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Digital system for frequency regulation and stabilisation of a four-frequency Zeeman laser gyroscope

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Abstract. **We report the results of theoretical and experimental studies of a digital system for regulating and stabilising the frequency of a four-frequency Zeeman laser gyroscope (ZLG), in which generation occurs simultaneously on two longitudinal modes with orthogonal circular polarisations; therefore, four light waves propagate in the optical ring resonator. In contrast to a two-frequency Zeeman laser gyroscope, a four-frequency ZLG does not use the intensity modulation signal of one of the light waves, which appears when an alternating magnetic field is applied to the active medium, to analyse the frequency detuning of longitudinal modes from the centre of the active-medium gain contour. At the same time, it is possible to construct a digital perimeter adjustment system (PAS), which allows the resonator perimeter to be adjusted so that the magnetic components of the zero drift of the gyroscope for two longitudinal generation modes are equal and opposite in sign. Due to this, almost complete compensation for the magnetic component of the zero drift of the gyroscope is realised in a four-frequency ZLG. The use of the digital system makes it possible to provide a higher accuracy of the resonator perimeter adjustment than that in a two-frequency ZLG, even with a relatively small bit capacity of the PAS digital-to-analogue conversion (DAC), and to fully take advantage of the capabilities of a four-frequency ZLG to reduce the magnetic sensitivity.**

Keywords: *Zeeman laser gyroscope, four-frequency ring laser, frequency stability, resonator frequency detuning, resonator perimeter adjustment, alternating frequency bias, zero drift.*

1. Introduction

The frequency of linearly polarised laser gyroscopes is traditionally stabilised using frequency stabilisation systems based on forced modulation of the laser perimeter and the use of the resulting modulation of the output radiation intensity as a search signal [1]. The main disadvantages of such systems are the difference in frequencies of counterpropagating waves that occurs with changing the perimeter, which increases the noise at the output of the laser gyroscope and the system's sensitivity to mechanical vibrations at frequencies that are multiple and half-multiple the modulation frequency of the perimeter [2]. Frequency stabilisation systems are used in two-frequency Zeeman laser gyroscopes (ZLGs) with circular polarisation of radiation, in which the modulation signal of the output signal intensity is used as a search signal. This signal appears when an alternating magnetic field is applied to the active medium [3], which makes it possible to significantly increase the vibration stability of the gyroscope exposed to mechanical vibration [2, 4]. In both cases, the systems are analogous according to the principle of operation, with their inherent disadvantages.

Nazarenko et al. showed [5] that the sum of the frequencies of the Zeeman beats in the modes, taking into account their sign, can be used as a search signal in the four-frequency lasing regime to stabilise the frequency of a Zeeman ring He–Ne laser. This made it possible to do without perimeter modulation and to build a digital frequency stabilisation system. It was shown that in the four-frequency generation regime, frequency stabilisation with a short-term stability higher than 10^{-11} is possible, which is 2–3 orders of magnitude better than in the case of stabilisation by the Lamb dip.

The results of investigating the operation of a four-frequency ZLG with a perimeter adjustment system based on the principle proposed in [5] are presented in [6, 7]. In these works, empirical dependences of the output signals of a four-frequency ZLG were used to analyse the algorithms for processing the information of the four-frequency ZLG.

The purpose of this work is to theoretically and experimentally substantiate the principles of constructing a system for regulating and stabilising the frequency and perimeter of a four-frequency ZLG, as well as to compare the efficiency of this system with an analogue system of a two-frequency ZLG. On the basis of the semi-classical theory of a ZLG [8–10], we have for the first time obtained theoretical expressions for the discriminatory characteristic of the system for regulating the perimeter of a four-frequency ZLG with an alternating frequency bias and estimated the error in measuring the angular velocity caused by the errors of the perimeter adjustment system. Using the obtained expressions and the experimental data, additional criteria are formulated for choosing the operating point of frequency stabilisation of a four-frequency ZLG, and the error of stabilisation of the four-frequency ZLG perimeter, due to the discreteness of regulation, and the error in measuring the angular velocity of the four-frequency ZLG are estimated. A comparison is made with an analogue perimeter control system traditionally used in two-frequency ZLGs [3].

2. Operation principle of a digital frequency tuning system

The study of Zeeman beats in a four-frequency ring laser was performed in [9]. It is shown that near the symmetric tuning of

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the cavity relative to the centre of the gain curve, the frequency differences of counterpropagating waves of a two-mode ring laser arising due to the Zeeman effect in an active medium of a 50% mixture of isotopes Ne^{20} and Ne^{22}, to which a constant magnetic field is applied, are determined by the expression

$$f_n = (-1)^n \mu \frac{\Delta\nu\eta}{\eta_0} \mathrm{Re}\Big[Z'\Big(\frac{\sigma}{2}\Big) + Z'''\Big(\frac{\sigma}{2}\Big)\frac{\xi_n^2}{2}\Big], \qquad (1)$$

where $n = 1, 2$ is the mode number; $\mu = \mu_B H/(ku)$ is the relative Zeeman splitting of gain contours; H is the longitudinal component of the magnetic field in the active medium; μ_B is the Bohr magneton; k is the wave number; u is the average thermal velocity of atoms; $\Delta\nu$ is the resonator bandwidth; η is the excess gain over losses; $\eta_0 = \mathrm{Im}Z(\frac{\sigma}{2})$; Z is the dispersion function of plasma [11]; Z' and Z''' are the first and third derivatives of the plasma dispersion function; σ is the relative isotopic shift of neon isotopes; and ξ_n is the relative detuning of the frequency mode n from the maximum of the gain curve.

The relative mode frequency detuning can be expressed in terms of the average mode frequency detuning y and the intermode distance δ:

$$\xi_n = (y + (-1)^n\delta/2)/(ku). \qquad (2)$$

Here $\delta = \nu_2 - \nu_1$; $y = (\nu_1 + \nu_2)/2 - \nu_0$; ν_1 and ν_2 are the mode frequencies; and ν_0 is the frequency of the gain curve maximum. The imaginary part of the Z-function argument, which is equal to the relative homogeneous linewidth γ_{ab} of the laser transition a → b, is hereafter omitted for brevity.

When deriving expression (1), it was assumed that μ, $y/(ku)$, $\delta/(ku)$, and $\gamma_{ab}/(ku)$ are much less than 1, and the difference in the parameters ku for isotopes was not taken into account.

In paper [9], as in [5], a constant magnetic field was applied in the research. In practical designs of the ZLG, an alternating magnetic field in the form of a meander is used [12]. In this case, in the positive half-period of the meander, the magnetic field in the active medium can be represented in the form

$$H = H_0 + h, \qquad (3)$$

and in the negative half-period, in the form

$$H = -H_0 + h. \qquad (4)$$

Here H_0 is the alternating component of the magnetic field, and h is the constant component (external field).

In the case of an alternating magnetic field, it is necessary to consider the frequency differences in each half-period of the field change. In the quasi-stationary approximation, when the duration of the half-period of the magnetic field sign switching is much greater than the strength of the laser limit cycle, formula (1) can be used for the frequency difference of counterpropagating mode waves far from the switching front.

Further, it is convenient to write down the frequency differences for each of the modes taking into account the splitting due to rotation f_Ω, assuming it to be the same for different modes and introducing the following notations:

f_A^+ for the frequency difference of mode 1 in the positive half period;

f_A^- for the frequency difference of mode 1 in the negative half period;

f_B^+ for the frequency difference of mode 2 in the positive half period; and

f_B^- for the frequency difference of mode 2 in the negative half period.

Then,

$$f_A^+ = f_\Omega + f_1\,(H_0 + h), \qquad (5)$$

$$f_A^- = f_\Omega + f_1\,(-H_0 + h), \qquad (6)$$

$$f_B^+ = f_\Omega + f_2\,(H_0 + h), \qquad (7)$$

$$f_B^- = f_\Omega + f_2\,(-H_0 + h). \qquad (8)$$

Introducing the scale factor for the angular velocity (K_Ω) and the coefficients characterising the dependences of the frequency splitting on the magnetic field (K_H) and on the average frequency detuning of the modes y (K_y), we have

$$f_A^+ = K_\Omega\Omega + K_H(H_0 + h) - K_y y, \qquad (9)$$

$$f_B^+ = K_\Omega\Omega - K_H(H_0 + h) - K_y y, \qquad (10)$$

$$f_A^- = K_\Omega\Omega - K_H(H_0 - h) + K_y y, \qquad (11)$$

$$f_B^- = K_\Omega\Omega + K_H(H_0 - h) + K_y y. \qquad (12)$$

For the coefficients K_Ω, K_H and K_y we obtain the expressions:

$$K_\Omega = \frac{4|\boldsymbol{S}|}{\lambda L} \qquad (13)$$

($\boldsymbol{S}$ is the vector of the area of the optical contour of the resonator, and L is the laser perimeter);

$$K_H = 2\mu_B \frac{\Delta\nu}{ku}\frac{\eta}{\eta_0} \mathrm{Re}\,Z'\Big(\frac{\sigma}{2}\Big); \qquad (14)$$

$$K_y = 2\frac{\eta}{\eta_0}\frac{\mu_B}{ku}\frac{\Delta\nu}{ku}\frac{\delta}{ku} \mathrm{Re}\,Z'''\Big(\frac{\sigma}{2}\Big)H_0. \qquad (15)$$

In this approximation, K_H and K_y are independent of y, K_y is proportional to the magnetic field strength, and K_H is independent of the magnetic field; in this case, the coefficient K_y changes the sign with changing the polarisation of the modes.

From equations (9)–(12), assuming h to be small and expanding the function K_y into a Taylor series in h, we, limiting ourselves to the first term of the expansion, can extract information about the rotation, resonator frequency detuning, external magnetic field, and frequency bias magnetic field:

$$(f_A^+ + f_B^+) + (f_A^- + f_B^-) = 4K_\Omega\Omega + 4yh\frac{dK_y}{dH}, \qquad (16)$$

$$(f_A^+ + f_B^+) - (f_A^- + f_B^-) = -4K_y y, \qquad (17)$$

$$(f_A^+ - f_B^+) + (f_A^- - f_B^-) = 4K_H h, \qquad (18)$$

$$(f_{\rm A}^{+} - f_{\rm B}^{+}) - (f_{\rm A}^{-} - f_{\rm B}^{-}) = 4K_H H_0. \quad (19)$$

Equation (17) describes the sum of the Zeeman beat frequencies in orthogonally polarised modes and shows that it is proportional to the average frequency detuning of the modes y. This dependence is used as a discriminatory characteristic of the resonator frequency control system of a four-frequency laser gyroscope. The operating point of the frequency control system is chosen at point $y = 0$, at which, with the approximations made, the error due to resonator detuning is simply zero.

For comparison with experiment, it is convenient to represent the average detuning of the mode frequencies in terms of the corresponding change in its length δL, expressed in fractions of the wavelength:

$$y = -\frac{c}{L}\delta L, \quad (20)$$

where $\delta L = \Delta L/\lambda$; λ is the wavelength of the laser transition; and ΔL is the change in the length of the resonator relative to its resonant length L.

In this case, equations (16) and (17) should be rewritten as

$$(f_{\rm A}^{+} + f_{\rm B}^{+}) + (f_{\rm A}^{-} + f_{\rm B}^{-}) = 4K_{\Omega}\Omega + 4h\delta L\frac{{\rm d}K_L}{{\rm d}H}, \quad (21)$$

$$(f_{\rm A}^{+} + f_{\rm B}^{+}) - (f_{\rm A}^{-} + f_{\rm B}^{-}) = 4K_L\delta L, \quad (22)$$

where $K_L = \frac{c}{L}K_H$.

3. Arrangement of a four-frequency Zeeman laser gyroscope

The arrangement of a ZLG was considered in Refs [6, 7, 12, 13]. In such gyroscopes, the sensing element is a ring He–Ne laser with a nonplanar optical cavity, the modes of which are circularly polarised, and the modes with orthogonal circular polarisations are separated in frequency. In two-frequency ZLGs, the parameters of the resonator and the active medium are chosen so that lasing occurs only on one of the longitudinal modes. The application of a magnetic field to the gas-discharge gap leads, because of the Zeeman effect, to splitting of the gain contours for counterpropagating waves of this mode. As a result, the frequency of each of the counterpropagating waves is pulled to the centre of its gain contour. The resulting frequency difference is called the Zeeman frequency bias [12]. When constructing a gyroscope based on such a laser, it is not required to install a vibrator to remove it from the lock-in zone; the gyroscope has no moving parts, which increases its resistance to mechanical shock and vibration [13]. However, a two-frequency ZLG has a significantly higher sensitivity to magnetic fields, which limits its accuracy [14, 15]. In a four-frequency ZLG, the cavity parameters are chosen such that not one, but two longitudinal modes with orthogonal circular polarisations are generated, and, accordingly, there are four waves in the laser (two waves propagate clockwise and two propagate counterclockwise) [6, 12]. Since the Zeeman frequency bias has opposite signs in modes with orthogonal circular polarisations, by adding the frequency differences of counterpropagating waves obtained for orthogonal modes, it is possible to exclude the influence of the magnetic field in the output signal of the four-frequency ZLG and weaken the requirements for magnetic shielding of the ZLG [6].

Figure 1a illustrates the schematic of a nonplanar optical resonator given in [12], and Fig. 1b shows sets of eigenfrequencies of longitudinal modes of such a resonator for different angles of rotation of the field distribution and polarisation of the light wave ρ_{Σ},, depending on the bend angle of the optical contour α. The case $\rho_{\Sigma} = 0$ corresponds to a two-frequency ring laser with linear polarisation; $\rho_{\Sigma} = 90°$, to a two-frequency ZLG; and $\rho_{\Sigma} = 189°$, to a four-frequency ZLG.

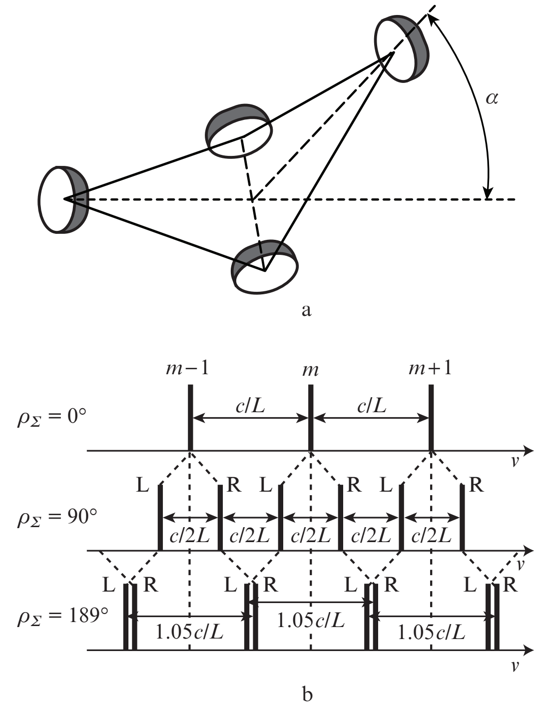

Figure 1. (a) Resonator of a ring laser with a nonplanar contour and (b) a set of eigenmodes of such a resonator as a function of the angle of rotation of the light wave polarisation ρ_{Σ}; L is the mode with left circular polarisation, and R is the mode with right circular polarisation.

For a laser gyroscope with linearly polarised light and a two-frequency ZLG, the task of the frequency stabilisation system is to tune the middle frequency of a mode to the centre of the gain curve, and for a four-frequency ZLG, to tune the centre frequency of a selected pair of orthogonal modes to the centre of the gain curve. The required arrangement of the generated modes with a longitudinal index m and two neighbouring modes with indices $m - 1$ and $m + 1$ in each of these cases is illustrated in Fig. 2, which shows the location of the longitudinal modes relative to the dependence of the excess gain over losses on the radiation frequency for traditionally used active media of a He–Ne laser: with a 50% mixture of $\rm Ne^{20}$ and $\rm Ne^{22}$ isotopes for a single-mode laser gyroscope with linearly polarised light (Fig. 2a) and a two-mode (four-frequency) ZLG (Fig. 2c), as well as with the $\rm Ne^{20}$ isotope for a two-frequency ZLG (Fig. 2b).

The half-width of the region of existence D of the corresponding generation regime of modes (mode) with one longitudinal index m depends on the excess of the gain over the losses at the maximum gain, but does not exceed $c/2L$ in the case of a laser with linearly polarised light, $c/4L - \mu_{\rm B}H$ in the case of a two-frequency ZLG and $c/2L - \mu_{\rm B}H$ in the case of a four-frequency ZLG.

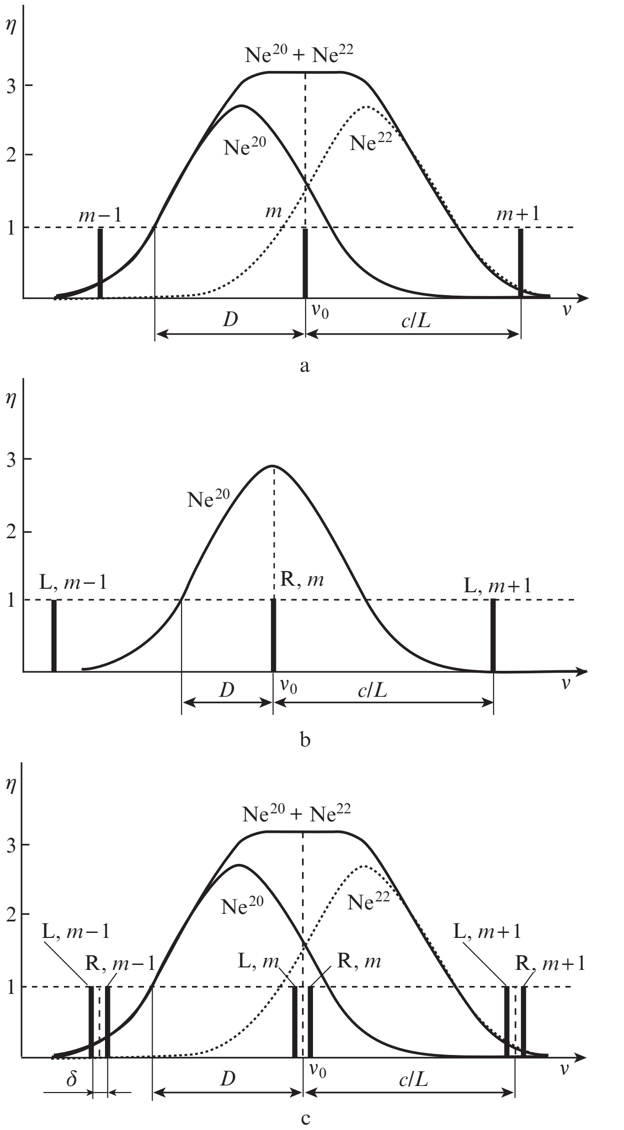

Figure 2. Location of the longitudinal modes of the resonator relative to the dependence of the excess of gain over losses on the generation frequency: (a) ring laser with linear polarisation; (b) single-mode (two-frequency) ZLG; (c) two-mode (four-frequency) ZLG.

Figure 3 shows the external view of a ring laser of a four-frequency ZLG, developed and manufactured at the Polyus Research Institute of M.F. Stelmakh and used to test the performance of the proposed frequency stabilisation system. The ring laser uses a four-mirror monoblock resonator with an asymmetric nonplanar optical contour. The resonator perimeter is 28 cm, and the angle of rotation of the polarisation plane is $\rho_\Sigma = 189°$. The asymmetry of the optical contour in the laser ensures a small angle (29°) between the beams incident on the output mirror and a relatively small distortion of the polarisation of the output light passing through the output mirror. The value of the period of the output signal of a laser gyroscope with such a resonator is 3″. The use of a symmetrical optical contour with angles of 29° between the beams incident on the mirrors and the resonator perimeter of 28 cm would lead to an increase in the value of the period by more than two times and, accordingly, to a deterioration of the gyroscope sensitivity, other things being equal (length and Q-factor of the resonator, gain, etc.).

Figure 3. External view of the ring laser resonator of a four-frequency ZLG.

The selectivity of the resonator ensures longitudinal mode generation at any tuning. The resonator is filled with a mixture of Ne^{20} and Ne^{22} isotopes. The laser is pumped by a dc discharge in two long channels with opposite current directions to compensate for the current drift. The discharge is formed between the hollow cathode shared by the gas-discharge gaps and two anodes. The frequency bias (the frequency difference between the counterpropagating waves in the laser) is produced with the help of solenoids wound on the working channels, due to the Zeeman effect [6, 12]. Membrane mirrors with piezoelectric drives are used to regulate and stabilise the perimeter. To extract information, a symmetric mixing prism is used, in which the opposite beams are mixed and then separated by modes using polarisation analysers. Interference patterns are recorded by slit photodetectors. To simplify alignment, sine and cosine signals are generated on different photodetectors. As a result, four sinusoidal signals are formed at the output of the photodetectors: two signals of mode A, shifted by 90° (hereinafter referred to as sinA and cosA), and two signals of mode B, shifted by 90° (sinB and cosB). Pulse signals of difference frequencies f_A^+, f_A^-, f_B^+, f_B^- are formed from these signals. Formation occurs according to a four-pulse logic: four pulses are formed at each signal period.

The scheme of a four-frequency ZLG is shown in Fig. 4. For the formation and maintenance of the discharge, a high-voltage ignition and pumping unit is used, which provides the positive voltage at the anodes necessary for ignition of the discharge and the negative voltage at the cathode required to maintain the discharge relative to the common bus ignition and pumping systems. An alternating bias is produced using a frequency bias unit (FBU). At the output of the ring laser photodetectors, sinusoidal beating signals of counterpropagating waves of orthogonal modes are formed. The resonator is tuned to the working pair of modes and the laser radiation frequency is stabilised by a digital perimeter adjustment system (PAS). It includes four-channel counters, a processor, a digital-to-analogue converter (DAC), a dc amplifier and an actuator, i.e. a membrane mirror with a piezoelectric drive. Four-channel counters and a processor are common to PAS and the system for generating the output information of the four-frequency ZLG. The discriminatory function of PAS is $f_y = (f_A^+ + f_B^+) - (f_A^- + f_B^-)$, which, as follows from equations (17) and (22), vanishes at the operating point. When tuned to this point, the angular velocity is determined by the formula $\Omega = [(f_A^+ + f_B^+) + (f_A^- + f_B^-)]/4K_\Omega$. The type of output information depends on the purpose of the laser gyroscope.

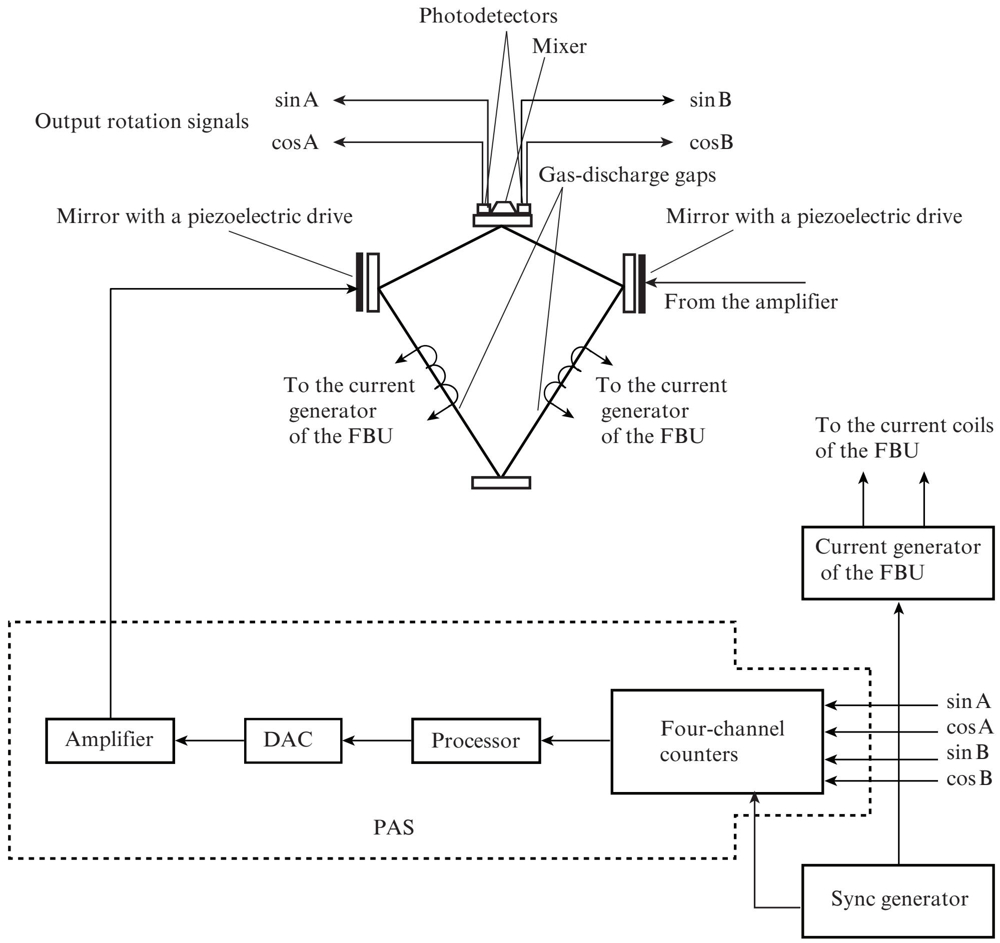

Figure 4. Schematic of a four-frequency ZLG.

Below we present the dependences of the amplitude of the Zeeman frequency bias for modes A and B (Fig. 5), the amplitudes of the beat signals sinA and sinB (Fig. 6), and the discriminatory characteristic, i.e. the sum of the frequencies of the Zeeman frequency bias in the modes (Fig. 7), on the adjustment of the resonator perimeter.

The analysis of Figs 5–7 leads us to conclude:

– there are two rows of resonator perimeter settings at which the amplitudes of the frequency bias of modes A and B are equal, and one row corresponds to the symmetric position of the modes of neighbouring longitudinal indices with an intermode distance of about 80 MHz with respect to the maximum gain, and the other to the symmetric position (relative

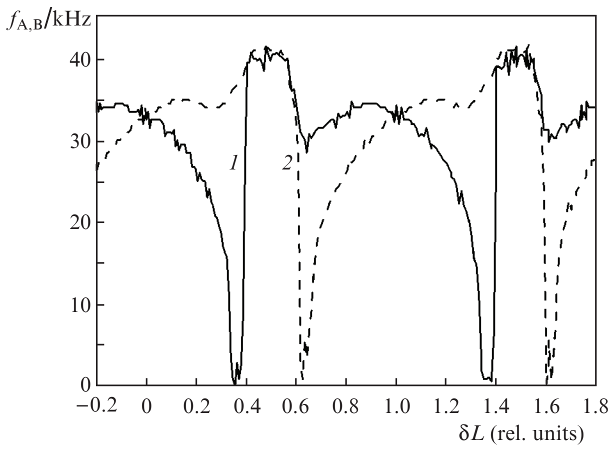

Figure 5. Dependences of the amplitudes of the Zeeman frequency bias of (*1*) A and (*2*) B modes on the adjustment of the resonator perimeter.

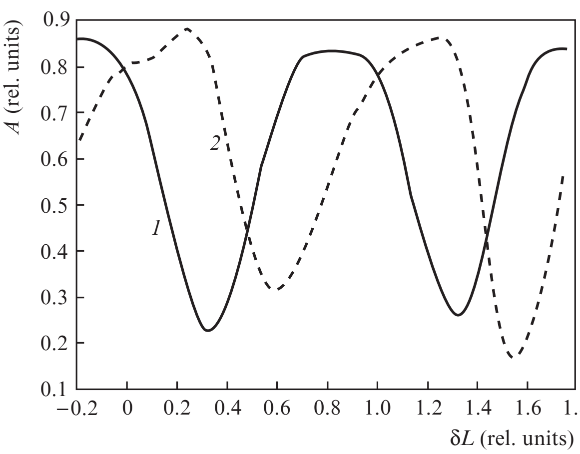

Figure 6. Dependences of the beat signal amplitudes (*1*) sinA and (*2*) sinB on the adjustment of the resonator perimeter.

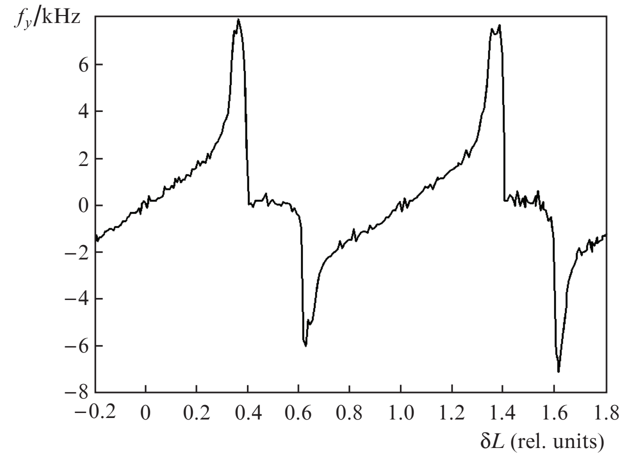

Figure 7. Dependence of the PAS discriminatory characteristics on the adjustment of the resonator perimeter.

to the maximum gain) of modes with longitudinal indices differing by 2, with an intermode spacing of about 1000 MHz;

– at the values of the first row settings, the intensities of the mode beat signals are greater and they are the same with the accuracy with which the gains of the amplifiers of the mode photodetectors are equalised;

– at the values of the second row settings, the intensities of the mode beat signals are less and, generally speaking, they are not the same, which is apparently due to the presence of waves with other indices in these cases;

– the region of existence of the four-frequency regime is much larger in the first case (by four times for a given excess of gain over losses); and

– the derivative of the discriminatory characteristic in the second case has the opposite sign.

Under these conditions, the operating point is selected from the first row of perimeter settings.

4. Analysis of theoretical results and experimental data

The error in determining the angular velocity $\Delta\Omega$ in the four-frequency regime directly depends on the accuracy of tuning to the operating point. Indeed, from equation (16) for the error $\Delta\Omega$ we have

$$\Delta\Omega = \frac{h}{K_\Omega}\frac{\mathrm{d}K_L(H)}{\mathrm{d}H}\delta L. \tag{23}$$

Tuning accuracy is determined by the DAC resolution and the noise of the counters that calculate the value of $(f_A^+ + f_B^+) - (f_A^- + f_B^-)$. It follows from equation (22) that if the errors in measuring the beat frequencies of counterpropagating waves f_A^+, f_A^-, f_B^+, f_B^- are assumed to be random and independent, then the error δL in tuning PAS can be calculated by the formula

$$\delta L = \frac{\sqrt{T}\,\Delta f}{2\sqrt{2\tau}\,K_L(H)}. \tag{24}$$

Here Δf is the error (noise amplitude) in measuring the beat frequency of counterpropagating waves for one longitudinal mode in each half-period of the alternating frequency bias; T is the period of the alternating frequency bias; and τ is the PAS time constant. For Zeeman laser gyroscopes, the measurement error of the beat frequency is determined mainly by the discretisation of the output information, i.e., taking into account the fact that four pulses of output information are formed during one beat period, $\Delta f = 1/8T$. As follows from formula (24), the error of the PAS adjustment can be decreased by increasing τ, as well as using digital filtering. The limitation on τ is related to the time constant τ_T of thermal heating (expansion) of the four-frequency ZLG resonator. For the stable operation of PAS, τ should be much less than τ_T. In our case, the constant τ can exceed 1000 s, $K_L(H) \approx 5000$ Hz (see Fig. 7), while the estimate by formula (25) gives $\delta L < 0.0001$.

The range of voltage variation on the piezoelectric drives of the mirrors is selected from the condition of ensuring the operation of the four-frequency ZLG on one pair of orthogonally polarised longitudinal modes with a possible change in the ambient temperature in a given range (about 100 °C). Considering that self-heating of the four-frequency ZLG can reach 15 °C, the full range of required voltages must correspond to a change in the perimeter by 6λ. The used DAC has a 14-bit output, i.e. the step of changing the voltage on the piezoelectric drives of the mirrors must correspond to the change in the perimeter by $6\lambda/16384$. In this case, the maximum stabilisation error is equal to 1/2 of the DAC step, i.e., it does not exceed 0.00018λ; in this case, it depends little on the level of the amplitude noise of the ZLG output signal, in contrast to analogue systems for stabilising the perimeter of ring lasers. For example, in a two-frequency ZLG, the detuning value is determined by the signal of synchronous noise induced on the PAS photodetector by an electric current in the ZLG nonreciprocity coils. With a pickup signal of 0.005 V, a useful signal of 1.5 V, and a perimeter detuning of 0.1λ, the perimeter stabilisation accuracy will be no more than 0.003λ, which is 16 times inferior to the digital PAS.

The error in stabilising the radiation frequency of a four-frequency ring laser, estimated from the stability of the resonator perimeter, is 0.019 Hz. Of course, in this case, we are not talking about the absolute stability of the laser radiation frequency, but about the accuracy of the locking to the zero position of the frequency difference of the Zeeman beats, which depends both on the parameters of the active medium (first of all, the ratio of neon isotopes) and on the perimeter parameters (first of all, Q-factor difference of modes) [14]. From the point of view of designing a four-frequency ZLG, the absolute frequency stability at which the difference in the frequencies of the Zeeman beats vanishes is not important for the approximations made, since the error in measuring the angular velocity at this point is minimal regardless of the absolute value of the average laser radiation frequency corresponding to this point.

Let us estimate the error in measuring the angular velocity by a four-frequency ZLG due to the residual field of the magnetic shields in the presence of a perimeter detuning caused by the discreteness of the stabilisation. In our case, $\mathrm{d}K_L(H)/\mathrm{d}H \approx 500$ Hz Oe^{-1}, the Earth's residual magnetic field h does not exceed 0.000016 Oe when using a three-layer magnetic shield with screening coefficients no worse than 40 for each layer. The typical residual magnetic field of the screens is 0.0002 Oe, and the scale factor is $K_\Omega = 0.33$ arcsec^{-1}. Then the error in determining the angular velocity $\Delta\Omega$ will not exceed 0.0006 arcsec s^{-1}, which is significantly less than in two-frequency ZLGs, and is negligible compared to other sources of errors in Zeeman laser gyroscopes, in which the error is up to 0.03 arcsec s^{-1} [16–18].

5. Conclusions

A four-frequency ZLG, in comparison with a single-mode (two-frequency) ZLG, is a more complex optical device, which is associated, first of all, with the need for polarisation separation of counterpropagating light waves with opposite directions of circular polarisations. This is what makes one introduce asymmetry into the optical contour of the resonator and use a complex photomixing device. This leads to complication not only of the design, but also of the laser manufacturing technology, since the requirements for the accuracy of manufacturing parts remain the same. However, the modern development of mechanical engineering makes allows one to solve these problems at an acceptable level of cost. At the same time, a four-frequency ZLG is a fully digital laser gyroscope, which has a number of other advantages over a two-frequency ZLG (first of all, much lower magnetic sensitivity).

In a four-frequency ZLG with an active gas mixture containing two isotopes Ne^{20} and Ne^{22}, it is possible to implement an accurate digital perimeter adjustment system that automatically selects the operating point at which the magnetic component of the zero drift of a four-frequency ZLG is zeroed. In the case when the static detuning of the PAS, due to the discreteness of control with a 14-bit DAC, does not exceed 0.00018λ, the error in measuring the angular velocity due to the external magnetic field and the residual field of the screens is no more than 0.001 deg h^{-1}. In a two-frequency ZLG, to reduce the magnetic sensitivity to such a level, it is necessary to resort to switching modes with orthogonal polarisations, which significantly reduces the ability to manoeuvre the objects on which it is installed and leads to an increase in the random component of the measurement error.

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