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On "incorrectness" of the rate equation for photon density of semiconductor lasers

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Abstract. **The authors of works [1, 2] published in Quantum Electronics assert that the rate equation for photon density with a term taking into account the contribution of spontaneous emission to the laser mode is erroneous (i.e., the spontaneous term should be excluded from consideration) and criticise works [3, 4]. In the present work, we analyse previously published papers and show that the necessity of taking into account the contribution of spontaneous emission to the laser mode follows from the quantum-mechanical rate equation for semiconductor lasers. The results obtained in [1, 2] are analysed.**

Keywords: *rate equations, photon density, spontaneous emission.*

1. Introduction

Today, it is accepted (see the list of books below) that the emission spectra of semiconductor lasers are calculated using the steady-state solution of the rate equation for photon density taking into account the contribution of spontaneous emission to the lasing mode. In papers [1, 2], it is asserted that this equation is erroneous and, therefore, the results obtained in works [3, 4] and reported in a number of books are incorrect. The authors of [1] write that "*this approach appears in a large number of books dealing with diode lasers (e.g. in Suhara [9]), so there is a need to separately analyse its incorrectness*" (Refs [9] correspond to [5] in the present work). Hereinafter, citations from [1, 2] are italicised. It is shown below that the conclusion about "incorrectness" of the rate equation for photon density that takes into account the contribution of spontaneous emission to the lasing mode follows from the fact that the authors of [1, 2] did not consider the quantum-mechanical model presented in [6–10]. It is shown that this rate equation follows from the quantum-mechanical laser theory. In the present work, we first consider the fundamentals of description of radiative processes in semiconductor laser and then analyse the results obtained in [1, 2].

2. Fundamental equations for description of radiative processes in semiconductor lasers

According to [6], the expression for the rate of a change in the number of photons $N_{\rm ph}$ in one lasing mode of a semiconductor laser has the form

$$\frac{dN_{\rm ph}}{dt} = -\frac{N_{\rm ph}}{\tau_{\rm mod}} + [r_{\rm sp}(E) + N_{\rm ph} r_{\rm st}(E)]\frac{1}{\phi(E)}, \qquad (1)$$

where $\phi(E)$ is the mode density, i.e., the number of modes per unit volume and unit energy, and E is the photon energy.

The first term in Eqn (1) gives the rate at which photons are lost from the mode and the two following terms determine the rates of arrival of spontaneous and stimulated emission photons into the mode.

Taking into account that $r_{\rm st}(E) = v_{\rm gr}\phi(E)g(E)$, $\tau_{\rm mod} = (v_{\rm gr}\alpha)^{-1}$, where α are losses and $v_{\rm gr}$ is the group velocity of light, Eqn (1) for the photon density can be written as

$$\frac{dS}{dt} = -\frac{S}{\tau_{\rm mod}} + g(E)S + \beta\frac{n}{\tau_{\rm e}(n)}. \qquad (2)$$

Here, g is the gain, β is the coefficient taking into account the contribution of spontaneous emission to the lasing mode, $S = N_{\rm ph}/V_{\rm a}$ is the photon density in the cavity, $V_{\rm a}$ is the active region volume, n is the carrier concentration, and $\tau_{\rm e}(n)$ is the lifetime of carriers.

Analysis of the literature shows that Eqn (2) follows from the quantum-mechanical rate equation for the photon density of a semiconductor laser [7–10].

It is mentioned in [9] that the fundamentals of the quantum-mechanical laser theory was formulated in the second half of the 1960s in works by Haken, Lamb, Lax, McCamber, and other authors. In these works, the spectral characteristics of semiconductor lasers were analysed in the approximation of rate equations. The quantum-mechanical rate equations are considered in [9]. The obtained rate equation for photon density is

$$\frac{dS}{dt} = -2\chi S + E_{\rm cv} + GS + F(t), \qquad (3)$$

where $S = b^+b$; b^+ and b are the photon creation and annihilation operators, χ is the field amplitude decay rate in the resonator, $E_{\rm cv}$ is the spontaneous emission rate, G is the gain, and F is the Langevin noise source. Equation (3) was obtained for the first time by Haug [7].

The first term in Eqn (3) is inversely proportional to the photon lifetime, the second term corresponds to spontaneous emission, and the third term describes stimulated emission. If we neglect the fourth term in Eqn (3), then Eqn (3) will almost coincide with Eqn (2).

H. Haken in [10, p. 77] writes: "We assume that there exists a certain set of modes in the laser resonator and we distinguish them by the index λ. Each mode can be occupied by a certain number of photons n_λ. Because the lifetimes of different modes in the resonator can be different we introduce

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decay constants χ_λ which in general will differ from each other. Because the individual atoms interact with the laser modes in a different way we have to consider the atoms individually. For simplicity we again consider the 2-level scheme leaving its extension to a 3-level scheme as an exercise to the reader.

We denote the occupation numbers of the atom μ in the states 1 and 2 by $N_{1,\mu}$ and $N_{2,\mu}$, respectively. The corresponding inversion of atom is described by $d_\mu = N_{2,\mu} - N_{1,\mu}$. Generalising the laser equation (2.1) we can immediately write down the laser equation for the mode λ

$$\frac{dn_\lambda}{dt} = -2\chi_\lambda n_\lambda + n_\lambda \sum_\mu W_{\lambda\mu} d_\mu + \sum_\mu W_{\lambda\mu} N_{2\mu} . \ (4.57) \qquad (4)$$

Though this equation was not derived exactly here (what we shall do later) its form is quite plausible. The temporal change of the number of photons of kind λ is given by:

(1) losses (first term on the r.h.s.);

(2) the stimulated emission and absorption processes by the individual atom μ (first sum on the r.h.s.);

(3) a term representing spontaneous emission (second sum on the r.h.s.)."

Coexistence of modes due to the spatial hole-burning effect is considered in section 4.10 of [10]. Two examples of simultaneous lasing of modes are analysed.

It is necessary to note that the contribution of spontaneous emission to the mode is often neglected in the case of steady-state lasing. However, spontaneous emission cannot be ignored when considering laser emission spectra.

In [11, p. 202] it is noted that "...spontaneous emission is absent in the classical theory based on the Maxwell equations. The quantum-mechanical approach shows that spontaneous emission gives a fluctuating addition to the photon density growth rate, whose average can be represented as a parameter proportional to the integral spontaneous emission rate, i.e., as $R_{sp} = N_e/\tau_e$ with a proportionality coefficient β_{sp} called "spontaneous emission factor". As a result, we can write the following equation for the photon density balance:

$$\frac{dN_\omega}{dt} = -\frac{N_\omega}{r_\omega} + G(N_e)N_\omega + \beta_{sp}R_{sp} + F(t) ." \qquad (5)$$

One can see that, neglecting term $F(t)$, Eqn (5) leads to formula (4).

Next, the author of [11] writes that " factor β_{sp} often provokes discussions, in particular, a discussion about its stability in a wide intensity range. The positive contribution of spontaneous emission to the photon balance allows the steady-state threshold condition to be met at a gain somewhat lower than loss rather than at their precise identity. In the lasing regime, gain continues to increase and asymptotically approaches the loss level. As is shows in [12], this process can be responsible for the continuing increase in the intensity of nonlasing modes at above-threshold pumping, which is frequently observed experimentally instead of precise saturation of these intensities."

The author of [11] analyses the stability of steady-state solutions and pays attention to "...the role of the time-averaged contribution of spontaneous emission to the photon balance. It is shown that the light–current characteristic (LCC) in this model is continuous and differentiable everywhere, including lasing threshold, at which it is smoothed by a superlinear region" (p. 209). The solutions are analysed and their images on the phase plane are presented. It is shown that "... transition from the metastable nonlasing state under action of intrinsic fluctuations in the laser has a step-like character and corresponds to the hard switching regime (in contrast to the soft regime, at which the radiation intensity gradually increases with increasing pump current and always only one stationary state exists). Hard switching is sometimes evidenced by an anomalously steep light–current characteristic near the lasing threshold" (p. 218).

It was shown in [13] that multimode rate equations make it possible to explain the dependences of laser spectra on the crystal length. It was reported that the calculated spectra coincided with experimental spectra.

Chapter 2 in [14] is written by K. Lau and A. Yariv and is devoted to studying high-frequency current modulation and static characteristics of injection lasers. The study was performed based on rate equations for the photon density taking into account the spontaneous term. In subsection 2.II.2., the authors analyse spatially-averaged rate equations and their range of validity. On p. 76, they write: "These results lead to the conclusion that the simple rate equations... will hold if the end-mirror reflectivity is greater than 0.2 and the laser is above the threshold."

It was pointed out that the rate equations can be used to analyse the modulation phenomena with frequencies not exceeding 60 GHz (p. 76). In subsection 2.II.4 the authors analyse the steady-state characteristics of injection lasers. In Fig. 4 (p. 82) they present the steady state solutions of rate equations for electron concentration and photon density versus the pump current density at coefficient β both equal to and different from zero. One can see that a linear dependence of the photon density on the pump current takes place in the case of $\beta = 0$.

Chapter 3 in [14] is written by C. Henry and is devoted to the spectral and noise properties of semiconductor lasers. On p. 170, C. Henry considers steady-state mode intensities. He writes: "In order to discuss mode intensity spectra, we must introduce spontaneous emission into the cavity. This is most easily done by approximating the facet losses as a distributed loss that is uniformly spread across the cavity. ...The steady-state solution of Eqn (9) is

$$I = R/(\gamma - G) = r/(\alpha\gamma - g), \ (28) \qquad (6)$$

where R is the average spontaneous emission rate, γ is the cavity loss rate, and G is the gain.

The mode intensities given by Eqn (28) are illustrated in Fig. 11."

In formula (6), I is the number of photons in the mode, which is related to the radiation power by formula (25) in [14]. It is seen that radiation consists of several longitudinal modes.

Rate equations and noise sources are considered in subsection 3.IV.14 of [14]. On p. 187, it is noted that the intensity changes corresponding to spontaneous emission events will be correctly described by motion equations if equation $dI/dt = (G - \gamma)I$ is added by terms of average spontaneous emission rate R and random Langevin forces $F_I(t)$:

$$\frac{dI}{dt} = (G - \gamma)I + R + F_I(t). \qquad (7)$$

In the same work, noise sources, frequency spectrum of intensity fluctuations, and a model of mode distribution noise are analysed based on rate equations. The obtained equations «…completely describe the fluctuation properties of lasers" (p. 189).

It should be noted that the rate equation for photon density with the spontaneous term is used in books [15–23]. In the list presented below, the first numeral corresponds to the consecutive number of the article in the list of references of the present work, the second numeral is the page, and the figures in parentheses correspond to the number of formula containing the equation for photon density with the spontaneous term: [15, 224, (6.2.14)], [16, 31, (2.74)], [17, 91, (5.21)], [18, 53, (2.11)], [19, 47, (3.78)], [20, 47, (1.43)], [21, 103, (6.86)], [22, 487, (11.1.2)], [23, Part. 5.2.8, (5.2.68b)]. Thus, in the opinion of the authors of papers [1, 2], the calculation results obtained in [15–23] are erroneous because the authors of these works use incorrect equations (whose numbers are given in parentheses) for the photon density with the spontaneous term.

The authors of all these books note that the calculated spectral characteristics of lasers coincide with experimental data. For example, Fig. 1 borrowed from [24] shows the spectral characteristics of a semiconductor laser with a fibre Bragg grating (FBG). Comparison of the experimental and calculated dependences presented in Figs 1a, 1c shows not only a qualitative agreement between them but also a satisfactory quantitative coincidence. Indeed, the temperature cycles of laser diode (LD) characteristics in theoretical and experimental dependences coincide ($\delta T_{\rm LD} \cong 2.6\,°$C). The intermode intervals determined from the experimental dependence in Fig. 1a are $\delta\lambda_{\rm LD} \cong 0.126$ nm for the LD and $\delta\lambda_{\rm ext} \cong 0.021$ nm for the external cavity (EC). The corresponding calculated values are $\delta\lambda_{\rm LD} = 0.141$ nm and $\delta\lambda_{\rm ext} = 0.0182$ nm. The rates of wavelength change with temperature for experimental curves are $\delta\lambda_{\rm LD}/\delta T_{\rm LD} \cong 0.0485$ nm °C^{-1} for the LD and $\Delta\lambda_{\rm ext}/\Delta T_{\rm ext} \cong 0.015$ nm °C^{-1} for the EC. The corresponding calculated values are $\delta\lambda_{\rm LD}/\delta T_{\rm LD} = 0.054$ nm °C^{-1} and $\Delta\lambda_{\rm ext}/\Delta T_{\rm ext} = 0.0265$ nm °C^{-1} (Fig. 1c). Coefficient $\beta_{\rm sp}$ used in calculations was 6.2×10^{-6}. The calculations were performed using formula (Part. 5.2.8, 5.2.70 a) from [23] taking into account the optical confinement factor. The coefficient $\beta_{\rm sp}$ is often calculated without taking into account the optical confinement factor [13, (16)], [14, chapter 2, (28)]. In this case, $\beta_{\rm sp} = 2.6\times10^{-4}$, and it is assumed that the astigmatism coefficient is $K = 1$. The typical value of $\beta_{\rm sp}$ for conventional lasers lies within the range 10^{-7}–10^{-5} [21, p. 104]. For semiconductor FBG lasers, the ratio between the volume of the mode in the laser to the total volume including the waveguide and the Bragg grating is 4.7×10^{-3}.

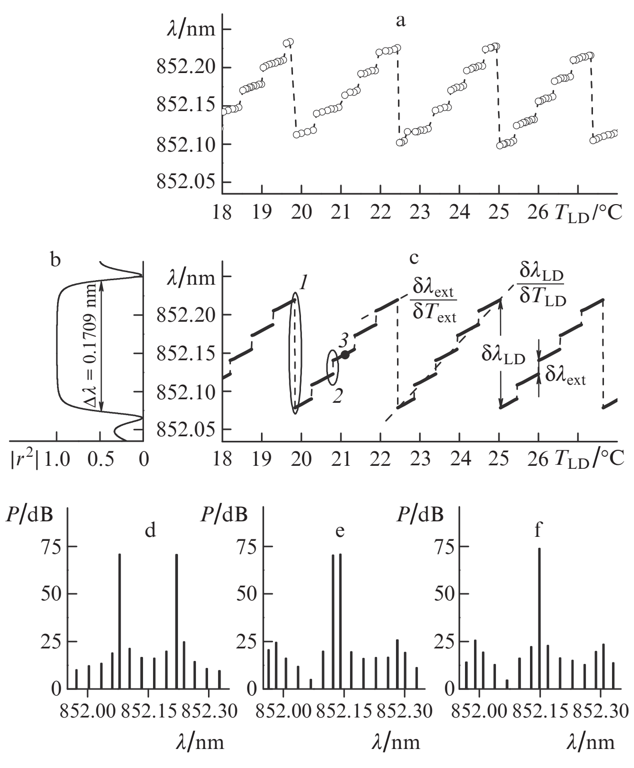

Figure 1. (a) Experimental and (c) theoretical dependences of the emission wavelength on the LD temperature at constant FBG temperature and pump current, as well as (b) dependence of the modulus of the FBG reflection coefficient on wavelength. (d, e, f) Emission spectra at points *1*, *2*, and *3*, respectively.

The use of rate equations allows one not only to calculate but also to optimise the characteristics of FBG LDs.

Thus, the analysis of the literature data shows that Eqns (2)–(7) are obtained as a result of consideration of quantum-mechanical emission processes, the correctness of which is confirmed by experimental results.

3. Analysis of the results obtained in [1, 2]

Work [1] analyses the role of spontaneous emission in the formation of the optical spectrum of diode lasers operating in the steady-state regime.

The authors of [1] consider the characteristics of semiconductor lasers within the semi-classical theory. They do not take into account spontaneous emission and derive a nonlinear second-order equation with the right-hand side equal to zero, obtain the van der Pol equation, and analyse its solution. They note: "*One important feature of the obtained solution $E(t) = A_0 exp(-i\omega_0 t + \varphi)$ (17) is that it does not require or contain any external field sources, e.g. such as produce spontaneous emission. This fundamentally distinguishes (17) from solutions presented in Refs [7–9] and other works based on the 'asymptotic lasing threshold*'" (p. 722). (Refs [7–9] correspond to [3–5] in the present work.)

Analysis of the results obtained in [1] shows that the used van der Pol equation yields a fixed lasing frequency and a zero laser linewidth, which does not correspond to experiment because the linewidth of any laser is finite.

The authors of [1] present on p. 723 a formula for calculating the pump threshold current. However, correctness of the obtained expression cannot be confirmed by experimental results. The experimental LCC does not allow one to implicitly determine the lasing threshold. This threshold is usually determined by extrapolation of the experimental LCC to zero. However, the threshold value in this case is determined taking into account the contribution of spontaneous emission to stimulated emission (Fig. 6.3 in [5]). The lasing threshold is sometimes determined as a point of intersection of the LCC regions corresponding to spontaneous and stimulated emissions (Fig. 3.3 in [21]). The lasing threshold can be also determined from spectral measurements as a point at which the stimulated emission rate exceeds the spontaneous emission rate (Figs. 4 in Chapter 3 of [14] or 2.11 in [17]). However, this cannot be done in [1] because the authors assume that spontaneous emission is absent, i.e., there is not any photon in the current range from zero to the threshold value, as well as there are no spontaneous photons at the lasing threshold. However, if spontaneous photons are absent, photon avalanche cannot arise due to stimulated transitions. Therefore, when calculating spectral laser characteristics, spontaneous emission must be taken into account.

As was mentioned above, the van der Pol equation yields a zero lasing linewidth. Because of this, the authors of [1] consider the equation for the cavity mode amplitude in a diode laser with allowance for spontaneous emission sources, which leads to a nonlinear second-order equation with the nonzero right-hand side. The authors write that allowance for spontaneous emission leads to lasing threshold broadening. "*But this is dynamic 'broadening' (fluctuating in time), rather than some constant difference between the gain and cavity loss, in contrast to what is assumed in phenomenological 'asymptotic threshold' models [7, 8] using rate equation.*" (p. 726) (Refs [7–8] correspond to [3–4] in the present work).

In [10, p. 7] we read: "Spontaneous emission of light is a typical quantum mechanical process. Quite evidently the semiclassical theory cannot treat this process. Thus it becomes necessary to develop a completely quantum mechanical theory of the laser."

In [16, p. 158] it is shown that spontaneous noises should be introduced by equation

$$\frac{\mathrm{d}S}{\mathrm{d}t} = \frac{S}{\tau_{\mathrm{ph}}}(G-1) + 2\,\mathrm{Re}(E_{\mathrm{sp}}(t)E^*(t)),$$

where $2\mathrm{Re}(E_{\mathrm{sp}}(t)E^*(t)) = R + F_{\mathrm{s}}(t)$, which leads to the rate equation for the photon density (7.23) [similar to (7)], while the Langevin noise source is determined as $F_{\mathrm{s}}(t) = 2\mathrm{Re}(E_{\mathrm{sp}}(t) \times E^*(t)) - 2\langle E_{\mathrm{sp}}(t)E^*(t)\rangle$.

A similar conclusion was made in [19, p. 49] and in works [7–10].

Thus, if the authors of [1, 2] performed calculations similar to those made in [16], they would obtain an equation for the photon density with a term taking into account the contribution of spontaneous emission to the laser mode similar to Eqns (2)–(7), i.e., the quantum-mechanical rate equation.

The necessity of taking into account spontaneous emission in calculation of laser noise characteristics is proved by the theoretical and experimental results demonstrated in Fig. 2 borrowed from our work [25], which presents the dependences of relative intensity noise (RIN) [curve (*1*)] and voltage U_0 [curve (*2*)] proportional to the LD power for frequencies f = 60 kHz, 1 kHz, and 167 Hz. The RIN was calculated using $\beta_{\mathrm{sp}} = 6.2 \times 10^{-6}$. One can see satisfactory coincidence of not only noise but also power characteristics.

The anomalously high experimental noise levels of the FBG semiconductor laser at pump currents of 70–75 mA observed in Fig. 2 were explained in [26].

In work [2], the rate equations and the range of their applicability are considered based on the results of [1]. The author writes: "*This paper can be viewed as a continuation of a previous one [1].*"

In the opinion of the author of [2], "*The most complete and typical form of rate equations is presented in Suhara [5] [Eqns (6.29a) and (6.29b)]:*

$$\frac{\mathrm{d}S_m}{\mathrm{d}t} = \Gamma_m\Big(G_m - \sum_j \xi_{mj}S_j\Big)S_m - \frac{S_m}{\tau_{\mathrm{ph}}} + \frac{C_{sm}N}{\tau_{\mathrm{s}}}.\ (1)\text{"} \tag{8}$$

In the steady-state lasing regime,

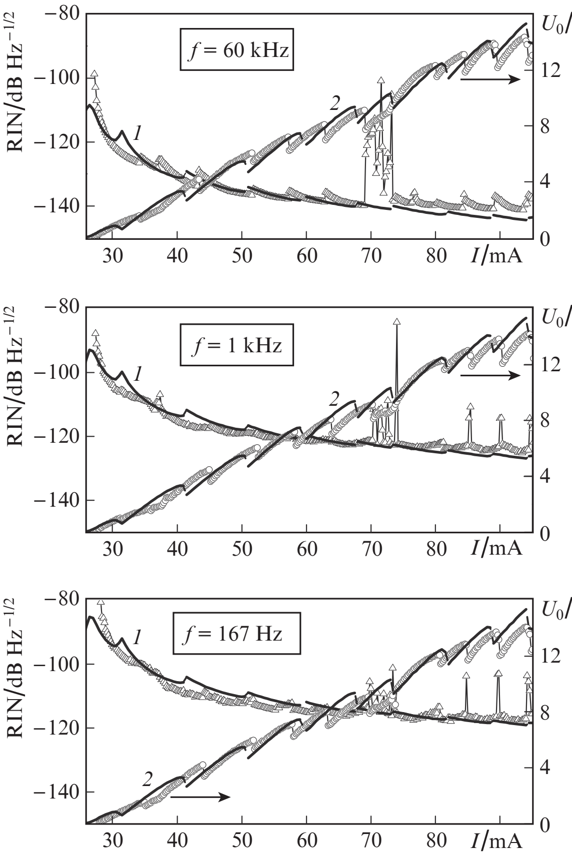

Figure 2. Dependences of (*1*) RIN and (*2*) voltage U_0 (proportional to the LD power) on the pump power for different frequencies f. Solid curves correspond to calculated results and points are experimental data. Vertical arrows denote anomalous noise levels.

$$S_m = \frac{\tau_{\mathrm{ph}} C_{sm} N/\tau_{\mathrm{s}}}{1 - \tau_{\mathrm{ph}}\Gamma_m\big(G_m - \sum_j \xi_{mj}S_j\big)}. \tag{9}$$

The authors of [1] write (p. 717): "*In these and other reports, use is made of an approach in which a laser field is considered essentially as amplified and spectrally filtered spontaneous emission. This physical meaning is suggested by theory proposed in Refs [7, 8], in which the spectral intensity distribution over modes can be represented as a fraction whose numerator is the spectral density of spontaneous emission and whose denominator is proportional to the difference between the loss and saturated gain.*" In other words, the authors of [1] criticise the use of formula (9) in works [3, 4] and consider this formula as incorrect.

Then, on p. 718, they write: "*but the approach that uses rate equations containing an additional term related to spontaneous emission often 'roams' from one paper to another in the literature. The process continues at present, and this approach appears in a large number of books dealing with diode lasers (e.g. in Suhara [5]), so there is a need to separately analyse its incorrectness.*" On p. 729 [2], they note that "*…spontaneous emission is taken into account without proper substantiation and, as a consequence, inadequately*."

It should be noted that the problem with the "incorrectness" mentioned by the authors of [1, 2] consists in the fact that they do not consider the quantum-mechanical laser theory. As was shown above, it is the quantum-mechanical calculation [7–10] that leads to the necessity of taking into account spontaneous emission in the rate equation for photon density.

The author of [2] writes on p. 733: "*That Eqns (6) are inadequate for analysing spectral characteristics of diode lasers was pointed out above. Kurnosov V.D. and Kurnosov K.V. [3] went even further. Using numerical simulation, they allegedly showed that introducing a negative term, $-\gamma S_m^2$, into (6a) leads to a transition from single-mode to multimode* lasing. *Below, it will be shown that this is another erroneous result*."

The assertion that "*Eqns (6) are inadequate for analysing spectral characteristics of diode lasers*" was answered by us above. Concerning the second objection, it is necessary to note that it is the presence of the nonlinear term that allowed the authors of [23 (Part 5.2.10)] to explain the multimode lasing regime Analysis is performed using the formula

$$S_m = \frac{\gamma \Gamma N/\tau_n}{1/\tau_p - \Gamma g_m (N - N_t)(1 - \varepsilon S_m)}. \tag{10}$$

Part 5.2.10 in [23] presents the spectral dependences of loss and gain taking into account spectral hole burning. This figure also presents the spectrum of longitudinal laser modes.

Thus, not "*…the authors of [3] went even further*", but this was done in [23, Part 5.2.10] as early as 1988. In [3], it was shown that this formula is appropriate for calculating multimode spectra and that the calculation results coincide with experimental data and term $G_m \varepsilon S_m^2$ is similar to term γS_m^2.

It is necessary to note that the system of rate equations used in [15–23] and some other publications makes it possible not only to obtain steady-state solutions for the photon density distribution coinciding with experiment (an example of which is presented in Fig. 1) but also to describe such phenomena considered in [21] as lasing spectrum asymmetry (Fig. 8.5, p. 139), hysteresis effects in lasers (Fig. 9.6, p. 149), and mode hopping to the second or third rather than nearest neighbouring mode (Fig. 9.10, p. 152). Figure 10.9 in the same work shows good agreement between theory and experiment for RIN in an AlGaAs laser.

Therefore, the conclusion of the author of [2] on p. 734 that "*Modelling a steady-state emission spectrum of a diode laser using rate equations, in particular, that in Ref. [3], should be considered erroneous*" follows from the fact that the authors of [1, 2] did not consider the quantum-mechanical laser theory and are obviously unacquainted with book [21].

To prove incorrectness of a calculation, it is necessary to perform a correct calculation and show the mistake. However, papers [1, 2] don not present calculations confirming incorrectness of the results obtained by modelling the laser spectrum using rate equations for the photon density with the spontaneous term and formula (9). Therefore, it is not clear why the authors of [1, 2] have concluded that the results obtained in [5] and other studies, in particular, in [3, 4], are incorrect.

Work [1] actually reproduces the results obtained in [27, 28] by K. Vahala and A. Yariv, who even by the titles of these papers indicate that they present semi-classical theory of noise in semiconductor lasers. In [29], A. Yariv with coauthors with reference to studies [7, 30], which are devoted to quantum-mechanical rate equations, use rate equation (1b) for photon density including a term taking into account the contribution of spontaneous emission to the mode, similar to chapter 2 of book [14].

It should be noted that, when analysing anomalously high noise levels in [26], we used the model of spectral burning of carriers [21, 31], which yields a better coincidence with experiment than the model of scattering on electron density waves [32]. It is shown in [32] that the anomalous mode interaction can be explained taking into account that two-mode beating leads to carrier density modulation and formation of a diffraction grating, from which the high-power mode scatters. The electron density modulation causes permittivity modulation, which, in turn, leads to additional (induced) gain.

Thus, the rate equations criticised in [1, 2] because they do not take into account beating of optical modes provide a better coincidence between theory and experiment than the model that takes these beating into account.

A calculation of multimode spectra upon direct laser radiation modulation by the pump current is performed in [33].

The asymptotic character of threshold conditions and the multimode laser radiation are theoretically considered for the first time in [34]. The correctness of conclusions made in this paper is doubtless.

The author of [2] discusses the incorrectness of rate equations but even does not mention publications [7, 8] and does not analyse quantum-mechanical rate equations.

It is necessary to note that the author of [2] found himself in a paradoxical situation. The entire section 6 is devoted to the two-photon absorption. On p. 733, he claims that paper [3] considers nonlinear losses due to two-photon absorption and criticises the modelling results obtained in [3] taking into account the two-photon absorption.

In particular, on p. 745 in paper [2] he writes: "*We have to conclude that Kurnosov V.D. and Kurnosov K.V. [3] seem to be unaware that two-photon absorption was proposed previously (almost 50 years ago), not quite properly, by Popov and Shuikin [35] as a mechanism of multimode operation of diode lasers.*" The paradox is that two-photon absorption is considered neither in work [3] nor in the referenced works.

The correctness of conclusions made in [2] when considering two-photon absorption is also doubtful, because it was necessary to consider not only the direct two-photon absorption determined by term γS_m^2 but also indirect two-photon absorption determined by term ξS_m^3 [36, 37].

However, it is necessary to note that, indeed, the rate equations cannot be used to analyse mode-locking and some other processes: "…some fast processes (propagation of 2π-pulses, emission of π-pulses, and induced self-transparency) fall out of consideration." ([11], p. 201).

4. Conclusions

1. "Spontaneous emission of light is a typical quantum mechanical process. Quite evidently the semiclassical theory cannot treat this process. Thus it becomes necessary to develop a completely quantum mechanical theory of the laser" [10, p. 7].

2. The correctness of the rate equation for photon density with the term taking into account spontaneous emission follows from quantum-mechanical calculations. This is confirmed by the coincidence between the calculated and experimental results not only for spectral but also for noise and power characteristics.

3. The results of modelling of laser emission spectra obtained in [3–5] and other works are not incorrect, because they are based on the use of quantum-mechanical rate equa-

tions, whose validity is proven by experimental results obtained in [10–26].

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