhomogeneous direction, depending on the magnitude of the interaction constant g, varies from half the sine period (ideal gas) to almost constant in the central part of the trap and rapidly decreasing to zero in narrow boundary regions (Thomas–Fermi limit). The role of the interaction for such 'flat' potentials is characterised by the ratio of the characteristic kinetic energy $\epsilon^* = \hbar^2\pi^2/(2mL^2)$ and the interparticle interaction energy $g\langle N_0\rangle/V$. In terms of spatial scales (more frequently used in the description of experiments), interparticle scattering is characterised by the healing length

$$\xi = \sqrt{\frac{\hbar^2 V}{2mg\langle N_0\rangle}}\,,$$

at which the condensate effectively shields the perturbation of the external potential. Relatively strong interactions bringing the system closer to the Thomas–Fermi limit correspond to the inequality $L/\xi \gg 1$, or, in terms of energies, $g\langle N_0\rangle/V \gg \epsilon^*$.

The considered trap configurations are remarkable for the fact that the characteristic function and cumulants for them are reduced to fairly simple explicit expressions (which has not yet been achieved for an arbitrary trapping potential because in the exponent the operator $\hat{N}_{\rm ex} = \hat{\psi}^\dagger_{\rm ex}\hat{\psi}_{\rm ex}$ is present, whose expression through operators $\hat{b}^\dagger_j$ and $\hat{b}_j$ is complicated). Namely, using the spectrum of eigenvalues of the so-called modified Schrödinger equation [11],

$$\left(-\frac{\hbar^2}{2m}\Delta + U - \mu + g\langle N_0\rangle\phi^2 + 2g\langle n_{\rm ex}\rangle\right)f = \epsilon f, \qquad (5)$$

the following result can be obtained:

$$\Theta(z) = \prod_j\left[(1-\zeta_j^{(+)}z)(1-\zeta_j^{(-)}z)\right]^{-1/2},$$

$$\tilde{\kappa}_m = \frac{\Gamma(m)}{2}\sum_j\left[(\zeta_j^{(+)})^m + (\zeta_j^{(-)})^m\right], \qquad (6)$$

$$\zeta_j^{(\pm)} = \frac{(E_j/\epsilon_j)^{\pm1}}{{\rm e}^{E_j/T}-1} + \frac{(E_j/\epsilon_j)^{\pm1}-1}{2}, \quad E_j \equiv \sqrt{\epsilon_j^2 + 2\Delta_{jj}\epsilon_j}\,,$$

where $\Gamma(m)$ is the gamma function. Here the characteristic function Θ is written in terms of the argument $z \equiv {\rm e}^{{\rm i}u} - 1$, the sums and products are taken over all nontrivial quasi-particles of the system (except for the Goldstone mode), $\Delta_{jj} \equiv g\langle N_0\rangle\int f_j^*\phi^2 f_j{\rm d}^3r$ are the diagonal overlaps of the solutions of Eqn (5) with the weight determined by the condensate wave function (coinciding with the lowest-energy solution of the same equation). The contribution of each quasi-particle to the fluctuations is represented by two expressions $\zeta_j^{(\pm)}$, which contain both thermal terms (with the Boltzmann exponent in the denominator) and quantum terms (associated exclusively with the nontrivial transformation of particles into quasi-particles and leading, in particular, to quantum depletion of the condensate).

For periodic boundary conditions along all directions, the result (6) is exact [12, 13], and the modified Schrödinger equation (5) is equivalent to the common single-particle Schrödinger equation in the initial trap without interaction. For the considered mixed boundary conditions, expressions (6) are a very good approximation based on a specific 'quasi-diagonal' structure of quasi-particles – each of them is mainly formed by only one mode of the modified Schrödinger equation (5), which determines at least 97% contribution to the quasi-particle norm. This feature was described in detail in [14] for a one-dimensional inhomogeneous problem, and its transfer to the 3D configuration under consideration is provided by the possibility of separation of variables in Eqns (2).

The difference of the characteristic function (6) from the characteristic function calculated for an ideal gas (see, e.g., [4]) is not reduced to replacing the spectrum of noninteracting particles with the spectrum of quasi-particles forming an ideal gas in the Bogoliubov–Popov approximation. The fact is that the direct calculation of the characteristic function for an interacting gas implies not only the transition from particles to quasi-particles, but also a subsequent return to the particle basis. This circumstance is reflected in the following fundamental property: the characteristic function (6) contains, along with the spectrum of quasi-particles $\{E_j\}$, also the spectrum of the single-particle modified Schrödinger equation $\{\epsilon_j\}$, i.e., it does not lose information about the initial particles that are transformed into the Bogoliubov quasi-particles.

Information on the single-particle spectrum is most fully preserved, as shown in [8], at the temperatures of the system $T \gg (\epsilon^*)^{1/4}\,(g\langle N_0\rangle/V)^{3/4}$, corresponding to the so-called thermal fluctuation regime. In this case, the cumulant $\tilde{\kappa}_2$, which determines the variance, as well as all the higher-order cumulants, are caused primarily by temperature factors. The thermal regime of statistics does not contradict the approaching of the system to the Thomas–Fermi limit, i.e., the inequality $L/\xi \gg 1$, written in terms of characteristic energies as $g\langle N_0\rangle/V > \epsilon^*$. Moreover, from the above criterion of the thermal regime of fluctuations it follows that, at an arbitrary intensity of interparticle interaction, it is exactly this regime that is invariably realised for the considered systems when passing to the thermodynamic limit, when at constant temperature $T \ll T_{\rm c}$, gas concentration N/V, and coupling constant g, the trap size L increases, enhancing the inequality $\epsilon^*/T \ll 1$. It is easy to verify that the quantum contributions in this case remain significant only for the average value, which corresponds to the expected depletion of the condensate (in this case quantum depletion may well prevail over the thermal one).

In the thermal regime of fluctuations, the first cumulant (mathematical expectation) and all the higher-order cumulants (including the second one associated with variance) are formed from the contributions of individual quasi-particles according to strongly differing scenarios. For the mathematical expectation of the number of particles outside the condensate, the contributions of individual energy levels decrease slowly with an increase in the corresponding energies. Thus, in the principal order, it is described by the expression

$$\langle N_{\rm ex}\rangle = \tilde{\kappa}_1 \simeq R\left(\frac{T}{\epsilon^*}\right)^{3/2} + \frac{\sqrt{2}\,\pi}{12}\left(\frac{g\langle N_0\rangle/V}{\epsilon^*}\right)^{3/2}, \qquad (7)$$

where the factor R is equal to $\pi^{3/2}\zeta(3/2)/8$ for a gas that is close to ideal (ζ is the Riemann zeta function) and decreases smoothly with increasing constant g, appreciably decreasing upon passing to the range of parameters in which the second term is comparable with the first one or exceeds it. Expression (7) is derived by integration over all quasi-particles and turns out to be independent of the imposed boundary conditions – so many excited levels make an effective contribution that changing a small group of them does not affect the result. At the same time, for higher-order cumulants, the contributions of individual quasi-particles rapidly decrease with increasing their energies, so that the resulting cumulant value coincides in order of magnitude with the contribution of a single level with low energy. Accordingly, in the limit of a large system,

$\epsilon^*/T \ll 1$, the values of the considered cumulants are as follows:

$$\tilde{\kappa}_{m\geqslant 2} \simeq S_m\left(\frac{T}{\epsilon^*}\right)^m,$$
$$S_m = \frac{\Gamma(m)}{2}\sum_j\left[\left(\frac{\epsilon^*}{\epsilon_j}\right)^m + \left(\frac{\epsilon^*}{\epsilon_j + 2\Delta_{jj}}\right)^m\right]. \tag{8}$$

It has been taken into account that for low-energy quasi-particles that make the largest contribution to fluctuations, no essentially exponential decay of population manifests itself, i.e., we can assume $\exp(E_j/T) - 1 \simeq E_j/T$. Expressions (8) coincide with those that determine the main orders of the cumulants of an ideal gas, up to redefinition of the coefficients S_m by the sums $\Gamma(m)\sum_j(\epsilon^*/\varepsilon_j)^m$ over the single-particle states of the trap with energies ε_j.

The obtained law of cumulant scaling (8) determines the non-Gaussian character of fluctuations, just as it occurs in an ideal gas [4, 7]. Indeed, introducing instead of the number of particles N_{ex} outside the condensate the random variable $x = (N_{\mathrm{ex}} - \langle N_{\mathrm{ex}}\rangle)/\sigma$, naturally centred and normalised by the standard deviation σ, it is easy to see that the corresponding normalised higher-order cumulants in the large system limit $\epsilon^*/T \to 0$ do not vanish, but tend to constants: $\tilde{\kappa}_m^{(x)} = \tilde{\kappa}_m/\sigma^m \to S_m/S_2^{m/2}$. Direct calculation shows that the numbers S_m depend on the boundary conditions imposed on the system, and therefore, the normalised probability distributions of the particle number in the condensate and outside it, as well as the principal orders of all the higher-order moments of these distributions, starting from the variance, also depend on the boundary conditions. This statement is illustrated in Fig. 1, according to which the statistical distribution of the particle number in the condensate at the interaction intensity corresponding to $L/\xi = 20$ looks wider and more asymmetric for boundary conditions that violate the homogeneity of the system.

It is interesting to consider how the evolution of the studied statistics occurs when the intensity of the interparticle interaction changes. In the case of a homogeneous trap with all periodic boundary conditions for which the energies $\{\epsilon_j\}$ do not depend on the magnitude of the interaction constant g, this evolution actually reduces to the effect of fluctuation squeezing, well known in quantum optics [15, 16] and found in relation to statistics of Bose atoms in Ref. [12]. This effect consists in the fact that with an increase in the interaction constant g in expressions (8), the terms containing the overlap integrals $\Delta_{jj} \equiv g\langle N_0\rangle\int f_j^*\phi^2 f_j\,\mathrm{d}^3r$ in the denominator significantly decrease. As a result, the higher-order cumulants (starting from the second one, which determines the variance) fall, decreasing with approaching the Thomas–Fermi limit ($L/\xi \gg 1$) to half of their values calculated in the absence of interaction.

In the case of a trap with zero boundary conditions along one of the axes, the effect of squeezing of fluctuations is also present, but does not completely describe the statistics behaviour. These boundary conditions violate the homogeneity of the system, because of which the existing quasi-particles and their energies turn out to depend on the magnitude of the interaction constant that determines the wave function of the condensate. With increasing interparticle scattering, the condensate density profile along the inhomogeneous direction becomes more flat in the central part of the trap and more and more sharply changing in the boundary region. Accordingly, the energies of the eigenstates of the modified Schrödinger equation $\{\epsilon_j\}$ decrease, which makes the excited energy levels of the system more accessible, facilitating, in turn, enhanced fluctuations and increase in cumulants (8) (the eigenstates of the projection of Eqn (5) onto the inhomogeneous direction with zero boundary conditions are described in detail in [14]). Such an effect of the transformation of quasi-particles acts oppositely to the squeezing of fluctuations.

The combined effect of both considered mechanisms is illustrated in Fig. 2, which shows the evolution of the standard (root-mean-square) deviation of the number of condensed particles. In a regime of almost homogeneous gas, the standard deviation decreases regardless of the boundary conditions in accordance with the effect of fluctuations squeezing. However, with an increase in the interaction constant to a value corresponding to $L/\xi \simeq 3$, the condensate in an inhomogeneous trap begins to experience substantial rearrangement, which activates the effect of transformation of quasi-particles. In the range of parameters $L/\xi \simeq 7$, it begins to prevail over the effect of fluctuation squeezing. As a result, in an inhomogeneous trap, the variance only increases with a further increase in the interaction (and does not fall, as in the case of a completely homogeneous system) and in the Thomas–Fermi limit tends to a value exceeding the variance in an ideal gas. Similar behaviour and similar differences for the considered boundary conditions are also demonstrated by the other higher-order cumulants.

It should be noted that for the traps being compared, the differences in statistics with scattering enhancement only increase (of course, provided that the system remain in the regime of thermal fluctuations). The maximum difference is achieved when approaching the Thomas–Fermi limit, which may look somewhat unexpected, because in this case, the condensate most efficiently shields the external potential and its perturbations. However, within the framework of a rigorous description no contradiction arises, since the information about the single-particle spectrum is preserved by a consequent transformation from particles to quasi-particles diagonalising the Hamiltonian of the system, and then back to the initial particles whose statistics are analysed.

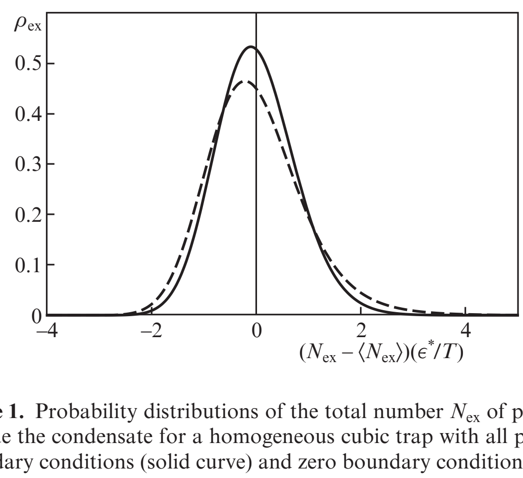

Figure 1. Probability distributions of the total number N_{ex} of particles outside the condensate for a homogeneous cubic trap with all periodic boundary conditions (solid curve) and zero boundary conditions along one of the directions (dashed curve). The distributions are centred at the corresponding average values and normalised to the characteristic variance scale $T/\epsilon^* \gg 1$, similar for the two trap configurations being compared. The calculations were carried out in the thermodynamic limit for the thermal fluctuations regime, the interparticle interaction corresponds to the ratio of the trap length and shielding length $L/\xi = 20$.

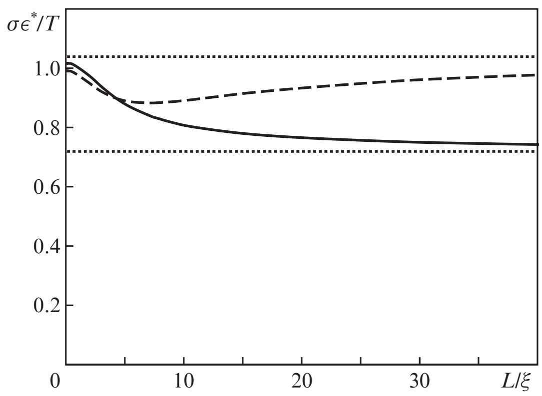

Figure 2. Dependence of the standard deviation of the number of particles in the condensate σ on the intensity of interparticle scattering for a large 3D Bose system, the statistics of which are determined by thermal factors: the case of all periodic boundary conditions (solid curve) and zero boundary conditions along one of the directions (dashed curve). The deviation is normalized to its natural scale $T/\epsilon^* \gg 1$, interparticle scattering is characterised by the ratio of the trap length L to the healing length ξ. Asymptotic values corresponding to the Thomas–Fermi limit $L/\xi \gg 1$ are shown by a dotted line for each boundary condition.

For trapping potentials of a more general form, the analysis of statistics is not so simple because of the nontrivial structure of quasi-particles, which leads to a much more cumbersome representation of the characteristic function. The effect of transformation of quasi-particles is also less transparent, since in the general case a variation in the interaction intensity can noticeably change the decomposition of each quasi-particle int a set of modes of the modified Schrödinger equation (5). For more complex traps the influence of boundary conditions can quantitatively differ significantly from that shown in the considered example; the dependence of statistics on the intensity of interparticle scattering can also be noticeably different. However, the order of magnitude of the effects of the boundary conditions that do not disappear in the thermodynamic limit is preserved.

3. Discussion of results

The studied example clearly demonstrates that the dependence of the statistical distribution of the number of boson particles inside and outside the condensate on the boundary conditions remains significant and is not suppressed in the presence of interparticle interaction, even strong enough when the system approaches the Thomas–Fermi asymptotic regime. The indicated dependence manifests itself in the principal order of magnitude for the variances of distributions and all the higher-order moments and cumulants. This conclusion can be generalised to the case of a trap that holds a Bose gas in a volume of arbitrary shape and, most likely, remains valid for any 3D atomic trap with a sufficiently rapid growth of the single-particle spectrum.

The nontrivial feature of the discussed statistics should be taken into account for the correct interpretation of the results of experiments with Bose systems carried out in nearly homogeneous traps, as well as analysing fluctuations of the Bose condensate and any other characteristics of the Bose gas that cannot be reduced to the average number of particles in the condensate and outside it. In addition, the study suggests an experiment on directly detecting the influence of boundary conditions on condensate fluctuations – switching boundary conditions similar to that described for model traps seems to be feasible in laboratory conditions, and the necessary measurement accuracy is considered achievable in the near future.

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