

CONTROL OF LASER RADIATION PARAMETERS

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Optomechanical interaction in fibre lasers with micro-optomechanical resonance structures

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Abstract. **We report the results of studies of fibre lasers with passive modulators based on light-excited micro-optomechanical resonance structures (micro-oscillators). It is shown that in fibre lasers based on active fibres doped with rare-earth elements (Er, Er/Yb, Yb, Nd), the optomechanical interaction of laser radiation with micro-oscillators of various types (fibre-optic, microvolume) leads to self-oscillations of the characteristics of laser radiation at frequencies of relaxation oscillations and intermode beats synchronised with the frequencies of elastic eigenoscillations of micro-oscillators. It is found that in an ultra-long erbium–ytterbium fibre laser with a nonlinear mirror based on a microcantilever, laser photothermal excitation of the second mode of elastic transverse oscillations of the microcantilever makes it possible to perform passive mode locking exclusively due to *Q*-switching of the laser cavity. Pulsed lasing with a controlled repetition rate (~76 kHz), a pulse duration of 2–5 μs and an output energy of 0.1 μJ per pulse is implemented. Based on a simplified physical model of the indicated fibre lasers with micro-oscillators, we have developed an approximate mathematical model describing the regimes of passive mode locking of fibre lasers with micro-oscillators that play the role of mirrors with a nonlinear reflection coefficient in the laser cavity. The prospects for the development and application of the considered laser systems are discussed.**

Keywords: *fibre laser, micromechanical resonator, optomechanical interaction, passive Q-switching, mode locking, resonance, self-oscillation.*

1. Introduction

Currently, a new scientific and technical direction of laser optomechanics is being born at the junction of laser physics and wave optics, and nanotechnology and microsystems technology. Laser optomechanics is associated with studies of the interaction of laser radiation with passive and active optical resonators based on micro-optomechanical resonant structures (MOMRS's) excited by light [1, 2]. Interest in this direction is due, in particular, to the fact that it opens up wide opportunities for the development of new methods and devices to control laser radiation [3]. From the standpoint of the oscillation theory, MOMRS's are acoustomechanical oscillation systems with distributed parameters, characterised by a wide set of modes of elastic eigenoscillations, which, due to optomechanical interaction (OMI), can be excited by the energy of laser (optical) radiation. Investigations of OMI in laser systems with MOMRS's made it possible to obtain a number of important results of a fundamental nature related to the manifestation of the quantum properties of macro-objects–micro-oscillators [3], to the transformation of thermal motion and dynamic cooling to ultra-low temperatures [4], and to nonlinear dynamics of complex systems (self-oscillations, chaos, frequency conversion of laser radiation) [5], which paves a new avenue for research in fields such as quantum macrophysics and optics, mesoscopy, computer science, and physical materials science including biological micro- and nanoobjects and structures [6].

Optomechanical interaction can be caused both by the ponderomotive action of radiation (light pressure, optical 'gradient force', and angular momentum transfer resulting in the Sadovsky effect), and parametric effects (photothermal, radiometric, electrostriction in the field of a light wave, photo piezoelectric effect, etc.) [7, 8], which manifest themselves in a wide spectral range in various materials and media, including artificial ones. Since the oscillations of a MOMRS cause modulation of the parameters of the light wave interacting with it, OMI leads to the dependence of radiation characteristics on its intensity. This optomechanical nonlinearity has a number of characteristic features: a low threshold of nonlinearity, a resonant character of the modulation depth of the parameters of a light wave near the eigenfrequencies of the MOMRS, the possibility of simultaneous modulation of several parameters of a light wave (amplitude, frequency, phase, polarisation state, and directional pattern), and a flat spectral characteristic (for a number of OMI mechanisms), which significantly distinguishes it from nonlinearities caused, for example, by the Kerr effect or absorption saturation of a medium that are widely used in lasers for passive *Q*-switching [9, 10]. In this regrard, it is of great interest to study the dynamics of lasers with intracavity optomechanical nonlinearity, which leads to the formation of an active (laser) system with self-regulating feedback.

As an example, Fig. 1 shows different types of MOMRS's (microvolume and fibre-optic), which are included in the optical cavity of a fibre laser (FL). Oscillatory elements (OEs) of a microvolume MOMRS are a micromembrane (Figs 1c and 1d), the reflective surfaces of which, together with semitransparent reflectors at the ends of the fibres, form nonlinear Fabry–Perot resonators (FPRs), which play the role of FL mirrors. In waveguide (fibre-optic) micro-oscillators, OEs can be implemented as, for example, a segment of a single-mode fibre waist (Fig. 1a) or a two-mode fibre with micro-

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bends (Fig. 1b). Laser excitation of elastic oscillation modes in the MOMRS, in particular, transverse oscillation modes of OEs, leads to modulation of the FPR base and, consequently, to modulation (self-modulation) of the amplitude and phase of radiation reflected from it. Similarly, in fibre-optic MOMRS's, OE oscillations result, for example, due to the tunnelling of light from a single-mode fibre waist or energy exchange between the fundamental and second modes in a two-mode fibre, to modulation of parameters (amplitude, phase, and polarisation state) of radiation in the resonator

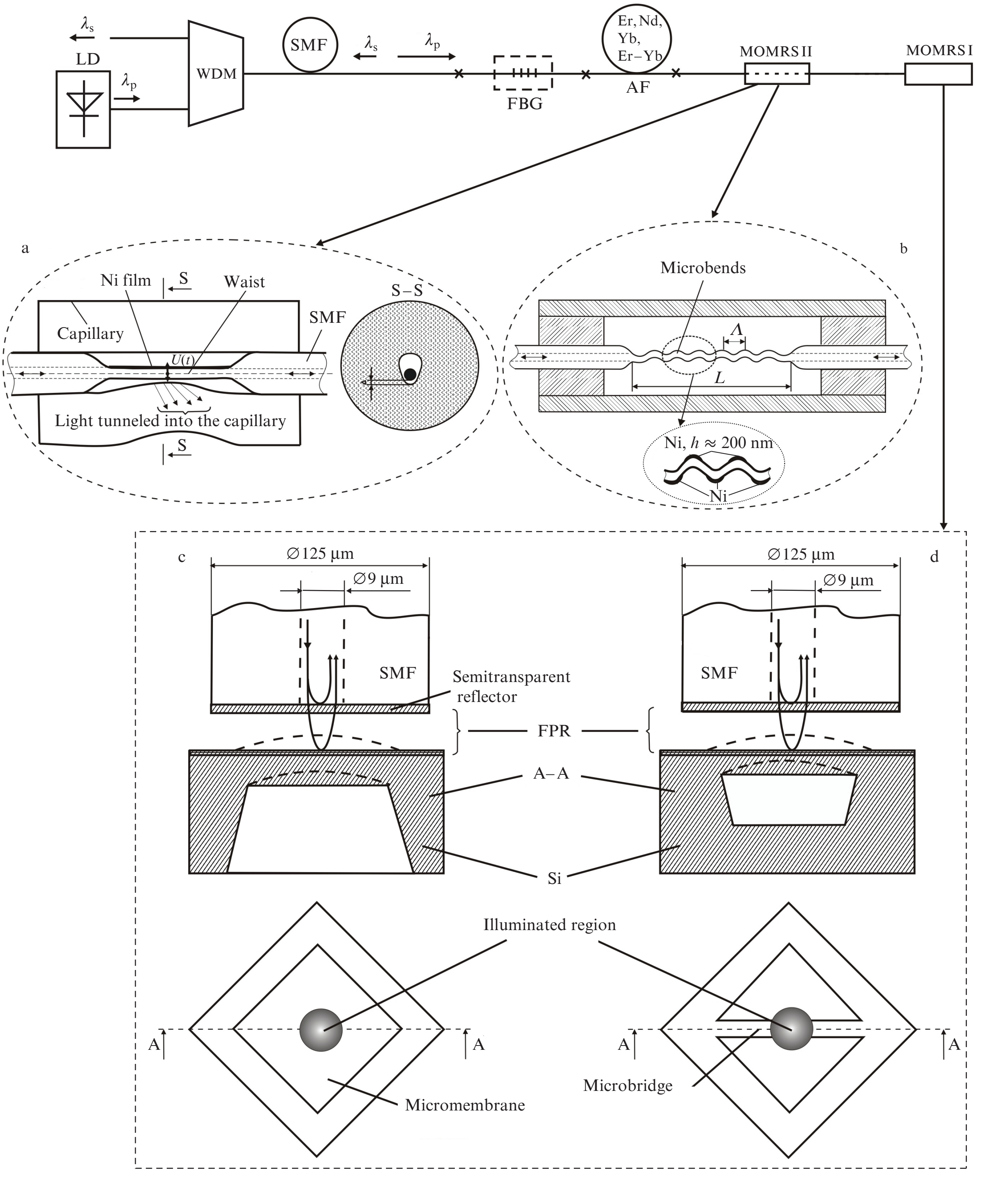

Figure 1. Schematic of a FL with micro-oscillators of (a, b) MOMRS II and (c, d) MOMRS I types:
(AF) active optical fibre (Er, Nd, Yb, etc.); (LD) pump laser diode (λ_p); (WDM) wavelength division multiplexer (λ_p/λ_s); λ_s is the lasing wavelength of the FL; (SMF) single-mode fibre; (FBG) fibre Bragg grating (mirror) of the FL cavity; (MOMRS I) microvolume silicon micro-oscillators; (MOMRS II) waveguide (fibre-optic) micro-oscillators based on special optical fibres.

and, thus, to passive modulation of a number of resonator characteristics (Q factor, phase incursion, and polarisation properties). It is important to emphasise that, in contrast to devices based on micro-electro-mechanical structures (MEMS's) [11–13] controlled using specified (external) electrical signals, the MOMRS dynamics in this case is determined exclusively by OMI in the laser system itself.

The efficiency of interaction of laser radiation with a MOMRS can be significantly increased under conditions of internal resonances, i.e. the coincidence of the MOMRS eigenfrequency with one or another characteristic (eigen)frequency in the laser. Taking into account the unique properties of FLs [14, 15], the study of fibre laser–micro-oscillator (FL–MOMRS) systems is of particular interest. Such unique FL features as a wide range of characteristic frequencies, effective optical coupling between the laser resonator and MOMRS (especially fibre-optic ones), the possibility of changes in a wide range of radiation power (energy), as well as flexible adjustment of the laser resonator configuration, mode composition, spectral and polarisation properties, etc., make it possible to implement different types of internal resonances in FL–MOMRS systems. The range of characteristic frequencies in FLs [relaxation oscillations (in-phase, antiphase), intermode beats, Raman interaction of modes, etc.], for example, in lasers based on active fibres (AFs) doped with rare-earth elements, can be on average from 1 kHz to 100 GHz. This range fully covers the MOMRS eigenfrequencies. This paper contains a generalising analysis of works in the field of studies of the interaction of radiation from erbium, erbium–ytterbium, and neodymium FLs with MOMRS's under conditions of internal resonances, as well as the results obtained recently in the course of studies of passive-mode-locking regimes implemented in erbium–ytterbium FLs with MOMRS-microcantilevers exclusively due to Q-switching of the laser cavity. A distinctive feature of these FLs belonging to class B lasers [9] is the significant inertia of the active medium, which leads, in particular, to a wide range of relaxation oscillation frequencies that have a pronounced resonance character. Internal resonances in such FL–MOMRS systems can lead to new interesting features in the lasing dynamics, opening up opportunities for developing multifunctional laser radiation sources (optical signal generators), new types of light-controlled optical elements, and precision fibre-optic sensors with a frequency output.

2. Self-oscillations under resonance conditions of frequencies of FL relaxation oscillations and MOMRS eigenoscillations

Lasers with micro-oscillators, which play the role of passive modulators of the optical cavity characteristics, were first studied in [16, 17]. It was shown that, in erbium–ytterbium FLs (EYFLs) with micro-oscillators, when the relaxation oscillation frequency $f_{\rm rel}$ of the laser coincides with the MOMRS eigenfrequency f, there arise self-oscillations with the intensity modulation of the generated radiation at the MOMRS eigenfrequency.

The authors of Refs [18, 19] demonstrated the dependence of the parameters of self-oscillations in a FL–MOMRS on the characteristics of the laser subsystem and the micro-oscillator; established the complex structure of the excitation zone of self-oscillations in the space of system parameters (pump level, spatial coordinates, optical-physical properties of MOMRS's, etc.); and showed the possibility of stabilising the repetition rate F of laser pulses using a MOMRS (analogue of quartz crystal frequency stabilisation in radio engineering), providing an accuracy of $\Delta F/F \approx 3 \times 10^{-6}$. It was found that self-oscillations also exist in FLs with intracavity fibre-optic MOMRS's based on special fibres that play the role of passive distributed modulators. Such all-fibre FL–MOMRS systems can be easily integrated into various fibre-optic circuits, which expands the possibilities of their applications. It was shown that, under resonance conditions ($f \approx f_{\rm rel}$), self-oscillations are possible in both linear and ring FL–MOMRS systems, regardless of the mode composition of the radiation and the design features of the laser cavity, including the case of nonresonant feedback implemented using diffuse mirrors [19]. This allows us to conclude that for $f \approx f_{\rm rel}$, the main features of self-oscillations in a FL–MOMRS can be considered within the framework of the concept of 'radiation as a whole' [20].

Of great interest is the study of the polarisation dynamics of radiation in a FL–MOMRS, caused by the dependence of OMI on the polarisation state and/or the dependence of the resonator anisotropy on MOMRS oscillations [21]. It was shown that under resonance conditions $f \approx f_{\rm rel\,a}$ ($f_{\rm rel\,a}$ is the frequency of relaxation antiphase beats of polarisation 'supermodes' in a FL), there emerge self-oscillations of the polarisation direction of laser radiation, which are caused by alternate generation of orthogonal 'supermodes' alternating with the resonant MOMRS frequency. This opens up prospects for designing sources of polarisation-modulated laser radiation with a high stability of the modulation frequency.

3. Passively mode locked FL–MOMRS system

In fibre lasers (as in any class B lasers [9]), the frequencies of relaxation oscillations are much lower than the cavity mode spacing: $\Omega = \nu_{n+1} - \nu_n \gg f_{\rm rel}$ (ν_n are the eigenfrequencies of the longitudinal modes of the FL), and from this point of view, self-oscillations arising at $f_{\rm rel} \approx f$ belong to the class of 'low-frequency' oscillations in lasers [10]. At the same time, under resonance conditions $f \approx \Omega$, micro-oscillators also make it possible to provide passive mode locking (PML) regimes in fibre lasers [22, 23], which are realised due to the nonlinearity of the MOMRS-based FPR mirror. Passive mode locking in the indicated FL–MOMRS with a composite (three-mirror) resonator can be due to various reasons: Q-switching and/or phase incursion in the laser resonator during MOMRS oscillations, as well as kinematic mode locking, which is ensured by the Doppler shift of the frequency of light reflected from the oscillating MOMRS surface. Although the field in an optical resonator with a moving mirror cannot be described (in the strict sense) within the framework of mode concepts [24], however, under certain assumptions [25], an approximate representation of the field with the help of 'quasi-modes' is possible that in the limit turn into modes of a static resonator, which justifies the use of 'mode' terminology in the consideration presented below. The simultaneous action of different PML mechanisms significantly complicates the dynamics of FL–MOMRS laser systems and makes it difficult to control the parameters of the generated laser pulses. As shown in this work, the unique combination of the properties of the modes of elastic eigenoscillations of a MOMRS in the form of a microcantilever (MC), which plays the role of a nonlinear mirror in a fibre laser with a simple (two-mirror) resonator (FL–MC), makes it possible to realise

PML regimes exclusively due to Q-switching of the laser resonator. A mathematical model is proposed that describes the dynamics of the considered laser systems under resonance conditions $f = \Omega$.

The experiments were performed using a laser system shown in Fig. 2. The active medium of the EYFL is a double-clad silica AF, the single-mode core of which is doped with erbium and ytterbium ($l_{af} \approx 4.5$ m, $N_{Er} \approx 5 \times 10^{18}$ cm^{-3}, and $N_{Yb} = 1.5 \times 10^{20}$ cm^{-3}). The AF (GTW) is optically pumped into a multimode cladding by radiation of a semiconductor laser with a wavelength of $\lambda_p \approx 976$ nm. The oscillating MOMRS element made of fused silica is in the form of a cantilever microbeam ($l \times b \times h = 1300 \times 320 \times 17$ μm), the reflecting surface of which plays the role of a mirror (M2) of the EYFL fibre-optic resonator. Optical coupling of the MC with the laser resonator is performed using a selfoc-based autocollimator, which forms a Gaussian beam with a diameter of $D \approx 380$ μm with a divergence angle $\varphi_0 \approx 2 \times 10^{-3}$ rad at the output. The total length of the resonator is $L = l_{af} + l_{smf}$, where l_{smf} is the length of a passive single-mode fibre (SMF-28). Laser excitation of elastic transverse oscillations of the MC is carried out due to the photothermal effect [26], which provides the possibility of selective excitation of bending oscillation modes [27]. In order to increase the amplitude value of the effective reflection coefficient of the autocollimator system, $|R_{2eff}|$, and the efficiency of laser excitation of oscillations, η, thin nickel films (200 nm) are deposited on the MC surface; in this case, due to the MC symmetry, the average absorbed radiation power does not lead to static bending deformation. Thus, R_{2eff} has only a real part, since MC oscillations do not lead to modulation of the reflected radiation phase.

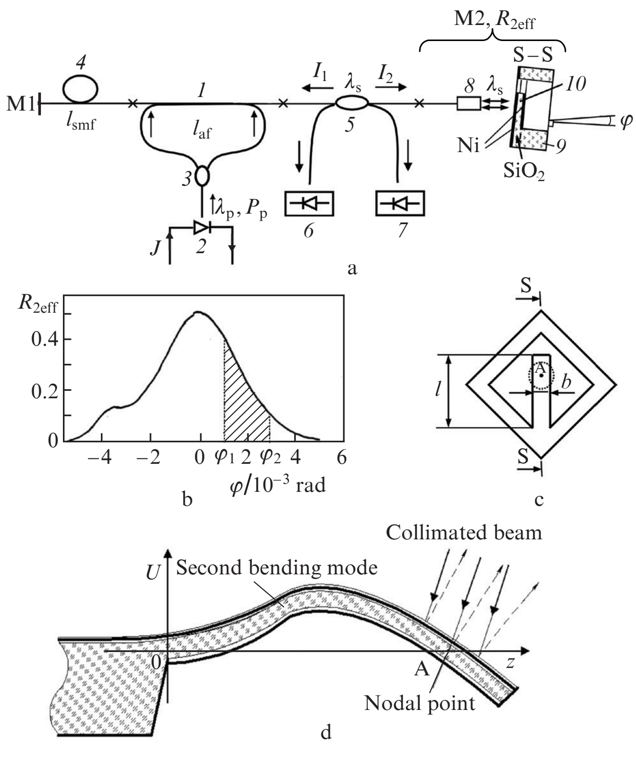

Figure 2. (a) Schematic of an ultra-long EYFL with a micro-oscillator, (b) angular pattern of an autocollimator with an MC-based reflector, (c) MC with a nodal point A, and (d) shape of the second mode of its bending oscillations [$U_2(z)$]:
(*1*) erbium–ytterbium double-clad silica AF (GTW); (*2*) semiconductor pump laser ($\lambda_p = 976$ nm); (*3*) multimode fibre Y-splitter (50 : 50); (*4*) section of a single-mode passive optical fibre (SMF-28); (M1) mirror with a reflection coefficient $R_1 = 85\%$ (multilayer interference reflector at the fibre end face); (*5*) single-mode fibre X-splitter (80 : 20); (*6, 7*) InGaAs photodetectors; (*8*) gradient rod collimator (selfoc); (*9*) micro-oscillator body; (*10*) oscillation element (MC).

Of the wide set of modes of elastic eigenoscillations of the MC [28], the second mode of transverse oscillations is of particular interest, the shape of which is presented in Fig. 2d. It has the following important properties: 1) the presence of a fixed (nodal) point A, the translational speed of which is $|V_A| = 0$; 2) curvature of the oscillation mode at the nodal point $\rho_A \approx |U''_2(z_A)|$ has a finite value $\rho_A^{-1} \neq 0$, which provides the possibility of photothermal excitation of this oscillation mode when illuminating the region of the nodal point (recall that the excitation efficiency is $\eta \approx \rho_A^{-1}$ [27]); and 3) the vicinity of point A performs only rotational oscillations around the axis passing through point A. Thus, under conditions of laser excitation of this oscillation mode by radiation incident in the region of the nodal point, the value of R_{2eff} is self-modulated, having only the real part $R_{2eff} = |R_{2eff}|$, since at $|V_A| = 0$ there is neither Doppler frequency shift ($\Delta\nu_d/\nu \approx |V_A|/c$) nor modulation of the reflected light phase (change in the length of the resonator $\Delta L \backsim |V_A|$).

The eigenfrequency and mechanical quality factor (in air) of the second mode of MC oscillations are $f_2 \approx 76$ kHz and $Q_2 \approx 70$ (for the fundamental mode $f_1 \approx 12.8$ kHz), while the resonance condition $f_2 = \Omega = c/2nL$ is fulfilled by using an ultra-long EYFL with the cavity length $L \approx 1.33$ km (n is the refractive index of the fibre). In the experiment, the frequency detuning ($f_2 - \Omega \neq 0$) was varied by changing both the intermode spacing by increasing (shortening) the length of the passive fibre Δl_{smf} and by shifting the resonance frequency Δf_2 as a result of heating the MC by cw optical radiation. Thus, the EYFL–MC system is a complex oscillatory system with a wide range of adjustable parameters: P_p, φ, η, Ω, f_2, etc. [P_p is the cw pump power, and φ is the angle between the axis of the incident laser beam and the normal to the MOMRS (see Fig. 2a).]

It was experimentally established that under resonance conditions $f_2 \approx \Omega$ at a pump level that exceeds a certain threshold value (second threshold) $P_{th2} \approx 1.8P_{th}$ (P_{th} is the laser generation threshold of the EYFL) and at values of the inclination angle in a certain interval ($\varphi_1 \lesssim \varphi \lesssim \varphi_2$) that corresponds to one of the branches of the autocollimator directional pattern (Fig. 2b), synchronous self-oscillations arise in the laser system, at which the generation of periodic laser pulses and MC oscillations occur simultaneously with a single (common) period T_m, close to the round-trip transit time for a photon in the laser cavity, $T_r = 2nL/c = T_m - \Delta T$, $\Delta T \ll T_r$. The oscillogram and the Fourier spectrum of the laser radiation intensity in the indicated self-oscillation regimes are shown in Fig. 3. Note that when the MC was replaced with an ordinary fixed mirror (similar to M1 in Fig. 2a), the self-mode-locking phenomenon was not observed in the EYFLs in question. The presented data, taking into account the dependence of the laser pulse duration on the pump level, allow us to conclude that the considered self-oscillations are due to passive mode locking of the EYFLs. Note that the connection of a passive single-mode fibre from the MC side leads to an increase in the self-oscillation threshold ($P^*_{th2} = 2.4P_{th}$). This can be explained by the fact that in order to achieve the required threshold amplitude of MC oscillations excited by

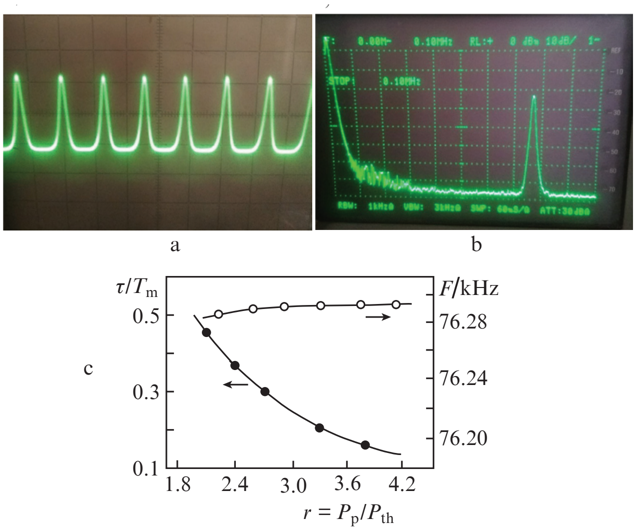

Figure 3. (a) Oscillogram and (b) Fourier spectrum of the laser radiation intensity in the self-oscillation regime (PML), as well as (c) dependence of the laser pulse duration and frequency on the excess of the pump level over the threshold in the PML regime.

laser radiation, compensation of radiation attenuation caused by losses in the passive fibre is required.

In self-oscillation regimes, the relative fluctuations of the laser pulse repetition rate $F = 1/T_m$ are $|(\Delta F/F)|_{fl} \approx 7 \times 10^{-5}$ (averaging time of 10^{-2} s); we also note that the measurements were performed under conventional (laboratory) conditions without the use of special methods of vibration-acoustic isolation and stabilisation of the operating conditions of the laser system. In particular, the instability of the pump power of the AF $|\Delta P_p/P_p|$ reached $\sim$3% (in the frequency range from 0.1 to 10 Hz). At a maximum pump level $r_{max} \approx 4.5$ ($r = P_p/P_{th}$) and a fixed MC eigenfrequency, self-oscillations are observed with relative changes in the cavity length $|\Delta L/L| \lesssim 0.3\%$. For a fixed length L, the frequency of laser pulses can be controlled (at least within the range $|\Delta F/F| \lesssim 2 \times 10^{-3}$) by shifting the resonant frequency of the MC, with $\Delta F = (1 + \alpha)\Delta f_2$, $|\alpha| \lesssim 3 \times 10^{-3}$. As can be seen from Fig. 3, the MC provides very high stability of the self-oscillation frequency with a significant (twofold) change in the pump power. For small variations in the angle equal to $|\Delta\varphi/\varphi| \lesssim 5\%$ and fixed L and P_p, a change in the self-oscillation frequency is $|\Delta F/F| \lesssim 2 \times 10^{-4}$. Note that in the PML regimes, the peak intensity of laser pulses and the average radiation power in the EYFL fibre cavity are $I_{s\,max} \lesssim 0.5$ MW cm^{-2} and $P_s \lesssim 50$ mW, respectively; the pulse energy $E_s \approx 100$ nJ (duration $\tau \geqslant 1.5$ μs). Note that $I_{s\,max} \ll I_{nl}$ (I_{nl} is the nonlinearity threshold of single-mode fibres) [29].

4. Mathematical model of the FL–MC system

The dynamics of FL–MC systems, which are multiparametric and complex nonlinear systems, is determined by the interaction of three subsystems: laser radiation, active medium, and micro-oscillator, each of which can be described with a varying degree of accuracy. Taking into account the key role of the phase of the light wave in the PML regimes, laser radiation can be described in the approximation of slow envelopes for a field satisfying the wave equation [9]. In this case, a sufficiently low radiation intensity ($I_s \ll I_{nl}$) and a significant duration of laser pulses make it possible to neglect both chromatic dispersion and optical nonlinearity (with the exception of the nonlinear properties of the AF) in the laser cavity. For this reason, the PML mechanisms, which are effective at high intensities and strong dispersion in an optical cavity (considered, in particular, in [30]), do not play a significant role in this case. In addition, the absence of self-mode locking of the EYFL indicates that one can also neglect the small-scale spatial dynamic inversion grating in the active medium (recall that the interaction of the generated modes with the inversion grating is one of the main reasons for self-mode locking in lasers [31]). This makes it possible to investigate the features of laser dynamics caused by OMI without taking masking effects into account.

Passive mode locking in the FL–MC system is caused by a number of physical effects and phenomena associated with significantly different aspects of the interaction of laser radiation with the MC (photothermal and thermoelastic effects, excitation and propagation of elastic waves and vibrations, thermal reflection, light modulation, etc.). In this case, the MC, which leads to the resonant nature of the Q-switching depth, differs significantly from the known types of passive modulators based on other physical principles (absorption saturation, Kerr nonlinearity, etc.). It should be noted that, as far as we know, the features of the PML related to the resonant nature of the Q-switching depth have not yet been considered. Taking this into account and the possibility of varying the parameters and configuration of the FL within a wide range, the considered FL–MC systems are also of interest as model objects convenient for studying these features. In this regard, we have proposed a simplified mathematical model that describes the 'high-frequency' dynamics of the FL–MC under resonance conditions ($f \approx \Omega$), including the PML regimes.

The photoinduced displacements and deformations of the MC [$\boldsymbol{U}(z, t)$], caused by OMI, lead under the lasing conditions in the FL–MC to modulation of the parameters of the collimated beam reflected from the MC, in particular, to modulation of the effective reflection coefficient of the 'micro-oscillator' mirror (M2): $R_{2\mathrm{eff}}(t) = R_2(\varphi + \theta(z_A, t))$, where $R_2(\varphi) = R_{20}\exp[-\varphi^2/(2\varphi_0^2)]$ is the angular characteristic of the autocollimator (Fig. 2b), approximated by the Gaussian function; and $\theta(z_A, t) = \theta_A(t)$ are the rotational oscillations of the MC section near the nodal point. Oscillations $\theta_A(t)$ lead to modulation of the reflection coefficient: $R_{2\mathrm{eff}}(t) = R_{20}\exp\{-[\varphi + \theta_A(t)]^2/(2\varphi_0^2)\}$. Since oscillations with frequencies close to the eigenfrequency of the second MC mode are of particular interest, taking into account the sufficient difference in the eigenfrequencies of the nearest modes ($|f_2 - f_1| \gg f_1$, $|f_2 - f_3| \gg f_2$) and the small width of the resonance curve for of the second mode ($f_2/Q_2 \ll |f_2 - f_{1,3}|$), when describing MC oscillations, one can restrict oneself to the approximation of a single-resonance linear oscillator with equivalent parameters m_{eff}, $\omega_2 = 2\pi f_2$, and Q_2 (m_{eff} is the effective mass, and Q_2 is the mechanical quality factor) [32]. In this case, angular deviations $\theta_A(t)$ caused by laser (photothermal) excitation of oscillations are determined by the equation:

$$\ddot{\theta}_A + \frac{\omega_2}{Q_2}\dot{\theta}_A + \omega_2^2\theta_A = \frac{1}{m_{\mathrm{eff}}}N_\theta, \qquad (1)$$

where $N_\theta = N_{\theta,s} + N_{\theta,p} + N_{\theta,sp}$ is the total force (moment of forces), which in the general case is due to both the generated laser radiation (I_s) and the residual pump radiation (I_p^*) incident on the MOMRS and transmitted through the AF, and spontaneous radiation (I_{sp});

$$N_{\theta,i} = \int_{-\infty}^{t} h_i(t-t')P_i(t')\mathrm{d}t', \; i = \mathrm{s, p, sp}; \tag{2}$$

$h_i(t-t')$ are the functions characterising the efficiency of photothermal excitation of oscillations and depending on the optical-physical, elastic, and thermomechanical properties and geometric dimensions of the MC [26]; and $P_i = SI_i$ is the radiation power ($S = \pi D^2/4$ is the beam area). For simplicity, let us assume that the pump radiation is completely absorbed in the AF ($I_{\mathrm{p}}^* \approx 0$), which is realised at $l_{\mathrm{af}} \gg [\sigma N_{\mathrm{Yb}}(N_{\mathrm{Er}})]^{-1}$ and leads to $N_{\theta,\mathrm{p}} \approx 0$ (σ is the absorption cross section). At a sufficiently high pump level, when $I_{\mathrm{s}} \gg I_{\mathrm{sp}}$, the contribution of spontaneous emission can also be neglected: $N_{\theta,\mathrm{sp}} \approx 0$. Since the MC material (fused quartz) is characterised by a rather small depth of the temperature wave decay at the self-oscillation frequency $\{\delta(F) \ll h$ [26]$\}$, then, taking into account $h \ll D$, we have $N_{\theta,\mathrm{s}}(t) \approx K\mathrm{Re}[P_{\mathrm{s}1}(t)\exp(\mathrm{i}\pi/2)]$, where $P_{\mathrm{s}1}(t) = P_{10}\exp(\mathrm{i}2\pi Ft)$ is the Fourier harmonic (fundamental) of the laser radiation power modulated with frequency F, and $K \approx \eta(1 - R_{20})$ is the efficiency factor. We note, running a few steps forward, that it is the phase shift $\pi/2$ in the expression for $N_{\theta,\mathrm{s}}(t)$, which arises due to $\delta \ll h$, that provides the phase relationship between laser pulses and Q-switching caused by photoinduced MC oscillations, which is a necessary requirement for the PML existence.

Oscillations $\theta_{\mathrm{A}}(t)$ lead to modulation of losses $\Pi(t) = -\ln[R_1R_2(\theta_{\mathrm{A}}(t))]$ and, consequently, to the laser cavity Q-factor:

$$Q(t) = \frac{2\pi L}{\lambda\Pi(t)}\frac{2\pi L}{\lambda}\left(\Pi_0^* + \frac{\varphi\theta_{\mathrm{A}}}{\varphi_0^2} + \frac{\theta_{\mathrm{A}}^2}{2\varphi_0^2}\right)^{-1}, \tag{3}$$

where $\Pi_0^* = -\ln(R_1R_{20})^{1/2} + \varphi^2/(2\varphi_0^2)$ [all passive losses are taken into account in the term $-\ln(R_1R_{20})$]. At oscillations $\theta_{\mathrm{A}}(t) \approx \cos(2\pi ft)$, where $f \approx \Omega$, Q-switching at the intermode beat frequency leads to a strong interaction of the generated modes with the possibility of their synchronisation. To study this regime, we used a space–time approach based on equations [33, 34] describing lasers with intracavity Q-switches, which in this case are MCs. Supplementing the above equations with terms and equations (1)–(3), which describe the Q-switching of the resonator using the MC and the motion of the latter under the action of laser radiation, we obtain a closed (self-consistent) system of equations describing the FL–MC:

$$T_{\mathrm{m}}\frac{\partial A(T,t)}{\partial T} = \left[g\left(1 - \frac{1}{\Omega_{\mathrm{g}}}\frac{\partial}{\partial t} + \frac{1}{\Omega_{\mathrm{g}}^2}\frac{\partial^2}{\partial t^2}\right) + (T_{\mathrm{m}} - T_{\mathrm{r}})\frac{\partial}{\partial t} - \left(\Pi_0^* + \frac{\varphi\theta_{\mathrm{A}}}{\varphi_0^2} + \frac{\theta_{\mathrm{A}}^2}{2\varphi_0^2}\right)\right]A(T,t), \tag{4}$$

$$\frac{\mathrm{d}g}{\mathrm{d}T} = \frac{g_0 - g}{\tau_{\mathrm{sp}}} - \frac{gnc\varepsilon_0}{T_{\mathrm{m}}I_{\mathrm{sat}}}\int_{-T_{\mathrm{m}}/2}^{T_{\mathrm{m}}/2}|A(T,t)|^2\mathrm{d}t, \tag{5}$$

$$\ddot{\theta}_{\mathrm{A}} + \frac{\omega_2}{Q_2}\dot{\theta}_{\mathrm{A}} + \omega_2^2\theta_{\mathrm{A}} = \frac{1}{m_{\mathrm{eff}}}N_{\theta,\mathrm{s}}, \tag{6}$$

where the envelope of the light wave amplitude $A(T,t)$ takes into account both slow ($T \geqslant T_{\mathrm{r}}$) and fast field changes (variable t); $g_0 \backsim (P_{\mathrm{p}}/P_{\mathrm{th}} - 1)$ is the unsaturated amplification of the AF (in the frequency band $\Omega_{\mathrm{g}} \approx 10^{11}$–$10^{12}$ Hz), depending on the pump level; g is the AF gain taking saturation into account (we assume that with one round-trip the change is $|\Delta g| \ll g$); $I_{\mathrm{sat}} = h\nu/(2\sigma_{\mathrm{las}}\tau_{\mathrm{sp}})$ is the saturation intensity; $h\nu$ and τ_{sp} are the energy and spontaneous photon lifetime; σ_{las} is the laser transition cross section; and N_θ is the photoinduced force (2).

A detailed study of the system of equations (2)–(6) (to be studied in the future) is a rather complex, independent task; therefore, we will restrict ourselves here to a qualitative consideration of stationary single-pulse PML regimes. In this case, the search for the corresponding approximate solutions is greatly facilitated due to the resonant nature of the MC oscillations. The fact is that, in contrast to passive modulators based on absorption saturation or the Kerr effect, in which a significant change in losses occurs precisely at the instant of a laser pulse action, in the case of the MC, due to mechanical inertia, the laser pulse action leads to a relatively small but sufficient long-term disturbance of the MC motion, which has the character of free oscillations. Due to the regularity and significant duration of free oscillations of the MC, the Q-switching caused by them, to a certain extent, has a 'specified' character, inherent in the regime of active Q-switching with forced sinusoidal modulation of losses. Thus, the MC, which plays the role of a passive Q-switch, exhibits the properties of an active modulator under conditions of resonant oscillations, and with an increase in the mechanical Q-factor of the micro-oscillator, this manifestation will only increase. Taking into account the foregoing, the shape of laser pulses in the considered PML regimes can be approximated by a Gaussian function characteristic of the active Q-switching of the cavity [35]. As a result, it is expedient to seek approximate solutions of system (4)–(6) in the form

$$A(T,t) \approx A_0\sum_{n=0}^{\infty}a(t - nT_{\mathrm{m}}), \tag{7}$$

$$\theta_{\mathrm{A}}(t) \approx \theta_{\mathrm{A}_0}\cos(2\pi Ft + \gamma), \tag{8}$$

where $T_{\mathrm{m}} = T_{\mathrm{r}} + \Delta T$ is the period of the pulses with constant peak amplitude A_0 ($\Delta T \ll T_{\mathrm{r}}$); θ_{A_0} is the amplitude of oscillations of the MC; γ is the phase shift between MC oscillations and laser pulses [harmonic $P_{\mathrm{s}1}(t)$)]; and $a(t) = \exp[-t^2/(2\tau^2)]$ is the Gaussian shape of laser pulses of duration $\tau \ll T_{\mathrm{m}}$. The desired quantities are T_{m}, A_0, θ_{A_0}, g, γ, and τ, with $P_{10} = \sqrt{2\pi}\,nB\varepsilon_0csA_0^2\tau/(4T_{\mathrm{m}})$ (s is the area of the AF mode, and $B = 0.4$–0.7 is the radiation power transfer coefficient in the section of the AF–MC resonator).

Substitution of (7) and (8) into the system of equations (2)–(6) leads to a number of relations presented below. The pulse duration is determined by the expression

$$\tau^4 = 2\bar{g}[mF^2\Omega_{\mathrm{g}}^2(1 + m\varphi_0^2/\varphi^2)]^{-1}, \tag{9}$$

where $m = |\varphi\theta_{\mathrm{A}_0}/\varphi_0^2|$ the depth of loss modulation due to MC oscillations; and

$$\bar{g} = g_0[1 + n\varepsilon_0cA_0^2\tau/(I_{\mathrm{sat}}T_{\mathrm{m}})]^{-1} \tag{10}$$

is the average gain of the AF per one trip, which is determined by integration (5) within the period T_{m}. Expression (10), in essence, is a balance condition: in the stationary oscillation regime, the resulting change in the AF inversion under the action of a laser pulse and pumping during the oscillation period T_m should be equal to zero.

Integrating equation (4) within the total period T_m, we obtain

$$\bar{g} - \Pi_0^* - \frac{\theta_{A_0}}{\varphi_0^2}\varphi(\cos\gamma) = 0. \quad (11)$$

Since lasing occurs at $\bar{g} > \Pi_0^*$, it follows from (11) that self-oscillations are possible ($\theta_{A_0} \neq 0$) for $\varphi(\cos\gamma) > 0$, i.e., only on one of the branches of the angular pattern (Fig. 2b), and this is consistent with the experiment. From equation (6) we obtain

$$\theta_{A_0} = \frac{KP_{10}}{4\pi^2\sqrt{(f_2^2 - F^2)^2 + 4f_2^2F^2/Q_2^2}},$$
$$\gamma = \frac{\pi}{2} - \arctan\frac{f_2F}{(f_2^2 - F^2)Q_2}. \quad (12)$$

It can be assumed that the generation of laser pulses occurs at the moments of minimum losses corresponding to $\gamma \approx 0$ [exact values are determined from Eqn (12)]. From (4) follows the relation that determines the relationship between the frequencies F and Ω,

$$\frac{F - \Omega}{\Omega} = 2\pi F^2\tau^2 m \sin\gamma - \frac{F}{\Omega_g}\bar{g}; \quad (13)$$

whence, taking into account F, $\Omega \ll \Omega_g$, $m \ll 1$, and $\sin\gamma \ll 1$, we obtain $F \approx \Omega$, which is consistent with the experiment. Relation (13) is fulfilled in a limited frequency detuning interval: $\Delta_1 \lesssim (F - \Omega)/\Omega \lesssim \Delta_2$, the boundaries of which are determined by the stability condition for stationary oscillations (7) and (8). The dependence of the self-oscillation frequency on the MC characteristics $F(f_2, Q_2, K, \ldots)$ is found as a result of the joint solution of the system of equations (7)–(13), which also determine the dependences $m(f_2, Q_2, K, \ldots)$, $\tau(f_2, Q_2, K, \ldots)$, $\gamma(f_2, Q_2, K, \ldots)$, etc. The system of equations (7)–(13) makes it possible to establish both the conditions of existence, i.e. the requirements for the parameters of the system, and the dependences of the characteristics of stationary single-pulse PML regimes on the main parameters of the FL–MC. In particular, preliminary research results show that an increase in the$Q$-factor of a micro-oscillator can lead to a decrease in the frequency difference $(F - f_2)/f_2$; therefore, high-Q MOMRS should be used to increase the stability of the laser pulse frequency.

5. Conclusions

The features of the dynamics of laser radiation caused by the optomechanical interaction of the FL radiation with light-excited MOMRS's are investigated. The experimental results obtained in this work allow us to conclude (taking into account the data of previous studies) that this interaction in the considered laser systems under conditions of internal resonances leads to self-oscillations and modulation of the characteristics of laser radiation with a frequency determined mainly by the eigenfrequency of elastic oscillations of the micro-oscillator. Due to the fact that at present much attention is paid to the development of FLs characterised, in addition to a high pulse energy, also by a long duration of laser pulses (hundreds of nanoseconds and more) [36–38], we note that the PML regimes implemented in the laser systems in question represent a new approach to solving this problem. It is noteworthy that such laser systems with a relatively simple scheme provide a sufficiently high stability of the characteristics of laser pulses and the possibility of their control (within a limited range) by controlling the system (in particular, the micro-oscillator) parameters. The dependence of the resonant frequency of the MC on external influences and environmental conditions opens up possibilities for designing new types of precision fibre-optic sensors with a frequency output on the basis of the FL–MC system, which are easily integrated into modern digital measuring systems. We note the relevance of further development of models of fibre lasers with micro-oscillators, taking into account the effect of noise and random external disturbances, allowing one to study fluctuations and limiting levels of instability of self-oscillation parameters in laser systems, and also to determine, in particular, the sensitivity threshold of sensors based on them.

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