Quantum Electronics **50** (9) 838–843 (2020)

INTERACTION OF LASER RADIATION WITH MATTER. LASER PLASMA https://doi.org/10.1070/QEL17261

Generation of a quasi-static magnetic field by a circularly polarised laser pulse due to tunnelling gas ionisation

I.M. Gabdrakhmanov, V.Yu. Bychenkov

Abstract. **We present a theoretical model of the quasi-static magnetic field generation in a laser channel, which is formed behind the front of a short laser pulse that ionises a gas. The generation of a magnetic field is caused by the appearance of the electron pressure anisotropy during tunnelling ionisation of atoms. In the considered case of subrelativistic laser light intensities, the generated magnetic field can reach ~1 MG with an energy transformation ratio of about 1 %, which paves the way for identifying the proposed mechanism when use is made of a wide class of ultrashort pulse lasers.**

Keywords: *quasi-static magnetic field, tunnelling gas ionisation, subrelativistic intensity, ultrashort laser pulses.*

1. Introduction

The generation of spontaneous magnetic fields in plasma has been widely discussed for more than half a century for solar/astrophysical plasma, laboratory plasma and laser thermonuclear fusion (LTF) plasma. In this case, a whole range of processes of self-generation and self-organisation of magnetic structures in plasma is due to the electron pressure anisotropy (temperature or energy distribution of electrons) on characteristic scales – from astrophysical, for example, with magnetic reconnection [1], to microscopic, for example, in the problem of the magnetostatic component of the field with Coulomb charge screening [2]. The explanation of the generation of quasi-static magnetic fields in a plasma with anisotropic pressure naturally follows from the tensor nature of the total electron pressure, which is taken into account, for example, in the vortex electron anisotropic hydrodynamics (VEAH) model [3] and leads to a change in the nature of the plasma thermal electromotive force (EMF). It is the latter (neglecting the possible plasma anisotropy) that was used to explain the mechanism of generation of spontaneous magnetic fields in laser plasma at the very beginning of research [4, 5].

The electron pressure anisotropy qualitatively changes the source of a thermal EMF, for which, in contrast to the classical case of isotropic pressure [5], the presence of crossed gradients of temperature and plasma density is not required, as shown in [6]. With regard to current experiments on laser-plasma interaction for LTF, the electron pressure anisotropy due to the inverse bremsstrahlung heating of laser-plasma speckles is considered as a possible cause of the magnetic field generation, which can lead to magnetisation of heat transfer in a speckle [7]. The electron pressure anisotropy resulting from ionisation of atoms by hard radiation is also an effective quasi-stationary source of EMF, leading to the generation of spontaneous magnetic fields [8, 9].

Along with the mechanisms of generation of a quasi-static magnetic field by a source of an anisotropic EMF, when its excitation is possible from a zero level, the electron pressure anisotropy of the plasma can lead to the magnetic field generation, which is characterised by instability. This process is, in essence, analogous to the development of the classical electromagnetic Weibel instability [10]. An example is a laser plasma anisotropically heated due to the inverse bremsstrahlung absorption of heating radiation [11, 12], or and anisotropic plasma formed as a result of the ionisation of atoms by a strong laser field [13–16] or as a result of the photoeffect when a substance is irradiated with X-rays [9, 17]. It has recently been demonstrated that fast optical ionisation of a gas by intense laser light is a good laboratory platform for studying kinetic plasma instabilities arising from the anisotropy of the electron energy distribution [18].

In this work, we develop a theoretical model for the generation of a magnetic field in a gaseous medium, which, due to tunnelling ionisation, transforms into a transparent plasma with an anisotropic electron energy distribution behind the front of a propagating laser pulse of subrelativistic intensity. The model in question substantially complements the concept of an anisotropic laser thermal EMF mechanism of magnetic field generation, first formulated more than 20 years ago [13], but still not developed. The point is that the theoretical approach of [13], although making it possible to estimate the rate of an increase in the magnetic field in the laser beam caustic, did not allow its steady-state distribution and magnitude to be found. Here we bridge this gap.

In Section 2, we present equations describing the generation of a magnetic field behind the front of a laser beam propagating in a transparent medium; in Section 3, their solution is given for the case of circular polarisation of light, which establishes the excited field distribution and magnitude, making it possible to estimate the coefficient of conversion of laser energy into magnetic field energy. Section 4 summarises the main conclusions of the work.

I.M. Gabdrakhmanov Lebedev Physical Institute, Russian Academy of Sciences, Leninsky prosp. 53, 119991 Moscow, Russia;
V.Yu. Bychenkov Lebedev Physical Institute, Russian Academy of Sciences, Leninsky prosp. 53, 119991 Moscow, Russia; Center for Fundamental and Applied Research, Dukhov Automatics Research Institute (VNIIA), ul. Sushchevskaya 22, 127055 Moscow, Russia; e-mail: bychenk@lebedev.ru

Received 30 January 2020; revision received 29 March 2020
Kvantovaya Elektronika **50** (9) 838–843 (2020)
Translated by I.A. Ulitkin

2. Basic equations

We consider the ionisation of a gas medium by a laser pulse of subrelativistic intensity, the quantum energy $\hbar\omega$ of which is less than the ionisation energy U of an atom (the case of nonlinear ionisation), and the frequency ω is less than the electron tunnelling frequency $\omega_t \approx eE_0\sqrt{2mU}$, where E_0 is the laser field amplitude; and e and m are the charge and mass of the electron. Therefore,

$$\omega < \omega_t \approx \frac{eE_0}{\sqrt{mU}} \approx \omega v_E\sqrt{\frac{m}{U}},$$

where $v_E = eE_0/(m\omega)$ is the electron oscillation rate in the laser wave field, and the Keldysh parameter is small, $\gamma = \omega/\omega_t = \sqrt{U/(mv_E^2)} < 1$, which, as shown in [19], corresponds to the case of tunnelling ionisation. Thus, here we restrict ourselves to the range of laser radiation intensities I corresponding to the condition $\sqrt{U/m} < v_E < c$. For visible-range lasers, this condition is fulfilled in the range from an intensity slightly exceeding 10^{16} W cm^{-2} to $\sim10^{18}$ W cm^{-2}, i.e., for sufficiently compact and widespread laboratory laser systems.

Under conditions of tunnelling ionisation in the subrelativistic limit of laser intensities, i.e., when the inequality $(v_E/c)^2 < 1$ is met, one can use approximate expressions for the electron distribution function $f(\boldsymbol{v})$. Depending on the laser light polarisation, which is determined by the ellipticity parameter α, it is represented in the form [13]:

$$f(\boldsymbol{v}) \propto \delta(v_y)\delta(v_z)\exp[-mv_x^2/(2T)],\ \alpha = 0 \text{ (linear polarisation)},$$

$$f(\boldsymbol{v}) \propto \delta(v_z)\delta(v_x^2 + v_y^2 - v_E^2),\ \alpha = 1 \text{ (circular polarisation)}, \quad (1)$$

$$f(\boldsymbol{v}) \propto \delta(v_z)\delta(v_x^2 + v_y^2/\alpha^2 - v_E^2)\exp[-mv_x^2/(2T)],\ 0 < \alpha < 1 \text{ (elliptical polarisation)}$$

Here the pulse propagates along the z axis and is polarised along the x axis. It is seen that in the case of elliptical polarisation, the tendency of α to zero or to unity ensures the transition to the cases of linear and circular polarisations, respectively. In the case of circular polarisation, the electrons formed during tunnelling ionisation are ejected in the transverse direction (with respect to the pulse propagation direction) with an energy equal to the oscillator energy in the laser field, $mv_E^2/2$. The value $T \ll mv_E^2/2$ introduced in (1) determines the thermal spread of electrons.

Let us now turn to the hydrodynamic description of a rarefied plasma, which arises as a result of ionisation of a gas by a laser pulse propagating in it, taking into account the anisotropic energy distribution of electrons during such photoionisation. Being interested in electrons of multikvolt energies, we neglect their collisions and write the standard (linear) equation for the electron current density $\boldsymbol{J}$ in the form of the generalized Ohm's law in a collisionless plasma:

$$\frac{\partial \boldsymbol{J}}{\partial t} = \frac{\omega_p^2}{4\pi}\boldsymbol{E} - \frac{1}{mn}\nabla\hat{P}. \quad (2)$$

Here ω_p is the plasma frequency of electrons ($\omega_p^2 = 4\pi e^2 n/m$, where n is the density of electrons), and the components of the electron pressure tensor $\hat{P}$ are expressed through the electron distribution function (1): $P_{ij} = m\int d^3v v_i v_j f(\boldsymbol{v})$.

Equation (2) is used to close Maxwell's equations,

$$\mathrm{rot}\boldsymbol{E} = -\frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t}, \quad \mathrm{rot}\boldsymbol{B} = -\frac{4\pi}{c}\boldsymbol{J} + \frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t}, \quad (3)$$

which are transformed into the system of two equations

$$\frac{\partial^2 \boldsymbol{E}}{\partial t^2} + c^2[\nabla\times[\nabla\times\boldsymbol{E}]] + \omega_p^2\boldsymbol{E} = \frac{4\pi e}{m}\nabla\hat{P}, \quad (4)$$

$$\frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t} = -\nabla\times\boldsymbol{E}. \quad (5)$$

This system completely describes the generation of an electromagnetic field under the action of a source due to the electron pressure anisotropy. The right side of equation (4) is the source of the anisotropic EMF. In the case $P_{ij} = P\delta_{ij} = nT$ (isotropic source), the system of Eqns (4) and (5) gives the well-known expression for the generation rate of the quasi-static ($\partial^2/\partial t^2$, $c^2\nabla^2 \ll \omega_p^2$) magnetic field $\dot{B} = (c/en)[\nabla n \times T]$, for example, in the LTF plasma.

Unlike the quasi-stationary case, we are interested in the rapid excitation of the magnetic field, when the characteristic time of the source change and the time l/c, where l is its characteristic spatial scale (for example, the laser beam radius), turn out to be less than the inverse plasma period $\sim\omega_p^{-1}$. Then, neglecting the term proportional to the plasma frequency ω_p in Eqn (4), we obtain the equation for the generation of a magnetic field with a source caused by the electron pressure anisotropy:

$$\frac{\partial}{\partial t}\left(\frac{\partial^2 \boldsymbol{B}}{\partial t^2} - c^2\Delta\boldsymbol{B}\right) = -\frac{4\pi ec}{m}\mathrm{rot}(\nabla\hat{P}). \quad (6)$$

Equation (6) is essentially similar to the equation describing the generation of a magnetic field in a vacuum by a given current.

3. Generation of a magnetic field by a circularly polarised laser pulse during tunnelling ionisation of a medium

Let us consider the obtained equation (6) in the case of gas ionisation by circularly polarised laser light in the laser beam caustic (waist) region, which has an approximately cylindrical shape. In this case, behind the front of the ionising pulse, an anisotropic cylindrical plasma is formed, in which the components of the electron pressure tensor have the form

$$P_{ij} = P_\perp(\delta_{ij} - \kappa_i\kappa_j) + P_\parallel\kappa_i\kappa_j. \quad (7)$$

Here $P_\parallel = nT_\parallel$ and $P_\perp = nT_\perp$ are the characteristic electron energy (pressure) densities along and across the laser pulse propagation direction; $\boldsymbol{\kappa}$ is the unit vector directed along the ionising laser pulse propagation; and

$$T_\parallel \equiv m\frac{\int d^3v v_z^2 f(\boldsymbol{v})}{\int d^3v f(\boldsymbol{v})} \quad \text{and} \quad T_\perp \equiv m\frac{\int d^3v v_\perp^2 f(\boldsymbol{v})}{\int d^3v f(\boldsymbol{v})} \quad (8)$$

are the average energies of electrons (temperatures), respectively, along and across the laser pulse propagation direction.

With circular polarisation, the problem is axially symmetric and can be described quite simply using cylindrical coordinates (r, φ, z), where the z axis corresponds to the direction of laser pulse propagation and $\partial/\partial\varphi = 0$. In this case, for the source on the right-hand side of equation (6) we have the expression

$$\mathrm{rot}(\nabla\hat{P}) = \left[0; -\frac{\partial^2(P_{\parallel} - P_{\perp})}{\partial r\partial z}; 0\right], \tag{9}$$

directly due to the presence of anisotropy of the electron energy distribution, $P_{\parallel} \neq P_{\perp}$.

The difference from zero only in the azimuthal component of the source causes the generation of a one-component magnetic field $\boldsymbol{B} = \{0, B_{\varphi}, 0\}$. According to (9), from (6) we obtain the equation for it:

$$\frac{\partial}{\partial t}\left[\frac{\partial^2 B}{\partial t^2} - c^2\left(\Delta B - \frac{B}{r^2}\right)\right] = \frac{4\pi e c}{m}\frac{\partial^2}{\partial r\partial z}(P_{\parallel} - P_{\perp}), \tag{10}$$

where $B \equiv B_{\varphi}$. Taking into account that the energy gained by the electron during tunnelling ionisation is much higher than the gas temperature, $T_{\perp}/T_{\parallel} \approx mv_E^2/T \gg 1$ [the latter is generally neglected in approximate formula (1)], we rewrite equation (10) in the form

$$\frac{\partial}{\partial t}\left(\frac{\partial^2 B}{\partial t^2} - c^2\frac{\partial^2 B}{\partial z^2} - c^2\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial}{\partial r}rB\right) = -\frac{4\pi e c}{m}\frac{\partial^2 P_{\perp}}{\partial r\partial z}. \tag{11}$$

We emphasise that we restrict ourselves to studying the generation of a magnetic field only in the region of a laser caustic with a length of the order of the Rayleigh one, where the interaction with the medium turns out to be the strongest and the magnetic field is expected to be maximum. In addition, our model is also applicable for such a self-focusing regime as the trapping of laser light, in which a laser pulse propagates in a channel of virtually constant radius over a distance significantly exceeding the Rayleigh length (cf. [20, 21]).

We neglect the difference between the propagation velocity of a laser pulse and the speed of light c in a low-density medium and its losses (depletion of the pulse along the caustic length), and also take into account that the source (right-hand side) in Eqn (11) is a function of the variables $\xi = z - ct$ and r. Then equation (11) takes a simple form,

$$\frac{\partial}{\partial\xi}\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial}{\partial r}rB(r,\xi) = -\frac{4\pi e}{mc}\frac{\partial^2 P_{\perp}(r,\xi)}{\partial\xi\partial r}, \tag{12}$$

and easily integrates:

$$B(r,\xi) = \frac{4\pi e}{mc^2}\frac{1}{r}\int_0^r \mathrm{d}r' r' P_{\perp}(r',\xi), \tag{13}$$

where $P_{\perp}(r,\xi) \equiv n(r,\xi)T_{\perp}(r,\xi)$. Here the effective electron temperature is directly proportional to the laser radiation intensity I:

$$T_{\perp}(r,\xi) = \frac{mv_E^2}{2} = 1.85\times10^{-16}\lambda^2$$

$$\times[I(r,\xi)\eta(\xi) + I_0(r)\eta(-\xi)]. \tag{14}$$

In expression (14), the temperature is expressed in keV; $\eta(\xi)$ is the Heaviside function of a single jump, and for definiteness it is assumed that the maximum laser pulse intensity $I_0(r)$ corresponds to $\xi = 0$; and λ is the wavelength of laser light in microns, and its intensity is measured in W cm^{-2}.

In the case of circular polarisation, the probability of tunnelling ionisation per unit time of atoms and ions, which is also determined by the laser pulse intensity [22], has the form

$$\omega_i = \omega_{\mathrm{a}}\frac{E_0}{8\pi Z E_{\mathrm{a}}}\left(\frac{4\mathrm{e}' Z^3 E_{\mathrm{a}}}{(n^*)^4 E_0}\right)^{2n^*}\exp\left(-\frac{2Z^3E_{\mathrm{a}}}{3(n^*)^3E_0}\right), \tag{15}$$

where $n^* = Z\sqrt{U_{\mathrm{H}}/U}$ is the effective principal quantum number of the electron; Z is the charge number of the atomic (ionic) core; $\omega_{\mathrm{a}} = me^4/\hbar^3$ is the atomic frequency; U_{H} is the ionisation energy of the hydrogen atom; $E_{\mathrm{a}} = m^2e^5/\hbar^4$ is the characteristic value of the nuclear field in the region of the electron orbit; and e$'$ is the base of the natural logarithm. Table 1 gives estimates of the probability of tunnelling from certain atomic levels (corresponding to the index at U) for atoms of widely used gases in the intensity range from 5×10^{16} to 5×10^{17} W cm^{-2} in the case of radiation from a Ti:sapphire laser ($\lambda = 0.8$ μm). Dashes in Table 1 denote the ionisation

Table 1. Estimation of the dimensionless ionisation probability $\omega/\omega_{\mathrm{a}}$ by formula (15).

Atom	Ionisation energy/eV	Intensity I_0/W cm^{-2}				
		5×10^{16}	7.5×10^{16}	1.0×10^{17}	2.5×10^{17}	5.0×10^{17}
He	$U_1 = 24.587$	–	–	–	–	–
	$U_2 = 54.418$	0.325	0.844	–	–	–
N	$U_5 = 97.8902$	0.13798	–	–	–	–
	$U_6 = 552.08$	0	0	0	0	0
Ne	$U_4 = 97.12$	0.0106	0.112	0.434	–	–
	$U_5 = 126.21$	7.15×10^{-5}	0.00268	0.0221	–	–
	$U_6 = 157.93$	7.51×10^{-8}	1.37×10^{-5}	0.000289	0.301	–
	$U_7 = 207.276$	5.72×10^{-14}	1.84×10^{-10}	2.14×10^{-8}	0.00128	0.24
	$U_8 = 239.099$	0	0	4.52×10^{-11}	4.46×10^{-5}	0.03
Ar	$U_7 = 124.323$	0.0222	0.580	–	–	–
	$U_8 = 143.460$	0.00100	0.0636	0.692	–	–

saturation from the indicated level ($\omega/\omega_a > 1$), and zero means a dimensionless probability less than 10^{-11}. Let us find the electron density after the passage of the laser pulse by using the values of the obtained ionisation probabilities. Usually, the tunnelling of several electrons is taken into account as a sequential (cascade) process, since at high intensities it prevails over simultaneous tunnelling [22]. Accordingly, the following system of ordinary differential equations arises to describe the appearance of free electrons during tunnelling ionisation:

$$\frac{\partial n_1(\xi)}{\partial \xi} = [n_1(\xi) - n_a]\frac{\omega_1(\xi)}{c},$$

$$\frac{\partial n_2(\xi)}{\partial t} = [n_2(\xi) - n_1(\xi)]\frac{\omega_2(\xi)}{c}, \qquad (16)$$

$$\ldots,$$

where n_i and ω_i ($i = 1, 2, \ldots$) are the partial densities of free electrons and the corresponding ionisation probabilities associated with the ionisation of an atom to charge $+i$; and n_a is the density of atoms.

For the above considered examples from Table 1, the average ionisation rate n/n_a of atoms calculated for the same range of intensities, which is achieved after the passage of the maximum of a laser pulse with a duration $\tau = 30$ and 60 fs, is given by the values presented in Table 2. Note that, according to this table, doubling the duration of the laser pulse almost does not increase the electron density of the plasma at a given intensity, i.e., to achieve a certain density, it is not rational to increase the pulse duration.

By numerically solving the system of equations (16), it is possible to obtain the longitudinal profile of the electron density shown in Fig. 1 for the Ne gas. In this case, we consider a laser pulse with an intensity $I_0 = 5\times10^{17}$ W cm^{-2}, duration $\tau =$ 30 fs, and an intensity distribution of the form $I(r,\xi) = I_0\cos^2[\pi\xi/(c\tau)]\exp(-r^2/r_0^2)$. It is seen that at virtually any distance from the beam axis, the electron density is saturated immediately behind the maximum of the ionising pulse. According to Table 2, a Ti:sapphire laser pulse with an intensity of 5×10^{17} W cm^{-2} and a duration of 30 fs, propagating in an Ne gas under normal conditions, produces an electron density $n = 0.09n_c$, which satisfies the condition of applicability of the developed theory $n \ll n_c$, where n_c is the critical density.

From the distribution of the electron density using expression (14) for the transverse temperature, the transverse electron pressure $P_\perp$ is found, which is the source during the magnetic field generation [see (12)]. This is illustrated in Fig. 2, which shows the longitudinal distribution of the dimensionless transverse pressure $p = P_\perp(r,\xi)/(\frac{1}{2}mn_ac^2a_0^2)$, where $a_0 = [2I_0/(mn_cc^3)]^{1/2}$ is the standard dimensionless laser field amplitude.

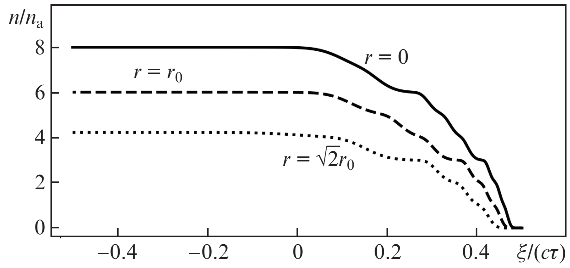

Figure 1. Dependences of the density of free electrons on ξ at various distances from the z axis for Ne atoms.

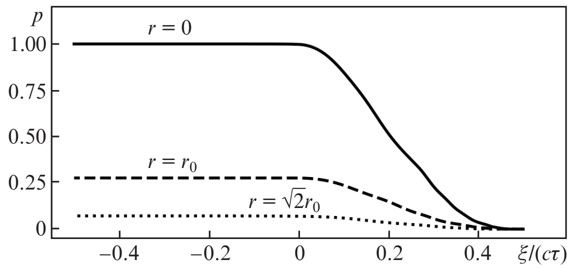

Figure 2. Dimensionless electron pressure p as a function of ξ in an ionising gas Ne at different distances from the z axis.

The electron pressure distribution has a rather smooth form with saturation near the maximum of the laser pulse intensity. This allows us to propose a simple analytical approximation to be used in expression (13) for the magnetic field:

$$P_\perp(r,\xi) = P_0\exp\left(-\frac{r^2}{r_0^2}\right)f\left(\frac{2\xi}{c\tau}\right),$$

where $f(\eta) \simeq 1$ for $\eta \leqslant 0$ and $f(\eta) \to 0$ for $\eta \gg 1$, and

$$P_0 \simeq \frac{\omega_p^2}{\omega^2}\frac{I_0}{2c}.$$

Table 2. Estimation of the average degree of ionisation n/n_a for some atoms.

Atom	τ/fs	Intensity I_0/W cm^{-2}				
		5×10^{16}	7.5×10^{16}	1.0×10^{17}	2.5×10^{17}	5.0×10^{17}
He	30	2.00	2.00	2.00	2.00	2.00
	60	2.00	2.00	2.00	2.00	2.00
N	30	5.00	5.00	5.00	5.00	5.00
	60	5.00	5.00	5.00	5.00	5.00
Ne	30	4.08	4.96	5.29	6.79	8.00
	60	4.16	5.02	5.50	6.96	8.00
Ar	30	7.70	8.00	8.00	8.00	8.00
	60	7.91	8.00	8.00	8.00	8.00

In this relation, the electron plasma frequency is determined by the electron density attained at the peak intensity of the laser pulse. In this case, we consider rather short laser pulses, the duration of which is less than the length of the caustic.

Then, according to (13), we have the following distribution of the generated magnetic field:

$$B(r,\xi) = \frac{2\pi e}{mc^2} P_0 \frac{r_0^2}{r}\left[1 - \exp\left(-\frac{r^2}{r_0^2}\right)\right] f(\xi). \quad (17)$$

Figure 3 illustrates the spatial (r, z) distribution of the generated magnetic field behind the leading edge of the laser pulse in comparison with the electron pressure distribution. Thus, the maximum value of the magnetic field (17) generated by a laser pulse during tunnelling ionisation of the medium is as follows:

$$B_m \simeq \pi \frac{ner_0}{n_c mc^3} I_0, \quad (18)$$

where $n_c \gg n$. Taking into account the condition of applicability of our theory, $\omega_p < c/r_0$, we can estimate the maximum possible value of the magnetic field:

$$B_{max} \simeq \frac{\omega_p}{4e} \frac{I_0}{n_c c^2}. \quad (19)$$

In accordance with formulae (18) and (19), it is possible to introduce the conversion factors K_m and K_{max} of the laser light energy into the magnetic field energy,

$$K_m = \frac{B_m^2/8\pi}{E_0^2/4\pi} \simeq \frac{\pi^2}{8} \frac{n^2}{n_c^2} \frac{r_0^2}{\lambda^2} a_0^2,$$
$$K_{max} = \frac{B_{max}^2/8\pi}{E_0^2/4\pi} \simeq \frac{a_0^2}{32} \frac{n}{n_c} \quad (20)$$

through the standard dimensionless laser field amplitude a_0, with $n \ll n_c$.

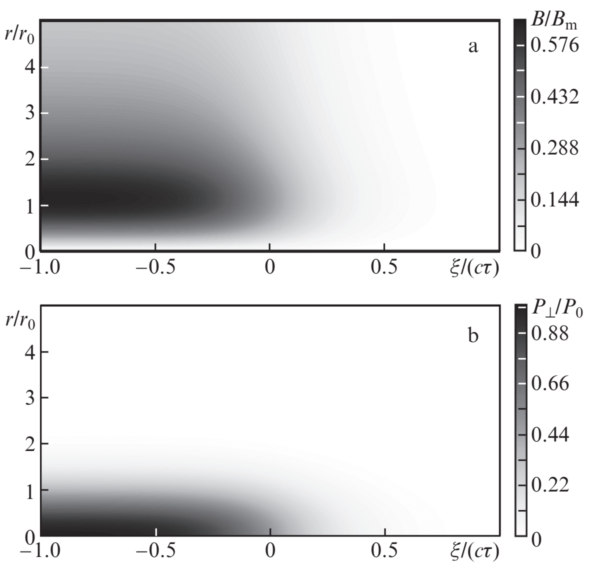

Figure 3. Spatial distributions of (a) the magnetic field and (b) the electron pressure as functions of r and ξ.

For radiation from a Ti:sapphire laser with an intensity of $I_0 = 5 \times 10^{17}$ W cm^{-2}, when the gas is ionised to a density of $n = 0.09n_c$, the magnetic field, according to (19), can reach $\sim$1 MG. It should also be noted that the transformation ratio is quite large (20 and can reach about 1%.

4. Conclusions

We have investigated the mechanism of generation of a spontaneous magnetic field due to the anisotropy of the electron distribution in a low-density plasma, which is formed as a result of tunnelling ionisation of a gas medium by a propagating circularly polarised laser pulse of subrelativistic intensity. Without pretending to fully describe the appearance of a magnetic field along the entire path of laser pulse propagation, we proposed a theoretical model of its excitation in the caustic region, where the generated magnetic field is maximum and it occupies the volume of a cylinder with a length of the order of the Rayleigh length and a diameter approximately equal to the waist diameter. At the same time, this model is also applicable in the case of self-focusing of laser light. Accordingly, a magnetic field arises in an extended plasma channel behind the leading edge of the laser pulse. In the considered case of nonrelativistic laser intensities, the magnetic field has only an azimuthal component caused by an ionisation thermoelectric source, similar to a dc current. The magnetic field is maximal at the channel edge and can reach $\sim$1 MG. A sufficiently high conversion rate, up to 1%, indicates a high efficiency of the considered mechanism of the laser anisotropic thermal EMF.

We have not touched here upon the issues of the possible role of electron collisions in the generation of the magnetic field and of the evolution of the magnetic field at long times, when relaxation dissipative processes take place in the produced plasma. Here we have considered the collisionless case, which has a wide range of applicability at densities lower than the critical one and at the studied intensity range corresponding to the tunnelling regime of ionisation. However, in order to formalise the corresponding condition for the applicability of our theory, we can explicitly state the conditions of the collisionless approximation used (in terms of the electron collision frequency ν_e and the electron mean free path λ_e), which have the form $\nu_e \ll \tau^{-1}$, $\lambda_e \gg R$, or, which is the same, in a practically convenient form: $3 \times 10^{-26}(\Lambda/a_0^3)n\tau \ll 1$, $10^{-28} \times (\Lambda/a_0^4)nR \ll 1$, respectively, where Λ is the Coulomb logarithm; and n is measured in cm^{-3}, τ – in ps, and R – in microns. In this case, the lifetime of a large-amplitude magnetic field behind the pulse, determined by the characteristic time of magnetic diffusion, is $t_B \approx R^2\omega_p^2/(c^2\nu_e) \gg \tau(R\omega_p/c)^2$. For Ti:sapphire laser radiation of femto- and picosecond duration with an intensity of $I_0 = 5 \times 10^{17}$ W cm^{-2} and $R = 10$ μm, this time is $\sim$100 μs.

In our theory, we also neglected the ponderomotive displacement of electrons from the beam axis, which is possible in principle. However, for the considered case of nonrelativistic intensity ($a_0 < 1$) and for a not quite low density, which are only of practical interest, the effect of ponderomotive displacement is insignificant. Indeed, the ponderomotive displacement of electrons is described by the force $F_{pm} \approx mc^2 a_0^2/(2R)$, which leads to a violation of the quasi-neutrality of the plasma, due to which the emerging Coulomb field self-consistently prevents such a displacement. Estimating the corresponding equilibrium-restoring Coulomb force $F_C = eE_C \approx 2\pi e n_e R$, from the condition $F_{pm} \ll F_C$ we come to the following condition of applicability of our model: $n/n_c \gg a_0^2 c^2/(R^2\omega^2)$, which is easily fulfilled by virtue of $a_0^2 \ll 1$ and $c^2/(R^2\omega^2) = 0.025(\lambda/R)^2 \approx 1$.

This investigation can serve as a starting point for continuing a detailed study of the proposed mechanism of magnetic field generation. In further studies, it is of interest to go over to higher laser intensities, $a > 1$, and higher densities, $n \approx n_c$. A study of the generation of a magnetic field by a linearly polarised laser pulse, similar to that carried out here, is also relevant. Finally, we note one more problem that develops this study. Here we have considered the generation of a magnetic field, neglecting the change in the laser pulse during its propagation in the medium. We examined the excitation of a quasi-static magnetic field behind the laser pulse, while taking into account the dynamics of the pulse change (its deformation, depletion, focusing/defocusing, etc.), it is possible to generate a low-frequency electromagnetic field in the form of output radiation (compare with [9]), most likely in the terahertz range. This is due to the nonstationarity of the source of the anisotropic laser-initiated thermal EMF, which has not yet been taken into account. Undoubtedly, the generation of terahertz radiation due to this mechanism of tunnelling ionisation of a medium by a laser field is of interest for future research.

Acknowledgements. This work was partially supported by the Russian Science Foundation (Grant No. 17-12-01283).

References

1. Hesse M., Winske D. *J. Geophys. Res.: Space Phys.*, **99** (A6) 11177 (1994).
2. Aliev Yu.M., Bychenkov V.Yu., Frolov A.A. *Fiz. Plazmy*, **8**, 1132 (1982).
3. Bychenkov V.Yu., Silin V.P., Tikhonchuk V.T. *Fiz. Plazmy*, **15**, 706 (1989); Bychenkov V.Yu., Silin V.P., Tikhonchuk V.T. *Phys. Lett. A*, **138**, 127 (1989).
4. Korobkin V.V., Serov R.V. *JETP Lett.*, **4**, 70 (1966) [*Pis'ma Zh. Eksp Teor. Fiz.*, **4**, 103 (1966)].
5. Stamper J.A., Papadopoulos K., Sudan R.N., Dean S.O., McLean E.A., Dawson J.M. *Phys. Rev. Lett.*, **26**, 1012 (1971).
6. Aliev Yu.M., Bychenkov V.Yu., Frolov A.A. *Fiz. Plazmy*, **8**, 125 (1982).
7. Dubroca B., Tchong M., Charrier P., Tikhonchuk V.T., Morreeuw J.-P. *Phys. Plasmas*, **8**, 3830 (2004).
8. Bychenkov V.Yu., Silin V.P., Tikhonchuk V.T. *Sov. Phys. J. Exp. Theor. Phys.*, **73**, 241 (1991) [*Zh. Eksp. Teor. Fiz.*, **100**, 440 (1991)].
9. Bychenkov V.Yu., Romanov D.V., Rozmus W., Capjack C.E., Fedosejevs R. *Phys. Plasmas*, **13**, 013101 (2006).
10. Weibel E.S. *Phys. Rev. Lett.*, **2**, 83 (1959).
11. Silin V.P., Uryupin S.A. *J. Exp. Theor. Phys.*, **84**, 59 (1997) [*Zh. Eksp. Teor. Fiz.*, **111**, 107 (1997)].
12. Thomas A.G.R., Kingham R.J., Ridgers C.P. *New J. Phys.*, **11**, 033001 (2009).
13. Bychenkov V.Yu., Tikhonchuk V.T. *Laser Phys.*, **2**, 525 (1992).
14. Leemans W.P., Clayton C.E., Mori W., Marsh K.A., Dyson A., Joshi C., Wallace J.M. *Phys. Rev. A*, **46**, 1091 (1992).
15. Arefyev V.I., Silin V.P., Uryupin S.A. *Phys. Lett. A*, **255**, 307 (1999).
16. Kraynov V.P. *J. Exp. Theor. Phys.*, **96**, 430 (2003) [*Zh. Eksp. Teor. Fiz.*, **123**, 487 (2003)].
17. Bychenkov V.Yu., Romanov A.Yu., Silin V.P., Tikhonchuk V.T. *Fiz. Plazmy*, **18**, 452 (1992).
18. Zhang C., Huang C-K., Marsh K.A., Clayton C.E., Mori W.B., Joshi C. *Sci. Adv.*, **5**, eaax4545 (2019).
19. Keldysh L.V. *Sov. Phys. J. Exp. Theor. Phys.*, **20**, 1307 (1965) [*Zh. Eksp. Teor. Fiz.*, **47**, 1945 (1965)].
20. Akhmanov S.A., Sukhorukov A.P., Khokhlov R.V. *Sov. Phys. J. Exp. Theor. Phys.*, **23**, 1025 (1966) [*Zh. Eksp. Teor. Fiz.*, **50**, 1537 (1966)].
21. Kovalev V.F., Bychenkov V.Yu. *Phys. Rev. E*, **99**, 043201 (2019).
22. Ammosov MV, Delone N.B., Krainov V.P. *Sov. Phys. J. Exp. Theor. Phys.*, **64**, 1191 (1986) [*Zh. Eksp. Teor. Fiz.*, **91**, 2008 (1986)].