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Acousto-optical analogue of a Fabry – Perot resonator

V.I. Pustovoit

Abstract. **A new method is proposed to increase the spectral resolving power of collinear acousto-optical filters by using optical feedback when the light leaving the region of its interaction with a periodic structure formed in the crystal by an acoustic wave returns to the input of the crystal, in which the diffraction process repeats itself many times. The light beam returned to the interaction region changes the boundary conditions of the parametric diffraction problem, because of which the amplitudes of the diffracted and passed light beams turn out to be strongly dependent on the feedback properties (similar to the processes occurring in a Fabry – Perot optical resonator). It is shown that such a combined acousto-optical Fabry – Perot filter with feedback is able to electronically tune the optical transmission wavelength and simultaneously has a higher spectral resolution than a conventional acousto-optical filter without feedback. It is also shown that the multiple radiation diffraction due to the feedback increases the diffraction efficiency with a comparatively small spatial change in the refractive index of the medium. Explicit analytical expressions for the instrumental functions of a combined acousto-optical Fabry – Perot filter are found and their properties are analysed. It is noted that a change in the feedback by any mechanism, i.e., by changing the returned wave phase or amplitude, leads to modulation of the measured signal, which makes it possible to create more precise methods of spectral measurements.**

Keywords: *acousto-optical filter, Fabry – Perot resonator, feedback.*

The search for new physical methods for increasing the spectral resolution [1, 2] and diffraction efficiency in modern acousto-optical (AO) spectrometers [3] based on collinear light diffraction on acoustic waves (or photonic crystals) is a topical problem. In the present work, we propose and analyse new approaches to this problem using optical feedback, when optical radiation passed through the region of its interaction with a periodic structure (formed, for example, by an acoustic wave) returns into the crystal, in which the diffraction process occurs over and over again. The light beam returned into the interaction region changes the boundary conditions of the parametric diffraction problem, which leads to a change in the amplitudes of the diffracted and transmitted optical radiation. It is physically clear that this change, like in the Fabry – Perot resonator [4], will depend on the amplitude and phase of the returned optical signal, which, in turn, strongly depend on the phase-matching conditions. As a result, it becomes possible to increase the efficiency and resolving power of such a combined AO Fabry – Perot (FP) filter.

Optical feedback can be achieved by different methods: first, when nondiffracted radiation returns (which leads to changes in the incident radiation spatial distribution satisfying the phase-matching conditions); second, when diffracted radiation returns (which, in contrast to the previous case, will cause changes in the incident radiation amplitude, phase, and frequency at the entrance to the interaction region, i.e., to the crystal); and, third, when both diffracted and nondiffracted radiation parts return. Of course, the aforementioned change in the diffracted wave frequency is insignificant and can be neglected in some cases, but, in the case of diffraction of narrowband laser radiation, when the absolute change in frequency is important, this effect must be taken into account, especially when considering noncollinear diffraction [5]. In the present work, in contrast to [6], we consider the cases of feedback for collinear diffraction, but it is clear that the use of light feedback extends the functionality of tuneable AO filters and spectrometers.

Examples of optical schemes of collinear tuneable AO filters with feedback are presented in Fig. 1. Polariser P1 separates from the initial radiation a light beam with ordinary polarisation, which falls into the AO crystal where collinear diffraction on an acoustic wave occurs and a new light wave with extraordinary polarisation appears. Mirrors M and a beam splitter BS form a feedback loop, and the diffracted radiation intensity is recorded by a photomultiplier PM. Polariser P2 separates the light beams with different polarisations so that the ordinarily polarised beam returns to the beginning of the interaction region (Fig. 1a) (this corresponds to the first feedback case). Figure 1b shows the scheme with feedback via the diffracted, i.e., extraordinary light wave. This feedback case, in contrast to the case shown in Fig. 1a, is interesting due to the occurrence of multiple shifts of the diffracted radiation frequency. It will be shown below that, under specific resonance conditions, the efficiency of such diffraction of, for example, laser radiation, can be rather high.

Let us derive analytical expressions for wave amplitudes and intensities under the conditions of collinear diffraction of optical radiation on an acoustic wave in the presence of feedback. From the Maxwell equations for an anisotropic medium whose permittivity varies according to a periodic low, it is possible to obtain truncated equations describing the collin-

V.I. Pustovoit Scientific and Technological Center of Unique Instrumentation, Russian Academy of Sciences, ul. Butlerova 15, 117342 Moscow, Russia; All-Russian Scientific Research Institute of Physical-Technical and Radiotechnical Measurements, 141570 Mendeleevo, Moscow region, Russia;
e-mail: vladpustovoit@gmail.com

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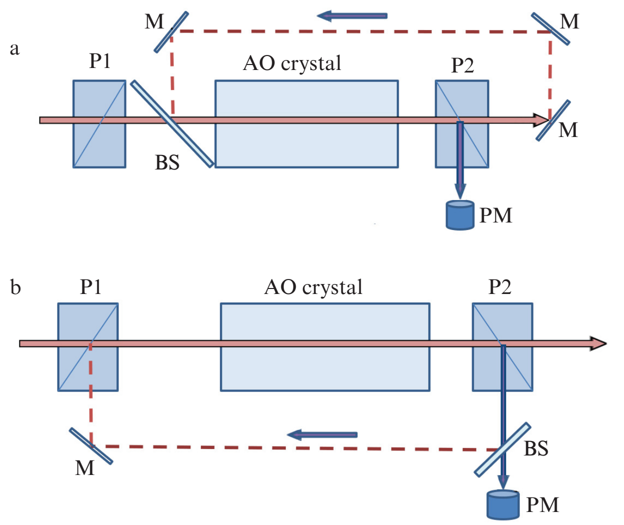

Figure 1. Optical schemes of AO filters with feedback based on (a) non-diffracted and (b) diffracted beams: (P1, P2) polarisers; (PM) photomultiplier; (M) mirrors; and (BS) beam splitter.

ear diffraction process under conditions close to phase-matching [7]. In the approximation of the theory of coupled modes, these equations have the form [7]

$$\frac{\mathrm{d}E_\mathrm{e}(x)}{\mathrm{d}x} = -\mathrm{i}\Gamma_\mathrm{e}\mathrm{e}^{\mathrm{i}\Delta kx}E_\mathrm{o}(x),$$
$$\frac{\mathrm{d}E_\mathrm{o}(x)}{\mathrm{d}x} = -\mathrm{i}\Gamma_\mathrm{o}\mathrm{e}^{-\mathrm{i}\Delta kx}E_\mathrm{e}(x). \quad (1)$$

Here, $E_\mathrm{o}(x)$ and $E_\mathrm{e}(x)$ are the amplitudes of the ordinary (incident) and extraordinary (diffracted) waves, respectively;

$$\Gamma_\mathrm{e} = \frac{\omega n_\mathrm{o} n_\mathrm{e}^2}{4c}pS(x), \quad \Gamma_\mathrm{o} = \frac{\omega n_\mathrm{o}^2 n_\mathrm{e}}{4c}pS^*(x) \quad (2)$$

are the coupling coefficients expressed via the crystal characteristics and the sound wave amplitude; p is the photoelastic constant corresponding to the considered interaction geometry; c is the speed of light in vacuum; n_o and n_e are the refractive indices of the ordinary and extraordinary light waves;

$$S(x,t) = \frac{1}{2}S(x)\exp(\mathrm{i}\Omega_\mathrm{s}t - \mathrm{i}q_\mathrm{s}x) + \mathrm{c.c.} \quad (3)$$
$$\left(\frac{1}{S(x)}\frac{\mathrm{d}S(x)}{\mathrm{d}x} \ll q_\mathrm{s}\right)$$

is the acoustic wave amplitude; Ω_s, q_s are the frequency and wavenumber of the acoustic wave; $\Delta k \equiv (\omega/c)n_\mathrm{o} - (\omega^*/c)n_\mathrm{e} + \Omega_\mathrm{s}/v_\mathrm{s}$ is the detuning from the phase-matching conditions. The relation between the frequencies of light waves followed from conservation laws has the form $\omega \equiv \omega^* + \Omega_\mathrm{s}$, where ω^* is the diffracted wave frequency; v_s is the speed of sound, and the crystal is assumed to be negative, i.e., $n_\mathrm{o} > n_\mathrm{e}$. The Γ_e and Γ_o coefficients slightly differ from each other due to a difference between the n_o and n_e refractive indices, but, as will be seen below, the final formulae always include the combination $\Gamma_\mathrm{e}\Gamma_\mathrm{o}$, the root of which will be denoted as Γ, $\Gamma \equiv \sqrt{\Gamma_\mathrm{e}\Gamma_\mathrm{o}}$. (Below, we will not distinguish Γ, Γ_e, and Γ_o, except for the cases in which this is necessary.) The considered medium (crystal) has no absorption and dispersion and, therefore, as is seen from system of equations (1), the law of conservation of energy of interacting light waves is fulfilled independently of the boundary conditions:

$$\frac{\partial(n_\mathrm{o}^{-1}|E_\mathrm{o}(x)|^2 + n_\mathrm{e}^{-1}|E_\mathrm{e}(x)|^2)}{\partial x} = 0. \quad (4)$$

Condition (4) is valid for waves inside the region of their interaction in the crystal, i.e., without taking feedback into account and, in addition, without taking into account negligible energy transfer from the light wave to the sound wave. This relation also indicates that the knowledge of the intensity of the wave with one polarisation makes it possible to determine the intensity of the wave with the other polarisation at any point in the interaction region. The phases of these waves obey the relations that can be found directly from system of equations (1) by dividing it into equations for the real and imaginary parts of the amplitudes of interacting light waves [8]. We will write the boundary conditions at the entrance to the interaction region in the most general form:

$$E_\mathrm{o}\big|_{x=0} = E_\mathrm{o}^0, \quad E_\mathrm{e}\big|_{x=0} = E_\mathrm{e}^0. \quad (5)$$

Then, the solution of the general problem (1), (5) has the form [7]

$$E_\mathrm{o}(x) = E_\mathrm{o}^0\exp\left(\frac{\mathrm{i}}{2}\Delta kx\right)\left[\cos(\Gamma\xi x) - \frac{\mathrm{i}\Delta k}{2\Gamma\xi}\sin(\Gamma\xi x)\right]$$
$$-\mathrm{i}\frac{E_\mathrm{e}^0}{\xi}\sqrt{\frac{n_\mathrm{e}}{n_\mathrm{o}}}\exp\left(\frac{\mathrm{i}}{2}\Delta kx\right)\sin(\Gamma\xi x),$$
$$E_\mathrm{e}(x) = -E_\mathrm{o}^0\frac{\mathrm{i}}{\xi}\exp\left(-\frac{\mathrm{i}}{2}\Delta kx\right)\sqrt{\frac{n_\mathrm{o}}{n_\mathrm{e}}}\sin(\Gamma\xi x) \quad (6)$$
$$+E_\mathrm{e}^0\exp\left(-\frac{\mathrm{i}}{2}\Delta kx\right)\left[\cos(\Gamma\xi x) + \frac{\mathrm{i}\Delta k}{2\Gamma\xi}\sin(\Gamma\xi x)\right].$$

Here, $\xi \equiv \sqrt{1 + (\Delta k/2\Gamma)^2}$, and as the wave amplitudes at the boundary $x = 0$ in expressions (5) we shall understand a complex value with allowance for phase. The general solution in the form of (6) does not take optical feedback into account. Similar to the case with a Fabry–Perot interferometer [4], solution (6) allows one to find scattering matrix $S_{\alpha\beta}(x)$, which relates the wave amplitudes at the entrance (i.e., at $x = 0$) to the amplitudes at any point x inside the interaction region. Solution (6) at the boundary $x = L$ can be written via scattering matrix $S_{\alpha\beta}(x)$ as [4]

$$E_\alpha(L) = \sum_\beta S_{\alpha\beta}(L)E_\beta^0, \quad \alpha,\beta \equiv \{\mathrm{o},\mathrm{e}\}, \quad (7)$$

where

$$S_{\alpha\beta}(L) = \begin{pmatrix} \exp\left(\frac{\mathrm{i}}{2}\Delta kL\right)\left[\cos(\Gamma\xi L) - \frac{\mathrm{i}\Delta k}{2\Gamma\xi}\sin(\Gamma\xi L)\right] & -\frac{\mathrm{i}}{\xi}\exp\left(\frac{\mathrm{i}}{2}\Delta kL\right)\sqrt{\frac{n_\mathrm{o}}{n_e}}\sin(\Gamma\xi L) \\ \frac{\mathrm{i}}{\xi}\exp\left(-\frac{\mathrm{i}}{2}\Delta kL\right)\sqrt{\frac{n_\mathrm{e}}{n_\mathrm{o}}}\sin(\Gamma\xi L) & \exp\left(-\frac{\mathrm{i}}{2}\Delta kL\right)\left[\cos(\Gamma\xi L) + \frac{\mathrm{i}\Delta k}{2\Gamma\xi}\sin(\Gamma\xi L)\right] \end{pmatrix}. \quad (8)$$

For anisotropic media without absorption, the elements of scattering matrix $S_{\alpha\beta}(L)$ satisfy the relations

$$S_{\rm oo}S_{\rm oo}^* + S_{\rm oe}S_{\rm eo}^* = 1,\ S_{\rm ee}S_{\rm ee}^* + S_{\rm eo}S_{\rm oe}^* = 1,$$
$$S_{\rm oe}S_{\rm oo}^* + \frac{n_{\rm o}}{n_{\rm e}}S_{\rm ee}S_{\rm eo}^* = 0, \tag{9}$$

which ensure the fulfilment of conservation law (4). The amplitudes $E_{\rm o}$ and $E_{\rm e}$ belong to the electric fields inside an anisotropic medium in which the refractive indices and wave group velocities are different for different polarisations, because of which the ratios of refractive indices appear in formulae (4) and (9) in contrast to the case of an isotropic medium [4].

Next, we consider the light wave diffraction on an acoustic wave propagating in the same direction as the light wave. Let the feedback to occur via the nondiffracted light wave, as is shown in Fig. 1a. Polariser P1 separates from the incident optical radiation a light beam with the polarisation corresponding to the ordinary wave in the chosen orientation of the AO crystal. A beam splitter BS splits the incident polarised radiation into two beams propagating in different directions, namely, parallel and perpendicular to the incident radiation direction. For the ideal beam splitter (without absorption), the condition $|E_{\rm o}^0|^2 = |(E_{\rm o}^0)_1|^2 + |(E_{\rm o}^0)_2|^2$ is satisfied, because of which the amplitudes of waves at the exit from the beam splitter satisfy the obvious relations

$$(E_{\rm o}^0)_1 = \gamma_{\rm bs}{\rm e}^{{\rm i}\varphi_{\rm bs}}E_{\rm o}^0, \quad (E_{\rm o}^0)_2 = (1-\gamma_{\rm bs}^2)^{1/2}{\rm e}^{{\rm i}\varphi_{\rm bs}'}E_{\rm o}^0.$$

Here, $\gamma_{\rm bs}$ is the beam splitter transmission efficiency and $\varphi_{\rm bs}$, $\varphi_{\rm bs}'$ are the phase jumps due to transmission through and reflection from the optical surfaces of the beam splitter. We will assume that, for the first feedback case, the direction denoted by subscript 1 corresponds to the initial incident radiation and the direction denoted by subscript 2 determines the feedback direction. Then, using the obvious recurrence relation for wave amplitudes at the entrance to the interaction region

$$(E_{\rm o}^0)_1^m = \gamma_{\rm bs}{\rm e}^{{\rm i}\varphi_{\rm bs}}(E_{\rm o}^0)_1 + (1-\gamma_{\rm bs}^2)^{1/2}{\rm e}^{{\rm i}\varphi+{\rm i}\varphi_{\rm bs}}(E_{\rm o}^0)_1^{m-1}S_{\rm oo},$$
$$m = 0,1,2,...,\ \ (E_{\rm o}^0)_1^{-1} = 0, \tag{10}$$

where integer number m determines the diffraction order, like in the case of a Fabry–Perot resonator [4], we obtain the total amplitude of the diffracted extraordinary wave in the form

$$E_{\rm e}\big|_{x=L} = \frac{\gamma_{\rm bs}{\rm e}^{{\rm i}\varphi_{\rm bs}}S_{\rm eo}E_{\rm o}^0}{1 - S_{\rm oo}{\rm e}^{{\rm i}\varphi+{\rm i}\varphi_{\rm bs}'}(1-\gamma_{\rm bs}^2)^{1/2}}. \tag{11}$$

Here φ is the total phase taking into account all possible jumps upon reflection from optical surfaces; it is also assumed that absorption in the feedback loop is absent, or, if absorption exists, phase φ will contain an imaginary part. The obtained expression (11) makes it possible to find the instrumental function (IF) of such an AO FP filter by the formula

$$T(\Delta k) \equiv \left|\frac{E_{\rm e}|_{x=L}}{E_{\rm o}^0}\right|^2 =$$
$$\frac{\gamma_{\rm bs}^2 T_0(\Delta k)}{\left|1-(1-\gamma_{\rm bs}^2)^{1/2}\exp\left(\frac{\rm i}{2}\Delta kL + {\rm i}\varphi - {\rm i}\Psi\right)[1-T_0(\Delta k)]^{1/2}\right|^2}, \tag{12}$$

where

$$T_0(\Delta k) \equiv \frac{\sin^2(\Gamma L\xi)}{\xi^2} \tag{13}$$

is the IF of a conventional AO filter without feedback and is the additional phase incursion due to the parametric coupling between the interacting light waves propagating in the crystal. When deriving (12), we assumed that phase jump at the beam splitter BS is absent. One can see from (12) that the IF of this AO FP filter is the product of the IF of a conventional AO filter [7] and the IF of a Fabry–Perot resonator [4] (with slight differences from the IF for a conventional Fabry–Perot resonator related to the fact that the optical feedback occurs as a result of unidirectional reflection of radiation to the entrance of the region of parametric interaction of light with an acoustic wave). At $\gamma_{\rm bs} = 1$, i.e., without feedback, formula (12) transforms to the IF of a conventional collinear AO filter. One can easily see that the maximum diffracted radiation intensity at the exit from the filter does not exceed unity at all T_0 values ranged from zero to unity. The phase incursion in the feedback loop φ in the simplest case can be expressed via the optical feedback path length l in the form $\varphi = kl \equiv (k_0 + \Delta k)l$, where k is the wavenumber of the optical radiation and $k_0 = q_{\rm s}/(n_{\rm o} - n_{\rm e})^{-1}$ is the wavenumber corresponding to the fulfilment of the phase-matching conditions.

Let us now consider the second case, when feedback occurs in the diffracted beam (see Fig. 1b). The recurrence relations between the wave amplitudes in this case will be obviously as follows:

$$(E_{\rm e}^0)^m = S_{\rm eo}(E_{\rm o}^0) + (1-\gamma_{\rm bs}^2)^{1/2}$$
$$\times\exp({\rm i}\varphi_{\rm bs} + {\rm i}\varphi(\omega))(E_{\rm e}^0)^{m-1}S_{\rm ee}, \tag{14}$$
$$m = 0,1,2,...,\ \ (E_e^0)^{-1} = 0,$$

where all values are related to the BS shown in Fig. 1b and the frequency dependence of phase $\varphi(\omega)$ emphasises the fact that the diffracted wave frequency changes due to the interaction with an acoustic wave. This slight change in frequency must be taken into account when considering laser radiation (below, we will not take this effect into account).

Using relation (14) and summing the terms of the geometrical progression, we can find the intensity of light fields inside the AO FP resonator as

$$|E_{\rm e}^{m\to\infty}|^2 = \frac{|E_{\rm o}^0 S_{\rm eo}|^2}{|1-(1-\gamma_{\rm bs}^2)^{1/2}\exp({\rm i}\varphi_{\rm bs} + {\rm i}\varphi(\omega))S_{\rm ee}|^2}. \tag{15}$$

The IF of this combined AO filter with feedback is determined by the light field intensity at the entrance to the photodetector and, as follows from (5), for the second case is

$$T(\Delta k) = \frac{\gamma_{\rm bs}^2 T_0(\Delta k)}{\left|1-(1-\gamma_{\rm bs}^2)^{1/2}\exp\left(\frac{\rm i}{2}\Delta kL + {\rm i}\varphi(\omega) + {\rm i}\Psi\right)[1 - T_0(\Delta k)]^{1/2}\right|^2}. \tag{16}$$

Instrumental functions (16) and (12) differ by the phase incursion sign for different feedback mechanisms and, in addition, in the latter case, the diffracted radiation frequency shifts by the acoustic wave frequency for each radiation trip over the feedback loop. One can clearly see that the maximum diffraction intensities of both IFs are identical, but the positions of the extrema are different. At a high AO filter efficiency, i.e., at $T_0 = 1$, the feedback ceases to affect the diffraction process due to the absence of radiation returned to the entrance. A similar situation is observed in the case of fulfilment of the condition $\gamma_{\rm bs} = 1$. At an optical path length satisfying the resonance conditions (multibeam interference)

$$\Delta kL/2 + \varphi(\omega) \pm \Psi(\Delta k) = 2\pi n, \quad n = 0, \pm 1, \pm 2, \ldots. \tag{17}$$

IFs (16) and (12) reach a maximum, which at $T_0(\Delta k) = \gamma_{\rm bs}^2$ becomes equal to unity, i.e., to the maximum possible value. This means that, in the case of continuous optical radiation incident on this combined AO FP filter, the energy of the polarised ordinary wave is completely transferred to the energy of the orthogonally polarised extraordinary wave. The line profile near the nth root of Eqn (17) can be represented in the form

$$T|_n = \frac{\gamma_{\rm bs}^2 T_0(\Delta k)}{(1-A)^2(1+\delta k_n^2/\sigma^2)}, \tag{18}$$

where $A \equiv (1-\gamma_{\rm bs}^2)^{1/2}[1 - T_0(\Delta k)]^{1/2}$; $\sigma \equiv (1/l)\sqrt{(1-A)^2/A}$ is the value determining the spectral width of the transmission window (or line) of the combined AO FP filter with feedback; and δk is the deviation from the transmission peak position, i.e., from the wavenumber satisfying Eqn (17). From formula (18), one can also see that the transmission window has a Lorentzian shape. At a 100% transfer of the incident radiation energy to the diffracted radiation, the spectral width of the transmission window at half maximum near each wave vector k_n determined by the root of Eqn (17) can be written in the form

$$\Delta k_n = 2\frac{\sqrt{2\gamma_{\rm bs}^4 - \sqrt{1-\gamma_{\rm bs}^2} + \gamma_{\rm bs}^2\sqrt{1-\gamma_{\rm bs}^2}}}{\sqrt{G(\gamma_{\rm bs}^2 + \sqrt{1-\gamma_{\rm bs}^2})}}, \tag{19}$$

where

$$G \equiv \frac{\partial^2}{\partial k^2} T_0\bigg|_{k=k_n}.$$

From Eqn (19), one can see that the spectral band with a 100% efficiency exists under the condition that $\gamma_{\rm bs} > 0.671$ and, hence, the required efficiency of the AO filter at $k = k_n$ should be $T_0 = 0.45$. If the last condition is not satisfied, then the minimum diffracted wave intensity exceeds 0.5, and the equation for determining the spectral width $T(k_n + \Delta k) = 0.5$ has no real roots. If the optical path length of the feedback loop at k_n^* close to k_n satisfies the condition

$$\Delta kL/2 + \varphi(\omega) \pm \Psi(\Delta k) = (2n+1)\pi,$$
$$n = 0, \pm 1, \pm 2, \ldots, \tag{20}$$

then the diffracted radiation intensity takes the minimum value

$$T|_{\rm min} = \frac{\gamma_{\rm bs}^2 T_0(\Delta k)}{|1 + (1-\gamma_{\rm bs}^2)^{1/2}[1 - T_0(\Delta k)]^{1/2}|^2}. \tag{21}$$

The $T_{\rm max}/T_{\rm min}$ ratio followed from formulae (18) and (21) determines the transmission window contrast, and, for $T_0(\Delta k) = \gamma_{\rm bs}^2$ is $\left(2-\gamma_{\rm bs}^2\right)^2/\gamma_{\rm bs}^4$, i.e., considerably exceeds unity.

The above formulae relate to the case when the radiation incident on the combined AO FP filter has a rather large spectral width considerably exceeding the spectral width of a conventional AO filter, and, therefore, there always exists such an incident wave that will satisfy the phase-matching conditions at any sound wave frequency. Mathematically, this means that boundary conditions (5) are satisfied for each spectral component of the incident radiation. If, in contrast, the incident laser radiation has a narrow bandwidth, then one can find the IF of a conventional AO by successively changing the acoustic wave frequency and measuring the diffraction efficiency. This is a frequently used approach. The natural question arises about the possibility of using the same method for measuring the IF of the combined AO FP filter. The answer is negative due to the existence of additional resonance conditions (17) and (20), due to which the diffracted radiation amplitude depends on one more resonance number p.

The problem of finding the amplitude of the diffracted wave of narrowband laser radiation in the case of multiple interference with allowance for the frequency shift at each diffraction circle turns out to be more complex than the above-considered case without taking the frequency shift into account. The problem is that, under the boundary conditions at $x = 0$, there exist waves with different frequencies and different amplitudes, and it is necessary for each of them to introduce its own scattering matrix $S_{\alpha\beta}$ with its own phase detuning $\Delta k^m = \Delta k + m\Omega_{\rm s}$. As a result, the recurrence relation of type (14) becomes more complex and does not reduce to a simple geometrical progression. Nevertheless, analysis shows that closed analytical formulae can be obtained in this case as well, but, due to the appearance of the new ‘quantum’ number p, which determines the resonance conditions and extends the range of possible diffraction conditions for each frequency, such analysis will be performed in a separate work.

The dependences of light field intensities on phase detuning Δk are shown in Figs 2 and 3 at different parameters of the combined AO FP filter. Figure 2 presents the dependence of IF (16) on phase detuning Δk at such a choice of the beam splitter parameters $\gamma_{\rm bs}$ and the conventional AO filter efficiency $T_0(\Delta k)$ that the condition for the highest diffracted beam intensity, i.e., $T_0(\Delta k) \approx \gamma_{\rm bs}^2$, is achieved near the point $\Delta k = 0$, because of which the most intense diffraction lines lie near the main maximum of the conventional AO filter. Figure 3 also shows the dependences of IF (16) on detuning Δk, but at such parameters that the condition $T_0(\Delta k) \approx \gamma_{\rm bs}^2$ is achieved at Δk corresponding to $T_0 = 0.04$. The latter shows that the existence of feedback considerably increases the diffraction efficiency. This makes it possible to use such combined AO FP filters for spectral measurements of radiation of weak sources or media with low absorption cross sections in separate spectral regions. Figure 4 shows the change in the position of the transmission window of the AO FP filter due to a change in the acoustic wave intensity. This feature of the AO FP filter opens additional possibilities not only for changing the frequency of the acoustic wave coupled into the crystal but also for controlling the transmission band of AO FP filters.

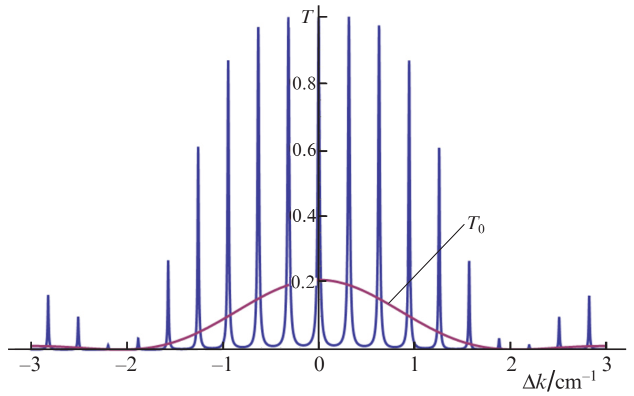

Figure 2. Dependence of diffracted radiation intensity T on phase detuning Δk corresponding to formula (12) at the following parameters: $l = 20$ cm, $\gamma_{bs} = 0.45$, $L = 3$ cm, $p = 0.3$, and $\Gamma = p\pi/2L$.

The above consideration and obtained formulae (12) and (16) related to the case when the coherence length of the incident radiation exceeds the optical feedback length, and the diffraction patterns shown in Figs 2–4 can be observed mainly for laser radiation. If the incident radiation has low coherence, i.e., the characteristic coherence length is smaller than the optical feedback loop length, then it is necessary to note the following. For a conventional AO spectrometer without feedback ($\gamma_{bs} = 1$), requirements for the incident radiation coherence are absent because the jump of the incident radiation phase drops from the final expression for the IF of the AO filter. The latter is directly seen from solution (6) provided that the optical train length exceeds the interaction length. If the latter is not fulfilled, it is necessary to use, instead of solution (6), the sum of solutions for several optical trains, and the interaction length for each train will be determined by the train length rather than by the crystal size. Mathematically this is similar to the problem of light diffraction on trains of acoustic waves [9, 10].

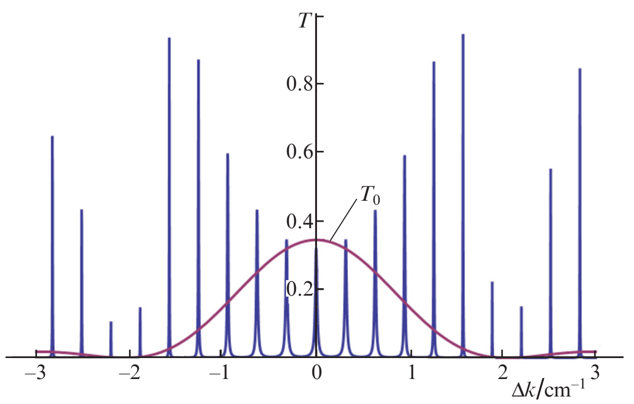

Figure 3. Dependence of diffracted radiation intensity T on phase detuning Δk corresponding to formula (16) at the following parameters: $l = 20$ cm, $\gamma_{bs} = 0.2$, $L = 3$ cm, $p = 0.4$, and $\Gamma = p\pi/2L$.

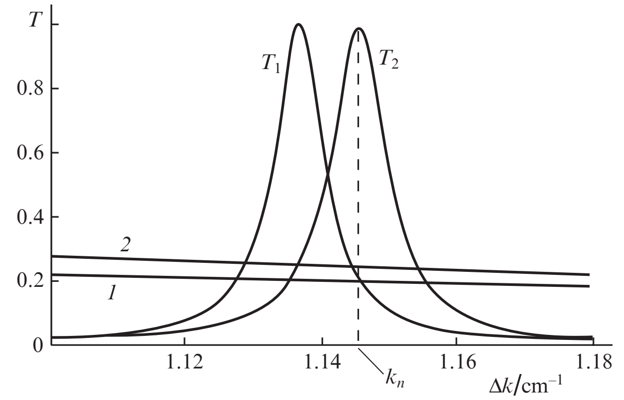

Figure 4. Absorption profiles of the AO FP filter according to instrumental function (16) at acoustic wave intensities $T_1 = T\,(T_0 = 0.22)$ and $T_2 = T\,(T_0 = 0.28)$ near the resonance condition $k = k_n$ (where $k = k_n$ is the root of Eqn (17) and Δk is the deviation from $k = k_n$) and dependences of the IF of a conventional AO filter corresponding to formula (13) at $p =$ (1) 0.6 and (2) 0.98. The other parameters are: $l = 20$ cm, $\gamma_{bs} = 0.45$, $L = 3$ cm, and $\Gamma = p\pi/2L$.

In contrast, the situation for diffraction of low-coherent radiation with feedback turns out to be much more complicated. In this case, the phase factor $e^{i\varphi}$ in expressions (10) and (14) contains the random additive s_{m-1} depending on number m, i.e., $\exp(i\varphi + is_{m-1})$, and the summation over all m with the use of the geometrical progression formula for deriving an expression for the IF becomes impossible. Then, it is necessary to restrict ourselves to a finite but rather large number of terms in the sum followed from recurrence relations (10) and (14) and then perform averaging over the values of the random additive, simultaneously introducing an assumption about the form of the random value distribution function.

Thus, the combined AO filter with feedback is able to electronically tune its optical transmission wavelength and simultaneously retain the possibility of high spectral resolution typical for FP resonators. It is also important that the feedback and multiple diffraction provide the possibility of achieving a higher diffraction efficiency with a relatively small spatial variation in the refractive index of the medium. Note also that any mechanism of changing the feedback efficiency, i.e., by changing the returned wave phase or amplitude, leads to modulation [9, 10] of the signal received by a photodetector, which opens the possibility of developing new, more precise methods of spectral measurements.

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