Quantum Electronics **50** (9) 801–815 (2020)

INVITED PAPER

https://doi.org/10.1070/QEL17348

Unipolar light: existence, generation, propagation, and impact on microobjects

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Contents

Abstract. **A review is presented of recent works on optical unipolar pulses, whose electric area (integral of the electric field strength over time) is nonzero, which determines the predominant direction of the electric field strength. It is shown that the existence of unipolar pulses does not contradict Maxwell's equations, and that unipolar pulses can propagate in waveguides. It is emphasised that, along with the spectral, energy, and polarisation parameters, the electric area of short light pulses is also an important characteristic. The unidirectional action of these pulses on microobjects indicates that it is promising to develop methods for generating radiation with such properties. We disciuss methods for the generation, propagation, and interaction of unipolar light with classical and quantum systems, as well as methods for recording the electric area of light pulses.**

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Received 10 April 2020; revision received 17 May 2020
Kvantovaya Elektronika **50** (9) 801–815 (2020)
Translated by V.L. Derbov

Keywords: *unipolar light, extremely short pulses, unipolar pulses, pulse electric area.*

1. Introduction

Immediately after the appearance of lasers, active research began on ways to reduce the duration of laser pulses [1]. The extreme importance of this area is evidenced, in particular, by the Nobel Prize in Physics 2018 [2]. To date, it has been possible to obtain pulses with a duration of the order of one oscillation period of a light wave in the femto- and attosecond ranges [3–7]. Extremely short pulses (ESPs), containing a few cycles of field oscillations, up to a single cycle, play an extremely important role in modern physics and other fields that use its achievements. The duration of pulses in the attosecond range is comparable to the characteristic periods of electron motion in atoms and molecules; therefore, attosecond pulses are a unique tool for monitoring and controlling the dynamics of electron wave packets in atoms and solids [8]. On the way to further shortening the duration, new and unusual problems for optics appear. For example, an unexpected question arose about what might be the features of the 'structure' of electric and magnetic fields inside such pulses, which is somewhat different from that of the usual multicycle pulses described in the framework of the standard concepts of carrier frequency, phase and slowly varying amplitude. We have to abandon these concepts here, and in the theoretical description, it is necessary to turn directly to the original Maxwell's equations. The limiting possibility from the point of view of reducing the pulse duration is the generation of subcycle unipolar radiation pulses with a significant single burst of the electric field strength, which, unlike the case of bipolar pulses, does not change direction during the entire pulse (a more rigorous definition is given below). Among potential consumers of ultrashort and extremely short laser pulses, the European Extreme Light Infrastructure (ELI) programme should be mentioned, the purpose of which is to obtain super-intense light fields with a peak intensity sufficient for the manifestation of essential quantum-electrodynamic properties of the electron--positron vacuum [9]. There is reason to believe that the efficiency of action on various objects is greater precisely for quasi-unipolar radiation pulses (see below).

In this review, the terminology is given, the important concept of the light pulse electric area is introduced, the objections that the authors had to face when discussing the problem of unipolar light are considered, and well-known qualitative, semiquantitative, and more rigorous mathematical substantiations of implementing possible schemes for generating unipolar light and its properties are presented. The results of so far few papers are presented, in which situations with the appearance of pulses with the property of unipolarity were directly analysed. Considering that most of the works in this area are of a theoretical nature, the presentation is structured in such a way that the essence of the problems involved is understandable not only to theoreticians, but primarily to experimental physicists who may be interested in the problem of obtaining unipolar light.

2. Electric area of a light pulse and the degree of unipolarity

A light pulse, e.g., generated by a mode-locked laser, is a limited or localised 'bunch' of electromagnetic field travelling in space, containing from a few to hundreds and thousands (depending on the duration) cycles of the electric (and magnetic) field strength variation. According to the usual concepts, in the case of linearly polarised radiation, the vector of the electric (and magnetic) field strength changes the sign at a distance of half the wavelength. Such a pulse, passing a point $\boldsymbol{R} = (x, y, z)$ in space, produces an alternating local field in it during the pulse duration. For each point $\boldsymbol{R}$ in space, the following quantity can be introduced, referred to as the electric field area:

$$\boldsymbol{S}_E(x,y,z) = \int_{-\infty}^{+\infty} \boldsymbol{E}(x,y,z,t)\,\mathrm{d}t \qquad (1)$$

(integral of the electric field strength vector $\boldsymbol{E}$ over time t [10, 11], which in accordance with the above definition is also a vector quantity). Note that for a conventional multicycle optical pulse, each component of such a vector and its length must be equal to zero. Thus, 'ordinary' light pulses, according to the introduced terminology, are bipolar. In unipolar pulses (UPs), which are the subject of this review, as a result of the integration of the electric field strength vector, there will be nonzero components of the electric area. For a quantitative characteristic, a quantity is introduced, which is called the degree of unipolarity [12–15] and shows the fraction of unipolar light in a pulse:

$$\xi(x,y,z) = \frac{\left|\int_{-\infty}^{+\infty} \boldsymbol{E}(x,y,z,t)\,\mathrm{d}t\right|}{\int_{-\infty}^{+\infty} |\boldsymbol{E}(x,y,z,t)|\,\mathrm{d}t}. \qquad (2)$$

A fully unipolar pulse corresponds to $\xi = 1$, and a usual bipolar pulse to $\xi = 0$. Both the electric area and the degree of unipolarity are quantities that change in space. Below it will be shown that the electric area obeys a certain conservation law (the so-called electric area conservation rule). The degree of unipolarity is not subject to any general conservation rules.

Figure 1 schematically shows the temporal behavior of the field strength at a certain point in space, when an extremely short light pulse passes through it, in situations of interest to us. The case in Fig. 1a corresponds to a bipolar single-cycle pulse. It has only one well-defined cycle of field strength oscillations. The electric area of such a pulse and the degree of unipolarity ξ are equal to zero. Figures 1b and 1c show a so-called subcycle, or halfcycle pulse. Its name is due to the pattern of the field change, in which there is a half cycle of oscillations, or, in other words, one half-wave with a large amplitude. The tail of the pulse, or the pedestal of the opposite sign, has a smaller amplitude. In this case, the integral of the field strength can also be zero, and if the area of the negative part is less than the area of the positive one, or vice versa, it is nonzero. Finally, Fig. 1d shows a unipolar pulse. Here the integral is not equal to zero and the degree of unipolarity is $\xi = 1$. Sometimes in the literature, a unipolar pulse is called a video pulse. This term is borrowed from radio engineering and seems to us not entirely successful for use in optics, since has no appropriate physical meaning.

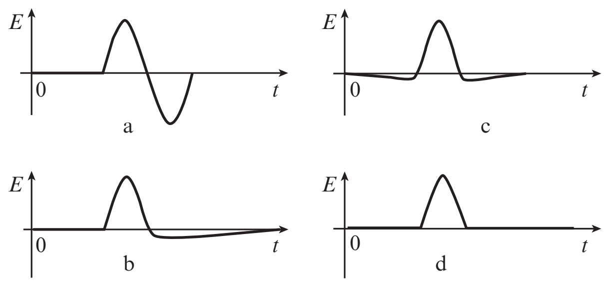


Figure 1. (a) Bipolar single cycle, (b, c) subcycle and (d) unipolar pulses.

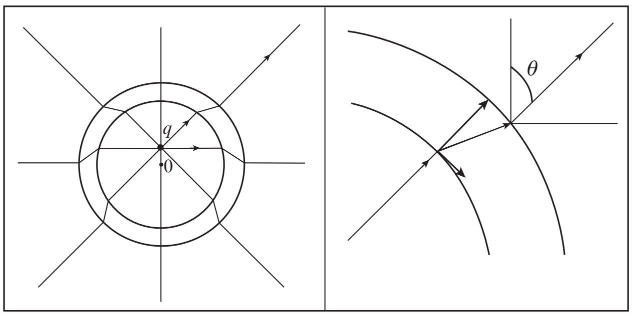
3. Pro and con arguments about the existence of unipolar light

It is the possibility of the existence of pulses with a nonzero electric area in the optical range that is often questioned. A number of arguments are presented here. One of them reduces to the statement that the existence of a component at a zero frequency in the Fourier expansion of the pulse indicates the presence of an electrostatic field, which a travelling wave cannot have. The argument is strange, since there is no such field in a travelling space-limited wave outside the pulse. The magnitude of the field is measured by its effect on probe charges. An example from mechanics suggests itself here. If a short-term force acts on the body, which does not change direction during the time of exposure, then in the Fourier expansion of such a force there will also be a component at a zero frequency. However, this fact does not give rise to the conclusion about the existence of a constant force that would act on the body. It is also impossible to agree with the objection that the presence of a field component at a zero frequency contradicts the wave properties of light. Maxwell's equations by no means prohibit the existence of such pulses. This circumstance follows already from the fact that the three-dimensional wave equation in vacuum, which follows directly from Maxwell's equations, admits, according to D'Alembert, the existence of a plane-wave solution in the form of pulses of arbitrary shape $\boldsymbol{E}(z,t) = \boldsymbol{E}(z-ct)$, which move in vacuum with the speed of light c along the z axis. Since $\boldsymbol{E}(z, t)$ is an arbitrary function, it corresponds to unipolar pulses, for which the sign of the electric field amplitude does not change during propagation.

The argument that a nonzero pulse area corresponds to a nonzero field amplitude with zero frequency, which, accordingly, 'cannot propagate' is also unacceptable. This is evidenced by the just-mentioned d'Alembert solution in the form of a traveling pulse, in which a zero-frequency component may well be present. The point is that the (phase) velocity is determined not simply by the radiation frequency, but by its ratio to the wave number. The latter in a vacuum is simply proportional to the frequency, which leads to a finite propagation velocity. Thus, unipolar pulses can propagate in a vacuum and even in a waveguide, which will be demonstrated below. The next question is whether there are sources of such waves.

4. Qualitative picture of the emergence of a unipolar light pulse

Faced with objections to the possibility of obtaining pulses of unipolar light, the authors can give an elementary example from the course of general physics, explaining the emergence of radiation when a charge moves. Indeed, free moving charges, which acquire a short-term acceleration, begin to radiate. In textbooks [16], this is usually illustrated as shown in Fig. 2.



a b

Figure 2. Schematic illustration of the unipolar pulse emergence: (a) a break in the force lines of the electric field of a positive point charge arising during its acceleration, and (b) the components of the electric strength vector in the break zone of the lines of force, indicating the appearance of a unipolar radiation pulse; θ is the angle between the directions of charge movement and propagation of radiation.

The charge q is at the origin of coordinates 0 until the moment of time $t = 0$. Then an external force acts on it during the time τ; at the end of the exposure time, the charge continues to move uniformly and rectilinearly with a velocity v. We consider the nonrelativistic case $v \ll c$. Let us imagine a picture of lines of force in space some time T after the termination of the force action. Then, in space far from the charge, three regions can be distinguished. Outside a sphere with a larger radius (of the order of cT), the field strength pattern will be the same as before the charge began to move. A field of a moving charge is formed inside a sphere of a smaller radius. In our case ($v \ll c$) the charge does not have time to go far from the point of origin of motion, and so the pattern of lines of force will coincide with that for the charge at rest. All changes in the direction of the lines of force will occur between these two spheres expanding at the speed of light. With such a break in the lines of force inside the sphere, one can distinguish a component orthogonal to the direction of the lines of force of the resting and moving charges. This component is responsible for producing a unipolar wave emitted by a charge during its acceleration. This is shown schematically in Fig. 2a. Details of the derivation are given in Ref. [16]. For the field strength $\boldsymbol{E}$ in a unipolar wave at a distance R from the origin of coordinates in the case of 'pulsed' acceleration $a = v/\tau$ during time τ, the following expression is obtained:

$$E(R,t) = \frac{qa}{c^2 R}\sin\theta\,. \tag{3}$$

Note that the duration of such a pulse is equal to τ, and it determines the spectral range and width of the radiation spectrum. The dependence on the angle θ between the direction of motion and the direction of radiation propagation is taken into account by the factor $\sin\theta$. Thus, the charge becomes a source of a unipolar spherical wave with an amplitude depending on the observation angle with respect to the direction of the charge movement. Strictly speaking, for the charge to produce only one unipolar wave and the electric area to be different from zero, the charge must continue to move. If the charge cannot leave the limited area, then it will have to stop, which means that a break in the lines of force will reappear, but in a different direction; the charge will again generate a unipolar wave, in which the vector of the electric field strength will have the opposite direction. As a result of acceleration, uniform movement and deceleration of the charge, radiation of two unipolar pulses with opposite signs will appear. Moreover, it is easy to verify that the electric area of such radiation at a considerable distance from the region of charge motion is zero, even if acceleration and deceleration occur during different times.

The conclusion about the existence of radiation from a charge moving with acceleration is confirmed by the well-known rigorous solution of the problem, based on Maxwell's equations [17]. It turns out that the low-frequency part of the spectrum of the vector potential is inversely proportional to the frequency, and this indicates the presence of a constant component in the radiation pulse accompanying the accelerated motion of the charge. The corresponding calculations are given in [18].

Speaking about the first significant works in the field of unipolar pulses, we should mention the work of Bessonov [19]. It draws attention to the well-known fact that a charge moving with acceleration emits a unipolar pulse. In Ref. [19], as far as we know, an expression of form (1) was considered for the first time. However, the author did not introduce the concept of the electric area; instead, for unipolar pulses he introduced the term 'strange electromagnetic waves'. The main processes in which acceleration of free charges takes place and such waves arise were listed: bremsstrahlung, radiation accompanying beta decay of nuclei, radiation of electrons accompanying Compton scattering of gamma quanta, radiation of charged particles in magnetic fields, radiation of cosmic rays in the magnetic field of the Earth, and radiation of positrons reflected from the surface of crystals.

In the subsequent papers [20, 21], Bessonov notes that, strictly speaking, unipolar pulses can arise only due to the infinite motion of particles. In other cases, conditionally strange waves may occur, when pulses with different polarities are separated in time. Then he schematically analyses examples of obtaining such conditionally strange waves and using interference schemes to manipulate such pulses.

In our opinion, the term ‘strange waves’, used in contrast to the usual ones, in which the sign of the field changes, in relation to unipolar waves is not quite good. The corresponding term ‘strangeness parameter’ as applied to the time integral of the electric field strength is not quite good as well. It does not reveal the physical meaning of the quantity. The terms ‘unipolar’, ‘bipolar’, ‘electric area’, and ‘zero electric area’ in combination with the word ‘pulse’ are more acceptable for opticians and laser physicists who are starting to deal with this issue. The term ‘degree of unipolarity’ defined by Eqn (2) is also appropriate and has a clear physical meaning.

In preprint [22] entitled ‘A bounded source cannot emit a unipolar electromagnetic wave’ it is noted that the problem was identified one and a half hundred years ago by Stokes, who also showed that a bounded sound source in the three-dimensional case cannot be unipolar. At the same time, a plane acoustic wave can have any shape and be unipolar. The title of the work already contains a statement, but one of the arguments is formulated in it not quite accurately. The presence of a constant component in the spectrum of a unipolar pulse is interpreted as the existence of an electrostatic component. The fact that this is not the case for a unipolar wave was already noted at the beginning of the review.

The assertion contained in the title of Ref. [22] is not quite correct: it denies the possibility of obtaining a wave that contains, e.g., two unipolar pulses separatedt in time, and the possibility of obtaining a nonzero electric area. Let us show that this is not the case with the following example. Let the negative charge first rest at point 1, then accelerate on a way to point 2, and move uniformly from point 2 to point 3. At point 3 it acquires negative acceleration, decelerates and stops at point 4. The distance between points 2 and 3 can reach several meters (of the order of the laboratory room size), and the path segments where acceleration and deceleration occur can be extremely small. In this case, the source is limited in space. The situation is illustrated in Fig. 3.

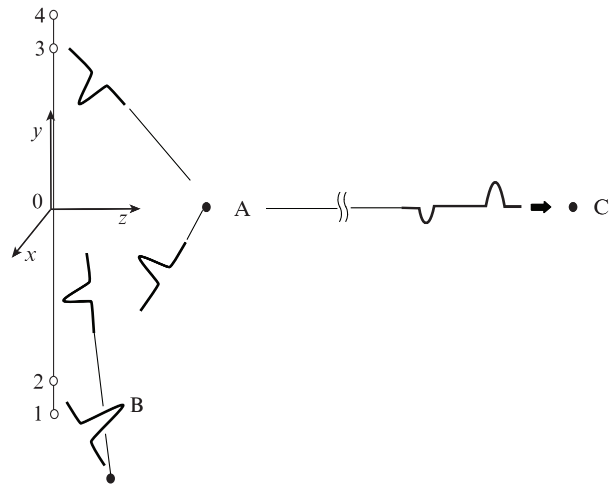


Figure 3. Schematic representation of the finite motion of a charge. The origin is located in the middle of segment 1–4 at point 0. Points A and C are located on the z axis: point A near the segment, point C at a considerable distance from it. Point B is located near the line segment in the area of negative values of the coordinate y.

If the accelerations in path segments 1–2 and 3–4 are the same and differ only in sign, then two unipolar pulses of the same shape will arrive at point A with some time delay. Their z-components have the same sign, and y-components have different signs. If we calculate the electric area of the pulse according to formula (1), its z-component will be nonzero, and the y-component will be zero. At point B, the electric area is also nonzero, since the pulses have different amplitudes due to different distances from the source and different directions of observation. At a significant distance, at point C, when the pulses can be assumed to propagate in one direction, they have the same shape, but opposite orientations of the electric field strength vector, and their area is zero. That is, on a laboratory scale, it seems quite realistic to create a limited source and obtain pulses with a nonzero area.

Looking ahead, it should be noted (Section 12 is devoted to these issues in the review) that in many problems of the radiation–matter interaction it is not the very existence of a nonzero electric area measured over an infinite time interval that is important, but the presence of single unipolar bursts of short-duration and large amplitude in the radiation. The condition of finite motion does not impose restrictions on the possibility of obtaining such pulses. The effect of unipolar pulses on charged particles, both free and bound, gives a different result than the effect of a bipolar pulse containing a few oscillation cycles. The action of a multi-cycle pulse is efficient only when the carrier frequency is in resonance with the transition in the quantum system. A single unipolar pulse can have a non-selective effect. Trains of two or more unipolar pulses can act like resonant radiation.

5. Possibility of generating unipolar light in the one-dimensional geometry of the experiment

The fundamental possibility of the existence of unipolar pulses in the above understanding (both in the rigorous and in the extended physical sense) is beyond doubt. However, there is a problem of their practical implementation in the optical and adjacent ranges, where the usual radiation sources are oscillating dipoles and varying alternating currents. The wave equation in the one-dimensional case has the following form [23–25]:

$$\frac{\partial^2 E(z,t)}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 E(z,t)}{\partial t^2} = \frac{4\pi}{c^2}\left[\frac{\partial^2 P(z,t)}{\partial t^2} + \frac{\partial j(z,t)}{\partial t}\right]. \quad (4)$$

We wrote the wave equation (4) in scalar form, since below we will consider linearly polarised radiation. The sources of the field in (4) are the second derivative of the medium polarisation P and the first derivative of the current density j.

The differences between the one-dimensional problem and the three-dimensional one were considered in Refs [26, 27]. Consider first the one-dimensional case. Note that we are interested in extremely short pulses, and accordingly, the movement of charges causing a change in the field will be short-time; also, due to the one-dimensionality, we assume the radiation to have linear polarisation, i.e., all charges move along one direction. Figure 4 illustrates the considered situation, when charges move in a thin flat layer of the medium M located in the x, y plane, and radiation occurs mainly along the z axis.

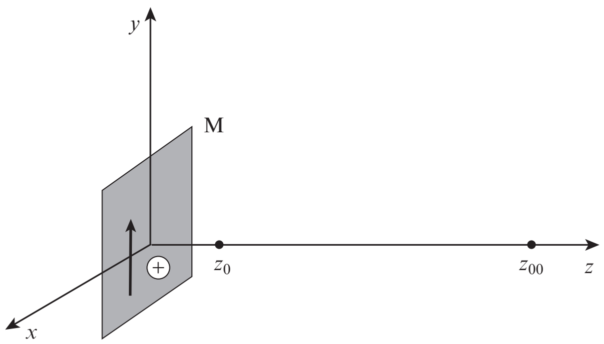


Figure 4. Illustration of the applicability of one-dimensional and three-dimensional approximations. In the x, y plane there is a layer of the medium M, in which the charges move parallel to the y axis.

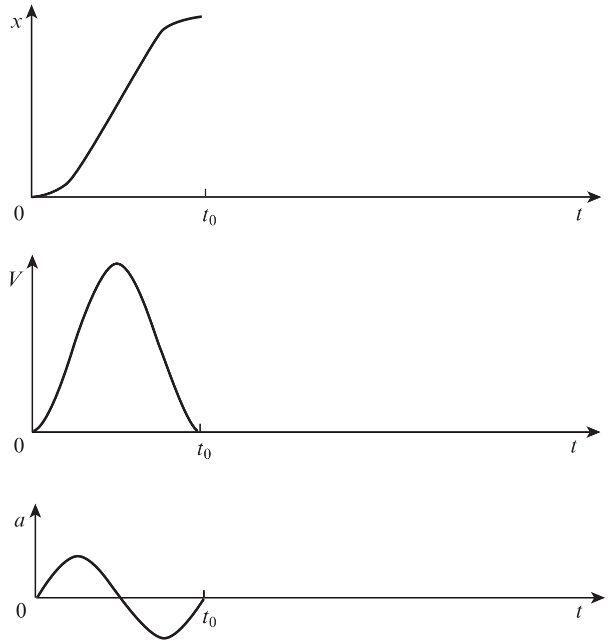


Figure 5. Dependences of displacement x, velocity V and acceleration a of the charge on time t.

In this case, at the point z_0 near the layer M one-dimensional wave equation (4) can be applied, which cannot be done at point z_{00}, which is located at a considerable distance from the medium. The expression for the field produced by dipoles and currents is as follows [26, 27]:

$$E(z,t) = -\frac{2\pi}{c}\int_{z_1}^{z_2} dz'\left[\frac{\partial}{\partial t}P\left(z', t - \frac{|z-z'|}{c}\right) + j\left(z', t - \frac{|z-z'|}{c}\right)\right]. \quad (5)$$

Here the integration is carried out over the layer between the points with coordinates z_1 and z_2. Let us pay attention to the fact that the field turns out to be proportional to the first derivative of polarisation and to the current density.

For clear illustration of the difference between a one-dimensional situation and a three-dimensional one, which, in principle, makes it possible to obtain a unipolar pulse, let us consider the time dependence of the charge displacement and its derivatives in a case of interest for the subsequent analysis. Figure 5 shows the situation when at the initial moment of time the charge is at rest. Then, if this charge is part of the dipole, the dipole moment is zero, and if the charge is a free carrier in the conductor, the current is zero.

Then, under the action of an external field, occurring in the interval from 0 to t_0, the charge begins to move, accelerates, slows down and stops. The magnitude of the velocity $V(t)$ in the case of a dipole will be proportional to the first derivative of the dipole moment, and in the case of interpreting this motion as a current in a conductor, to the magnitude of the current j. The acceleration $a(t)$ is proportional to the second derivative of the dipole moment and the first derivative of the current, respectively. If the charges of the dipoles remain in a displaced position, then the substance is polarised. Stopping charges in a conductor means termination of the current. Obviously, in a three-dimensional problem, when we calculate the fields far from the source, such a movement of charges will produce a single-cycle bipolar pulse with a zero electric area if the dependence of acceleration on time is bipolar. In a one-dimensional problem, formally, according to Eqn (5), we can obtain a strictly unipolar pulse, since the first derivative of polarisation and the current are proportional to the speed of the charge. This consideration should be accompanied by the following comment. In a one-dimensional problem, we neglect those fields that arise, e.g., when the source of the field is a pulse of current in a real system. Due to the requirement for the circuit closedness, i.e., the presence of a section of the circuit, albeit far from a conducting flat layer of large dimensions, where the current should flow in the opposite direction, there is a radiation field source with a different sign.

It is clear that the one-dimensional problem is a mathematical idealisation, applicable for calculations when, e.g., waves can be considered plane and the diffraction of radiation can be neglected. Such situations are encountered in practice, and one-dimensional problems are a good approximation for many problems in nonlinear optics and laser physics. An example of obtaining a unipolar pulse in a linear medium with a one-dimensional geometry of the experiment is given in [26]. It considers a thin flat layer of matter, which is excited by a single-cycle pulse with a plane wavefront propagating along the z axis. The medium is excited by a single-cycle pulse (Fig. 6a), consisting of two half-waves of opposite polarity. The first half-wave will accelerate the charges of the medium, and the second will stop their movement. If the charges stop and do not return to their original position, as shown in Fig. 5, then the velocity of the charge does not change sign. In the one-dimensional case, this behavior of charges will provide a source of a unipolar pulse, i.e., the reflected field will be a subcycle pulse (Fig. 6b). In the direction of the transmitted radiation, this pulse will add up with the transmitted bipolar one. Numerical calculations of reflected radiation performed in [26] confirm this conclusion.

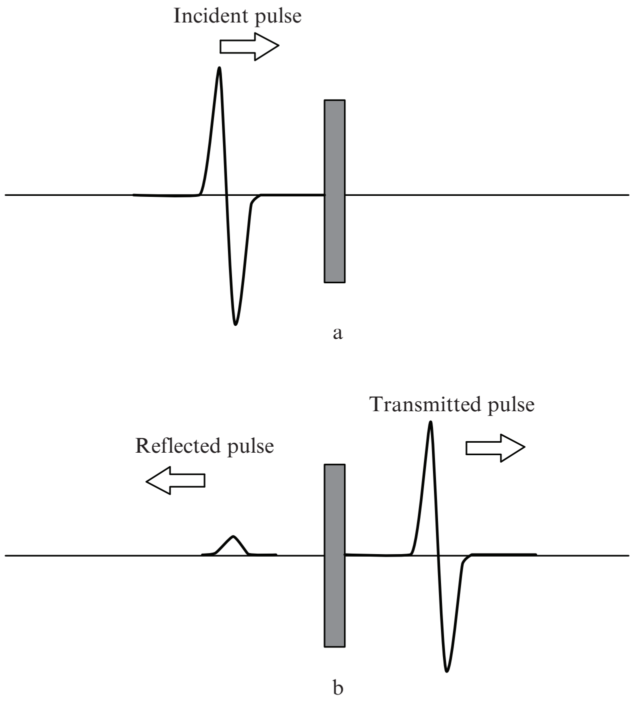


Figure 6. Reflection of a single-cycle pulse from a thin metal layer according to [26]: (a) single-cycle pulse incident on the layer, and (b) transmitted and reflected subcycle pulses.

This example is a clear illustration of the possibility of obtaining unipolar and subcycle pulses in a fairly simple and practically realisable situation. Note that the one-dimensional geometry considered in this section is valid near a layer that has finite dimensions. At a distance greater than the dimensions of the layer, the one-dimensional approximation is invalid, and it is necessary to take into account the three-dimensional geometry when calculating the field. This will lead to the disappearance of unipolarity, because the field will be proportional to the acceleration of charges, rather than their velocity (see Fig. 5).

6. Basic properties of unipolar light pulses and the rule of conservation of the pulse electric area

To derive the properties of unipolar light pulses, we use Maxwell's equation for the electrodynamics of continuous media, which relates the curl of the electric field strength $\boldsymbol{E}$ with the change in magnetic flux density $\boldsymbol{B}$ [28]. Maxwell's equations relate the strengths of electric and magnetic fields to the motion of charges. Let us pay attention to the equation relating the electric field strength $\boldsymbol{E}$ to the magnetic flux density $\boldsymbol{B}$:

$$\mathrm{rot}\boldsymbol{E} = -\frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t}. \quad (6)$$

Imagine a system of charges localised in space, onto which a sufficiently short radiation pulse is incident. If the system is dissipative, i.e., the motion of charges is accompanied by energy losses leading to deceleration and stopping, we assume that the pulse acts on a steady system that does not produce changing fields. Since the duration and energy of the pulse are finite, then after its interaction with charges, which should be accompanied by re-emission and dissipation of the pulse energy through nonradiation channels, the electric and magnetic fields in the system will become zero, and in the case of the existence of static fields, they will stop changing in time. This means that before the impact of the pulse and after the termination of its effects, the derivatives of the magnetic flux density will be equal. If we integrate both sides of Eqn (6) over the specified time interval, then on the left-hand side of the equation we obtain the electric area of the pulse, from which the derivatives with respect to spatial coordinates are taken according to the rules of the curl operation. Since we are integrating the time derivative of the magnetic induction, the right-hand side will be zero. Thus, for each point in space in the selected area,

$$\mathrm{rot}\boldsymbol{S}_E = 0, \quad (7)$$

where $\boldsymbol{S}_E$ is the electric area of the pulse (1). Note that we consider dissipative media and that the radiation began to act on an isolated system, in which all the processes of energy dissipation had ended before. In this case, the integration interval ends after the end of all processes that have arisen in the system under the action of a radiation pulse. This means that the static fields on the right-hand side of the equation, if they exist, must be equal at the time moments of minus/plus infinity (if this is not the case, then no energy dissipation has occurred in the system), which are indicated as the limits of integration. We emphasise that equation (7) is valid for any media in which radiation propagates – linear and nonlinear, isotropic and anisotropic, and with absorption and amplification.

Equation (7) turns into a rule for the conservation of the electric area in the plane wave approximation, in which the field strengths depend only on one Cartesian coordinate z (the longitudinal coordinate along the direction of preferential propagation of radiation). In this case,

$$\frac{\mathrm{d}}{\mathrm{d}z}\boldsymbol{S}_E = 0. \quad (8)$$

The one-dimensional version is the most illustrative, and it is easier to check it during simulation. At first glance, the result looks paradoxical. It would seem that amplification or absorption of radiation should lead to a change in the pulse area. However, as shown in direct numerical calculations of the propagation of unipolar pulses in amplifying and absorbing media [11], relation (8) is satisfied. In this case, the degree of unipolarity (2) is not preserved. When a unipolar pulse passes through an absorbing and amplifying medium, its value decreases. As noted in [11], intuitive expectations of a certain behavior of pulses in typical situations of their absorption or amplification, which are based on the known solutions for multicycle bipolar pulses, do not work in the case of unipolar pulses. For example, a decrease in the peak amplitude of a pulse in an absorbing medium is accompanied by a proportional increase in its duration, so that the area of the pulse is actually conserved. The area conservation rule has predictive power. Thus, a necessary condition for the emergence of unipolar pulses from pulses with a zero area in one-dimensional geometry is the appearance of a counterpropagating wave, for example, when a pulse is reflected from a medium. This is illustrated by the above example of reflection of a single-cycle pulse from a thin layer (see Fig. 6). During the propagation of light in a medium in the direction of the initial bipolar pulse, its transformation into a unipolar pulse is impossible. If such a result arises in the course of calculations, then there is an error in the theoretical model or calculations. Note also that the constancy of the electric area was violated in simplified one-dimensional models, when their authors assumed the pulse to be sufficiently short and neglected relaxation. Such assumptions led them to the conclusion that the electric area of the pulse changes during the one-dimensional propagation of light in the medium (see, e.g., [29–31]), which contradicts the conservation rule.

In fact, discussing relations (7) and (8), we deal with a new conservation law, first formulated in [10]. Undoubtedly, this law, which we call the rule, will play an important role in the physics of unipolar light, being a criterion for checking the correctness of calculation and experimental results.

Naturally, these relations, like any conservation law, are insufficient to determine the electric area itself under the conditions of a specific experiment. To do this, for the medium it is necessary to specify the type constitutive relations between the electric displacement $\boldsymbol{D}$ and magnetic flux density $\boldsymbol{B}$, on the one hand, and the electric and magnetic field strengths $\boldsymbol{E}$ and $\boldsymbol{H}$, on the other hand. Thus, an important mechanism for generating electromagnetic radiation is the accelerated motion of electric charges. In a vacuum, the electric displacement coincides with the electric field strength, so that the relevant Maxwell's equation has the form

$$\mathrm{div}\boldsymbol{E} = 4\pi\rho, \tag{9}$$

where ρ is the density of electric charges, which we will assume to be given, neglecting the effects of the emerging radiation on the distribution of the charge density.

Following Ref. [32], we integrate Eqn (14) over time also in infinite limits:

$$\mathrm{div}\boldsymbol{S}_E = 4\pi Q, \tag{10}$$

where

$$Q(\boldsymbol{r}) = \int_{-\infty}^{\infty} \rho(\boldsymbol{r}, t)\,\mathrm{d}t$$

is the density of the charge flowing through the vicinity of the point $\boldsymbol{r} = (x, y, z)$ for the entire infinite time interval. We assume the finiteness of this value, which excludes, e.g., the presence of immobile electric charges.

Equations (7) and (10) make it possible to determine the electric area $\boldsymbol{S}_E$ from a given distribution of the charge density Q. Formally, they coincide with the basic equations of electrostatics [17, 28] when replacing $\boldsymbol{E} \to \boldsymbol{S}_E$ and $\rho \to Q$.

7. Propagation of unipolar light, influence of diffraction, and optical waveguides for the transmission of unipolar light

Consider the process of propagation of a unipolar pulse in a space free of matter. It can be often heard that during propagation, a unipolar pulse should turn into a bipolar one. The statement that the propagation of a unipolar wave always leads to the loss of unipolarity and its transformation into a bipolar one is not entirely true and requires clarification. Therefore, the issue of changes in the degree of unipolarity and their causes should be considered separately. Suppose, for example, we have a certain source that produced a spherical unipolar wave (similar to the accelerated charge in the example mentioned above, see Fig. 2). Such a wave will not lose its unipolarity. Indeed, in free space, it travels at the speed of light, and there is no reason for any changes to appear.

The situation will change if such a wave meets an obstacle on its way. Then we should deal with the phenomenon of diffraction and a change in unipolarity associated with the fact that the interaction of the wave with the material medium causes the movement of charges in it. It should be emphasised here that the reaction of the charges of the medium will be the cause leading to a change in unipolarity. In optics, they usually deal with holes of various shapes in screens. The space–time Fourier transform allows considering any spatially limited wave after passing through a hole in the screen, regardless of whether it is bipolar or unipolar, as a superposition of monochromatic waves with different frequencies and propagation directions. Waves traveling at an angle to the main direction of propagation have a delay relative to waves travelling in the main direction. This leads to the appearance of a component of the opposite sign in the initially unipolar fragment of the wave. This approach was demonstrated in [33].

Understanding just such a physical picture, leading to the disappearance of unipolarity, is important, since sometimes it is replaced by formal mathematical constructions, behind which the physics of the phenomenon is lost. In Refs [34, 35], the scalar diffraction problem is considered as applied to an extremely short unipolar pulse. The authors replace the real pulse with a finite duration and the time dependence of the field with a delta function. The approach using delta functions is criticised in Ref. [36], since such a pulse has an infinitely wide spectrum and infinitely high energy. Therefore, nothing can be said about the range of applicability of the results of this approach. In Ref. [36], the change in the temporal and spatial wave profiles due to diffraction was calculated for a plane wave with a unipolar time dependence of the intensity in the form of a Gaussian profile, which is incident on a screen with 'soft edges', which also form a Gaussian spatial profile. The paper shows the main changes that should occur with unipolar pulses when they are diffracted by a hole in the screen. This is primarily the loss of unipolarity. The time dependence of the field strength in the far zone becomes proportional to the time derivative of the field strength in the near zone.

The conclusion that the unipolar field becomes bipolar in the far zone can be made based on the qualitative reasoning presented at the beginning of the review. In the one-dimensional problem, the field is proportional to the current or the first derivative of the medium polarisation, and in the three-dimensional problem, it is proportional to the derivative of the current and the second derivative of the polarisation. Consequently, the far field will be proportional to the time derivative of the near field. Accurate calculations clarify the details of the dependence of the pulse on time and coordinates and confirm the fundamentally important conclusion about the loss of unipolarity. Worth mentioning are the results of Ref. [37], where the action of a focusing lens on a single-cycle pulse was considered. The calculation showed that, at the focus, the initially unipolar pulse becomes bipolar. Paper [38] should also be mentioned, where it was shown that in the case of unipolar pulse diffraction, the field in the far zone is proportional to the time derivative of the near-field distribution. Note that the approaches that find application in the analysis of temporal behaviour of single-cycle terahertz pulses during focusing and diffraction [39–41] can also be applied to unipolar pulses.

An inevitable spreading of a radiation packet in a vacuum and in any homogeneous linear medium as it propagates, as well as the presence of obstacles in the path of unipolar pulses, lead to the possible loss of unipolarity. The traditional way of suppressing the spreading of pulses in space is the use of waveguides (optical fibres). However, conventional waveguides, both metallic and dielectric, have frequency dispersion, due to which the pulse duration gradually increases and the wave packet spreads in the longitudinal direction. In dielectric fibres, the waveguide dispersion is complemented by the material dispersion of the fibre material.

The authors of Ref. [42] propose a possible way of solving the problem of transporting quasi-unipolar pulses by using hollow metal waveguides with a nonsimply connected cross section. In fact, this is an optical analogue of the well-known coaxial electric cable, which successfully transmits pulses of any polarity in the radiofrequency range. In such a waveguide, radiation propagates in the space between two concentric metal surfaces, in which, unlike a standard coaxial electric cable, there is no dielectric. For ideal metals with infinitely high conductivity, the electromagnetic field of the principal wave (mode) is purely transverse and, regardless of the radiation frequency, such waves propagate with the same phase velocity equal to the speed of light in vacuum [17]. Thus, frequency dispersion is suppressed in coaxial waveguides, and the dynamics of the transverse field components, as in the plane-wave approximation, is described by a one-dimensional wave equation. Accordingly, in principle, it is possible to propagate pulses of arbitrary shape, including unipolar ones, over considerable distances.

8. Diffraction radiation as a source of unipolar light

Diffraction radiation arises when free charges move near the edges of metal screens. For the first time, a correct theoretical description of the phenomenon was obtained in Ref. [43]. The physical cause of radiation is the currents in the conductor, which are induced by a rapidly moving charge [18]. Note that by its nature it is close to the Smith–Purcell radiation [44], which arises when charges move above a periodic metal structure. Diffraction radiation is a source of information about the characteristics of charged particle beams. There are a sufficient number of studies devoted to calculating the spectral and energy characteristics of such radiation [45]. They depend on the energy and spatial characteristics of the beam; therefore, diffraction radiation can be observed in the optical range [46].

Diffraction radiation is interesting in that the pulses of the electromagnetic field arising when a charged particle passes near the edge of the screen can be unipolar. This occurs in the case of the so-called backward diffraction radiation (BDR) at a certain observation angle, namely, perpendicular to the edge of the screen and the direction of movement of charges (Fig. 7).

The details of the experiment are described in Ref. [47], the authors of which report on the observation of unipolar subnanosecond pulses during the BDR process. Although the region of the radiation spectrum in Ref. [47] was far from the optical range, this work, in our opinion, is an important attempt to experimentally confirm the existence of such radiation. The transition to the region of high frequencies in such an experiment is associated only with a change in the characteristics of the electron beam. Study of the generation of diffraction radiation poses interesting problems that are not typical for the optical range. For example, is it possible to amplify unipolar radiation and create a kind of a laser that generates unipolar pulses? If we talk about the possibility of realising stimulated radiation in the case of diffraction radiation of charged particles, then a similar attempt was made in Ref. [48], in which, in the author's opinion, it was possible to experimentally observe stimulated coherent diffraction radiation. However, the author does not report on attempts to obtain unipolar stimulated emission, although there are no fundamental obstacles to the realisation of this possibility. It should be recalled that stimulated emission arises not only in purely quantum systems with pronounced discrete energy levels, in which optical lasers are traditionally used. Stimulated emission can be observed in classical systems and have a classical interpretation [49]. The issue of stimulated unipolar radiation is extremely interesting and requires study.

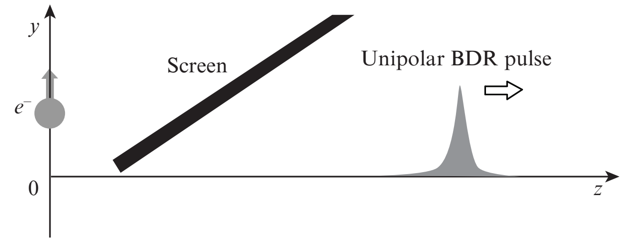


Figure 7. Schematic of the experiment for observing a unipolar BDR pulse. The y-axis coincides with the direction of motion of the charge e^-, the z axis coincides with the propagation direction of a unipolar BDR pulse, the x axis is perpendicular to the plane of the figure and, accordingly, is parallel to the edge of the metal screen.

9. Formation of unipolar solitons in nonlinear media

Another example that can underlie the appearance of unipolar pulses is soliton unipolar solutions of the equations of nonlinear optics in media with different types of nonlinearity. The existence of a unipolar soliton in a resonant two-level absorbing medium was shown for the first time in the theoretical paper [50]. The existence of such a solution can be associated with the phenomenon of self-induced transparency (SIT). In this phenomenon, a short laser pulse with the duration shorter than the times of longitudinal (T_1) and transverse (T_2) relaxation of the medium (for which the interaction of light with matter is coherent), can propagate in a medium without losses, like a 2π soliton [51]. Recall that the first theoretical and experimental studies of the SIT phenomenon were carried out for multi-cycle pulses using the approximations of slowly varying amplitudes and a rotating wave in the theoretical description [51–55], which corresponded to the experimental capabilities of those years when only nano- and subnanosecond pulses were available.

In Ref. [50], an analytical solution of the Maxwell–Bloch equations was obtained without using slow-amplitude and rotating-wave approximations in the form of a unipolar pulse with a hyperbolic secant shape. The very fact of the existence of soliton solutions of the equations of nonlinear optics in the form of unipolar pulses was established, but no schemes for obtaining these solitons were proposed in practice. Interest in such problems revived some time later, when the generation of femtosecond pulses containing several oscillation cycles was practically realised, and the theoretical description required the solution of the exact Maxwell–Bloch equations without the approximation of slow envelopes.

Solutions in the form of unipolar pulses in the one-dimensional geometry of light propagation were obtained in subsequent works, where the problem of the propagation of a short pulse was solved using the Maxwell–Bloch equations. It was shown [29, 30] that a unipolar pulse can propagate in a two-level medium with a duration that is both longer than the period of the resonant transition in a two-level system, and much shorter. In the case of an absorbing medium for a very short pulse, the authors again come to a unipolar 2π pulse of self-induced transparency. In an amplifying medium, the pulse becomes bipolar, and as it propagates, the spectrum shifts towards higher frequencies. In [56], the propagation of an initially bipolar pulse containing several oscillation cycles in a Raman-active medium was considered. It was shown by a numerical solution that during propagation, the pulse turns into a unipolar one. The next theoretical work [57] considers the propagation of a high-power femtosecond pulse in a metal, where the field acts on conduction electrons. The authors analytically show that the existence of unipolar solitons similar to 2π SIT pulses is possible.

Unipolar solitons were found in a multilevel quantum system [58]. Also, unipolar solitons were found in a medium consisting of nonlinear oscillators with quadratic and cubic nonlinearities [59]. Unipolar pulses, in the opinion of the authors of Ref. [60], can be formed when a very short laser pulse propagates in an atomic gas, when high-intensity radiation actively ionises atoms. In the methodological note [31], the propagation of short pulses in a medium consisting of two-level absorbing and amplifying particles is considered, and the existence of unipolar pulses is also shown. It should be noted that in a number of the above-mentioned works, in the course of analytical transformations of the original Maxwell–Bloch equations, assumptions were made and terms taking into account the reaction of the medium to the action of the electric field of a light pulse were discarded, after which it was possible to obtain an equation of the sine-Gordon type for the electric area of the pulse. Then the conclusion followed about the change in the electric area of the pulse during propagation in the medium. As noted in Ref. [11], this conclusion contradicts the conservation rule for the electric area of a pulse in a one-dimensional situation.

Numerical calculations free of simplification of the original equations are more correct. In [61], a half-cycle pulse was obtained by numerical simulation of the coherent propagation of a multi-cycle pulse in a two-level resonant medium. The system of Maxwell–Bloch equations was solved numerically without the approximations of slowly varying amplitudes and rotating wave. It was shown that during propagation, the initial pulse is split into three pulses. The first one is bipolar, consisting of several cycles, the second one is unipolar and the third one is unipolar, but with a different sign. The last two pulses are unipolar solitons. Since the pulse reflected from the medium remains bipolar, the pulses propagating in the medium should all together have zero electric area.

Similar schemes for obtaining a half-cycle pulse in the form of solitons during the coherent propagation of an initially multi-cycle bipolar pulse in a two-level medium have been considered by other authors. For example, in Ref. [62] this possibility was studied theoretically in the case when the initially bipolar pulse propagates in an asymmetric two-level medium with a constant dipole moment. The revealed mechanism for the formation of a subcycle soliton consists in nonlinear self-shaping and frequency conversion during resonance coherent interaction of a pulse with a medium. Later, the authors demonstrated the possibility of obtaining a subcycle soliton in an inversion symmetric medium [63] using a similar mechanism. In the papers mentioned above, the study of the formation of a half-cycle pulse due to the SIT phenomenon was carried out mainly for a two-level model of a resonant medium. Recently, the possibility of the appearance of unipolar pulses in the form of solitons was demonstrated theoretically [64] in a medium of four-level atoms. Although the above schemes are difficult to implement in practice, soliton mechanisms for producing unipolar pulses may turn out to be attractive for the practical generation of unipolar pulses (see also [65–68]).

The next group of theoretical studies, worth mentioning in this review, is devoted to the possibility of dissipative solitons in the form of unipolar pulses. Many laser systems include an amplifier and an absorber, which allows the generation of short radiation pulses through mode locking. This is a particular situation of a more general case, when energy is supplied to the system to compensate for the losses existing in it. For example, in the case of SIT solitons in a real experiment, damping will always occur, which will inevitably lead to their disappearance. If the damping is compensated for by amplification, then such a dissipative soliton can exist indefinitely.

The possibility of obtaining a half-cycle pulse in the form of a dissipative SIT soliton was demonstrated in Refs [69–71]. In this scheme, it was proposed to use a mixture of absorbing and amplifying centres embedded in the matrix and having approximately a twofold difference in the transition dipole moments of absorbing and amplifying particles. The dynamics of the formation of extremely short self-induced transparency solitons in such a system is described in sufficient detail in monograph [72]. During propagation, a femtosecond pulse is a π-pulse for amplifying centres that removes the inversion from the medium and transfers the population to an unexcited state, which leads to a decrease in the pulse duration and an increase in amplitude. For absorbing centres, however, such a pulse transforms into a 2π SIT soliton. An example of numerical simulation of the described situation is presented in Fig. 8.

In this example, the resulting extremely short SIT solitons are realised in a system where amplifying and absorbing centres are mixed. When such a system 'amplifier + coherent absorber' is placed into a cavity and the amplifier compensates not only the losses in the absorber, but also the cavity losses, the generation of a single soliton can turn into the generation of a train of extremely short pulses. In this case, the so-called coherent mode locking (CML) is realised. Since in the CML regime the gain and absorption band widths do not limit the pulse duration [73], it was proposed in theoretical

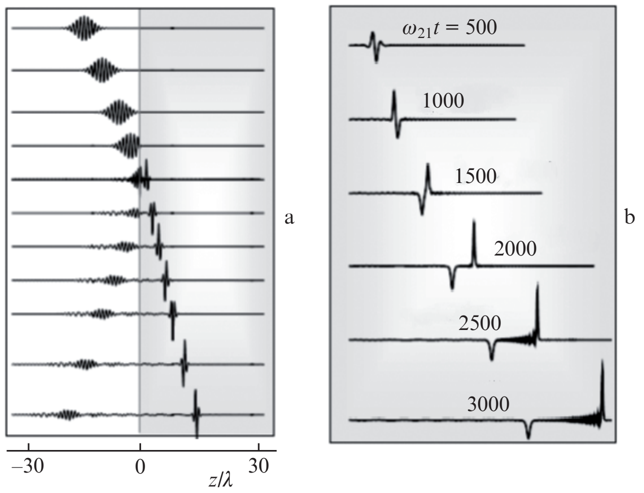


Figure 8. Formation of unipolar solitons from a multi-cycle femtosecond pulse in a medium with resonance gain and absorption nonlinearity. Instantaneous profiles of the field strength near the boundary of the medium (a) when a pulse enters the medium and (b) during its further propagation in the medium with the formation of a single-cycle pulse splitting into two unipolar pulses of different polarity [71].

papers [74, 75] to use this regime to generate extremely short pulses in ring-cavity lasers. Recently, Ref. [76] theoretically demonstrated the generation of extremely short pulses due to the CML regime in a laser with an ultra-short linear cavity. The existing results indicate the fundamental possibility of obtaining subcycle pulses in lasers using the CML regime. It was noted that the CML regime itself was obtained experimentally quite recently and it was shown that the duration of generated pulses decreases with an increase in their intensity [77–79].

In this part of the review, we considered, in our opinion, the main theoretical works using the one-dimensional approximation in the framework of the Maxwell–Bloch equations. Once again, we note that the results of some papers do not agree with the rule of conservation of electric area. These issues were also raised in an earlier brief review [12].

10. Obtaining attosecond and terahertz unipolar pulses

Let us now proceed to pulses of attosecond duration and pulses in the terahertz region of the spectrum. Although they differ in duration by four to six orders of magnitude, they are related by the use of femtosecond IR lasers, which initiate processes leading to the appearance of radiation in these ranges, the physics of these processes being different.

The duration of attosecond pulses (1 as = 10^{-18} s) is comparable to the characteristic periods of motion of electrons in the electron shells of atoms and molecules, and therefore they are successfully used to study and control the motion of electrons on atomic time scales [8, 80, 81]. To date, attosecond pulses have been experimentally demonstrated in the extreme ultraviolet and X-ray frequency ranges [6, 7]. Attosecond pulses are obtained by generating high-order optical harmonics when atoms are exposed to an intense IR pulse from a femtosecond laser. According to a three-step model of the formation of high harmonics, during the action of a femtosecond pulse, an atom is ionised, then a free electron is accelerated by a laser field and, with a change in the sign of the field strength during a cycle of oscillations, recombination with the parent ion occurs [5]. In such a process, the motion of an electron is confined in space, and so the radiation pulses contain a few oscillation cycles and are bipolar.

Attosecond pulses can also be obtained from radiation with an ultra-wide (from IR to UV) spectrum (coherent supercontinuum) as a result of dividing the radiation into portions followed by summation of different portions of the spectrum [82]. Here, the summation of bipolar fields is also unable to produce unipolarity, although it will make it possible to obtain pulses of subcycle shape. This applies to any linear wave synthesis method.

Half-cycle pulses with a duration of ~380 as and an amplitude of $\sim 10^{7}$ V cm^{-1} in the optical range were obtained in [83]. In this work, the authors successfully applied these pulses to study the dynamics of bound electrons in a krypton atom, demonstrating the inertia of bound electrons. It was also possible to show an increase in the efficiency of the action of a half-cycle quasi-unipolar pulse on atoms in comparison with a single-cycle bipolar pulse. In [84], a method was proposed for obtaining a quasi-unipolar half-cycle attosecond pulse in the visible and ultraviolet spectral regions (duration ~100 as) with a high peak amplitude (exceeding 10^{12} V m^{-1}) in a gas, where high-power femtosecond laser pulses form a thin layer of relativistic electrons, which then pass through the slanted target. A similar method for the formation of a high-intensity single half-cycle attosecond pulse upon irradiation of a thin foil with a powerful femtosecond pulse is described in [85]. In this paper, single half-cycle pulses with a duration of 10 as and an amplitude up to 10^{13} V m^{-1} are reported.

In the terahertz region, where the period of electromagnetic waves and, accordingly, the duration of extremely short pulses lies in the picosecond range, from the point of view of obtaining unipolar pulses, methods using optical rectification in nonlinear crystals are of interest. The very phenomenon of optical rectification was demonstrated at the beginning of the laser era [86]. On metal plates applied to a crystal in which the second harmonic of a ruby laser was generated, the appearance of a unipolar voltage pulse was recorded, which coincided in duration with the laser pulse. Several years later, after the implementation of the reduction of the pulse duration due to mode locking, there were proposals to use the effect of optical rectification to obtain radiation pulses in the microwave range [87]. Naturally, these field pulses were unipolar. After the appearance of femtosecond laser pulses, the method of optical rectification was used in modern schemes for generating extremely short terahertz radiation pulses, described in a large number of publications, e.g., in [88, 89].

It should be noted that the question of whether short pulses are unipolar was not raised in studies on terahertz radiation. The fact is that when detecting them, focusing is used, and this should lead to the loss of unipolarity. Focusing is also often used to increase the intensity, and in this case, the observed pulses are already bipolar. Therefore, we can only give examples of studies where unipolar half-cycle pulses were obtained in the terahertz frequency range. In addition to optical rectification, a common mechanism is nonlinear photoionisation of gases or liquids by the field of a femtosecond laser pulse. Subcycle terahertz pulses can be obtained by generating a unidirectional current pulse in the process of photoionisation, as well as by the action of femtosecond pulses on semiconductor structures [90–95]. Highly efficient generation of half-cycle terahertz pulses due to nonlinear photoionisation was recently achieved in water and other liquids [96]. Methods have been proposed for generating quasi-unipolar terahertz pulses in the form of precursors propagating ahead of high-power laser pump pulses in electrooptical crystals [97–99]. The shape of such pulses is close to rectangular. Noteworthy is the recent theoretical work in which the possibility of obtaining a unipolar pulse due to the directed motion of electrons under the action of a femtosecond pulse on a silicon plate is discussed [100].

Another approach to obtaining unipolar pulses of a controlled shape suggests using low-frequency oscillations in Raman-active media. The idea is to use a pair of femtosecond pulses. The first pulse excites a low-frequency vibration, and the second, arriving after half a period, stops the movement. Because of polarisation, the time dependence has the form of half a cycle and can be a source of single-cycle pulses in one-dimensional geometry. At the same time, it is proposed to use excitation travelling through a medium at a speed different from the speed of light, for example, an oblique incidence of a plane wavefront on a layer of matter. The simultaneous use of these two techniques makes it possible to affect the shape of the resulting radiation and ensure unipolarity [101–108]. A survey of these works is given in Ref. [12].

11. Effect of quasi-unipolar pulses on a classical charged particle

The problem of particle acceleration by laser pulses is becoming increasingly important in connection with the progress of laser physics and technology [109]. For example, an approach with intermediate conversion of targets into laser plasma and subsequent acceleration of charges by a wake wave is being intensively studied [110–112]. However, the inclusion of an intermediate conversion reduces the efficiency of laser acceleration of charges. In a vacuum, direct acceleration of charged particles by conventional laser pulses is ineffective, since the field strength changes direction many times during the pulse duration, thereby acting on the charge in opposite directions. An approach is known with the acceleration of particles by tightly focused extremely short laser pulses with radial polarisation [113]. In this approach, acceleration is achieved due to the longitudinal (along the beam axis) component of the field, which, as a rule, is much smaller than the transverse components.

At the same time, progress in obtaining strong laser fields using extremely short and even subcycle pulses makes a different approach to the problem of laser acceleration of charged particles possible in principle. We mean the use of quasi-unipolar laser pulses with a noticeable electric area for these purposes. It was shown [114, 115] that the acceleration of particles to high energies is completely determined by the electric area of the pulse. Analytical expressions are presented for the energy and momentum of a charged particle accelerated in vacuum by linearly polarised radiation pulses (plane-wave approximation). The results presented are valid for a classical particle in neglect of quantum effects and the Lorentz force causing radiative deceleration of a particle [28]. The latter is justified outside the narrow region of ultrarelativistic motion; otherwise, acceleration of charged particles by a radiation pulse with zero electric area is possible [116]. While maintaining these limitations, the results obtained confirm the possibility of direct acceleration of a single charged particle by pulses of electromagnetic radiation using pulses of unfocused (plane-wave) radiation. A necessary condition is a significant electric area of the radiation pulse, which is the only quantity that determines the kinetic energy and mechanical momentum of the accelerated particle.

12. Effect of unipolar light on the simplest quantum systems

Let us consider the effect of an extremely short pulse on a quantum harmonic oscillator, the model of which is used, for example, to describe molecular vibrations. An exact solution of the Schrödinger equation is known for a quantum harmonic oscillator with frequency ω_0, mass m, and charge q, driven by an electric field with an arbitrary time dependence [117]. If the pulse duration is less than the period of eigenoscillations of the oscillator, then in the approximation of sudden perturbations, the population of the nth energy level of the oscillator w_n also depends on the electric area of the pulse S_E [118]:

$$w_n = \frac{1}{n!}\left(\frac{q^2 S_E^2}{2m\hbar\omega_0}\right)^n \exp\left(-\frac{q^2 S_E^2}{2m\hbar\omega_0}\right). \quad (11)$$

The decisive role of the electric area of the pulse in the efficiency of the effect of subcycle pulses on quantum objects is again visible. Similarly, for a hydrogen atom, the probability of remaining in the ground state is determined by the electric area of a short pulse [119]:

$$w_0 = \left[1 + \left(\frac{\hbar S_E}{2mq}\right)^2\right]^{-4}. \quad (12)$$

For a multilevel quantum system other than a harmonic oscillator, it is possible to analyse the effect of a short unipolar pulse using an approximate solution of the Schrödinger equation based on perturbation theory, when the incident field can be considered weak. The probability of the system transition from the ground state of the discrete spectrum to the kth state can be calculated in the first order of the perturbation theory [120]:

$$w_{0k} = \frac{1}{\hbar^2}\left|\int V_{0k}\exp(\mathrm{i}\omega_{0k}t)\,\mathrm{d}t\right|^2. \quad (13)$$

Here $V_{0k} = -d_{0k}E(t)$ is the matrix element of the perturbation operator; d_{0k} is the transition dipole moment; and ω_{0k} is the frequency of the resonant transition.

If the pulse duration is less than the period of the considered transition $T_{0k} = 2\pi/\omega_{0k}$, then the oscillating exponent in the integrand of Eqn (13) does not have time to change noticeably during the pulse action, and the pulse can be approximately considered delta-shaped. In this approximation, for the transition probability we have [121]

$$w_{0k} = \frac{d_{0k}^2}{\hbar^2} S_E^2\,. \quad (14)$$

It can be seen from Eqn (14) that the transition probability w_{0k} depends exclusively on the electric area of the pulse, increases in proportion to its square, and vanishes if the pulse has a zero area. This circumstance once again shows the specificity of the effect on a quantum system of unipolar pulses in comparison with bipolar single-cycle ones. In the case of a single pulse, the action is nonresonance and changes the populations of all levels. However, if the system is exposed to a pair of short unipolar pulses (for simplicity, we consider a pair of delta-shaped pulses) having electric areas S_{E1} and S_{E2}, the time delay between which is Δ, then for the transition probability from formula (13) it is easy to obtain [121, 122]

$$w_{0k} = \frac{d_{0k}^2}{\hbar^2}\left(S_{E1}^2 + S_{E2}^2 + 2S_{E1}S_{E2}\cos\omega_{0k}\Delta\right). \tag{15}$$

It can be seen that in the case of a pair of subcycle pulses in the first order of perturbation theory, the result is determined by both the electric area of the pulses and the delay between them. The formula implies the possibility of controlling the populations of the system energy levels by varying the delay between incident pulses [121, 122]. This allows selective control of the medium state, despite the nonresonant nature of the interaction of each of the pulses with it.

Note that in this case it becomes possible to record information about an object holographically using pulses of subcycle unipolar radiation [123], when a resonant medium with a long phase memory time T_2 is used as the recording medium. An interference pattern of a subcycle or unipolar pulse reflected from an object with a medium polarisation wave created by the same short pulse is recorded in the medium. Coherence is provided by a polarisation wave, which, when interacting with radiation reflected from an object, induces a population lattice in the medium according to (15), repeating the interference pattern in a similar holographic process with a monochromatic source with a wavelength equal to the wavelength of the resonant transition in the medium.

The important role of the electric area in changing the direction of the spin of an electron interacting with a quasi-unipolar radiation pulse was also revealed in Ref. [124].

13. Detecting the unipolar nature of radiation and measuring the pulse electric area

For operations with unipolar light, it is required to have detectors that allow not only to determine the fact of unipolarity, but also to measure the electric area of the pulses. Moreover, there is no doubt about the applicability of conventional photodetector systems for recording the energy characteristics of such radiation. However, they most likely will not allow solving the indicated problem. Note that, until now, the task of developing systems for recording the electric area of a pulse and creating the corresponding devices has not been posed.

The problem of recording the electric area of radiation arises in the optical and shorter wavelength ranges. For the RF range, it is not difficult to register a unipolar pulse using a broadband oscilloscope and an appropriate coaxial cable with a capacitive sensor at the input. In the shorter-wavelength range of electromagnetic radiation (as, e.g., in the above experiment on recording the unipolar nature of diffraction radiation of relativistic charged particles [47], where the pulse duration, according to the authors' estimates, was several tens of picoseconds), the experimenters used a balanced scheme that included two strip lines with two microwave diodes. With a nonzero unipolarity of the microwave pulse, the signal in one channel will be larger. A similar scheme is used to control the position of a charged particle beam [125].

In the terahertz range, free-space electrooptical sampling schemes [126] are widely used, which, in principle, make it possible to register the unipolar nature of terahertz subcycle pulses. However, when terahertz radiation is focused into a nonlinear crystal, the unipolarity of the initial radiation can be lost.

Such systems are not applicable in the optical range of pulse durations. An optical pulse cannot produce surface currents in the strip line; the abovementioned scheme for detecting terahertz radiation is also inoperative. At first glance, one can use the fact that unipolar pulses can induce a current of free carriers, such as electrons, in a vacuum. However, the motion of free electrons is subject to uncontrolled external electric fields, and the measurement system is unlikely to be immune to interference without proper shielding.

Taking into account the results of works on evaluating the action of unipolar pulses on the simplest quantum systems (which were discussed in Section 12), quantum systems that do not experience changes under the action of bipolar pulses with a zero electric area, but change their state in the presence of a pulse of a nonzero electric area can be promising. For this aim, as shown in Section 12, the period of eigenoscillations for the transition between energy levels in a quantum system must exceed the pulse duration. A change in state can be detected by a change in the electrical or optical properties of a substance, its glow, which is the main sensitive element of the 'detector of unipolarity and electric area'. For a detector to be sensitive to the direction of the field, it must have anisotropy of optical properties.

It is also worth noting that a unipolar subcycle pulse with a nonzero area, after its passage, changes the magnitude of the vector potential of the space. Hence, unipolar light could be used to observe the Aharonov–Bohm effect and thus register the electric area of the pulse. In Ref. [127], the scheme of an electron-optical version of the experiment for observing this effect is considered. In this experiment, the source of the vector potential in one arm of the electronic interferometer, which exists in the region of a zero electric field strength, is a subcycle unipolar light pulse with a nonzero electric area. At first glance, using such a layout it would be possible to determine the electric area of the pulse. However, in view of Eqn (7), the vector field of the pulse electric area is potential; therefore, the Aharonov–Bohm effect does not manifest itself in the discussed scheme [127].

14. Conclusions

The issue of unipolar light until recently arose in purely academic discussions. The direct task of producing sources of unipolar radiation, which may have practical sense, has not yet been posed. There was no mention of applications where unipolar light would be needed. Recently, in connection with the extensive development of methods for generating extremely short pulses, the problem of obtaining unipolar light in the optical and adjacent ranges begins to attract more and more attention.

In our opinion, the results presented above convincingly indicate that the existence of unipolar electromagnetic radiation does not contradict the existing fundamental concepts, Maxwell's equations and the electromagnetic theory of radiation based on them. The opinion about the non-physicality and impossibility of the existence of such radiation is unfounded.

Electromagnetic radiation used in scientific and practical applications extends from radio to X-ray and gamma ranges and is of a common nature, so that the problem of obtaining unipolar radiation can be raised for all these ranges. Sources of unipolar radiation, of course, can be systems using the motion of free charges. In systems of bound charges, quasi-unipolar subcycle radiation can be obtained, mainly in highly nonlinear media, under the action of short intense electromagnetic pulses.

Analysis of the situation of using unipolar light revealed a number of problems. It is not completely clear how to most effectively transport such radiation, transmit it over a distance and focus without a significant decrease in the degree of unipolarity. The coaxial fibres mentioned in this review are theoretically capable of transporting and maintaining unipolarity; however, the ways of practical implementation of such devices are not clear. The situation is better with the understanding of the features of the effect of unipolar radiation on quantum systems. Unipolar radiation, which is no longer resonant to any of the transitions between energy levels in a quantum system, can nevertheless change the populations of its states selectively and extremely rapidly.

There is also a problem of monitoring unipolarity. For these purposes, special detectors are needed that respond to the nonzero electric area of radiation pulses, which are not available today. The creation of unipolar radiation sources should be accompanied by the development of appropriate registration systems. The absence of 'detectors of unipolarity' was, in our opinion, one of the reasons for the lack of interest in experiments with unipolar radiation.

Finally, we would like to draw attention to the rule of conservation of the pulse electric area. In fact, this rule is a recently discovered conservation law. When solving problems associated with unipolar radiation and analysing its propagation, it will play an important role. This rule is still little known to researchers. There are practically no publications providing examples of the implementation of this conservation law.

Acknowledgements. The authors are grateful to I. Babushkin for the discussion of the issues considered in the review.

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