Interference caused by electromagnetic interactions

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Abstract. **We consider interference fluxes of energy, momentum and angular momentum, describing the interaction of electric charges with constant and variable electromagnetic fields. Absorption and stimulated emission of atoms in the field of free and evanescent waves are described. It is shown that in the case of interaction with reactive fields, the momentum of an atom during absorption and stimulated emission does not change. A wave derivation of Planck's formula based on the interference of the fields of classical thermal radiation and quantum zero-point oscillations is presented.**

1. Introduction

In classical electrodynamics, conservation laws for a system of electric charges in an external electromagnetic field are formulated using energy, momentum and angular momentum fluxes passing through a closed surface surrounding this system. The fluxes that change the dynamic characteristics of the charges are interference ones, since they depend both on the external field and on the field of the charges themselves. The properties of such interference fluxes (IFs) arising in the total field can differ markedly from the properties of fluxes in original interfering fields.

The IF calculations presented in this work show that they can be used to describe such diverse physical phenomena as the Coulomb interaction of charges, the exchange of energy, momentum and angular momentum between charges and a constant electric field, the lateral shift of a light beam during its total internal reflection, direction of elementary acts of interaction of atoms with fields of free and evanescent waves, nonradiative energy transfer between atoms, and absorption and stimulated emission of atoms without changing their momentum.

To calculate the IFs, it is necessary to use all the elements of the energy-momentum tensor of the electromagnetic field, which significantly expands the concept of traditional phenomena of second-order interference in the field and allows one to interpret interference as a mechanism for the formation of a new quality in linear systems.

2. Complete set of quantities required to describe interference

Let a superposition of two harmonic waves with given same frequency electromagnetic fields E_1, H_1 and E_2, H_2 forming a

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certain interference pattern be produced in a vacuum. Here $E_{1,2}$ and $H_{1,2}$ are the vectors of the intensity of the electric and magnetic fields, respectively. Let us introduce the oscillation period-averaged interference densities of energy,

$$w^{\text{int}} = \frac{1}{8\pi} \text{Re}(E_1 E_2^* + H_1 H_2^*), \tag{1}$$

momentum,

$$\boldsymbol{p}^{\text{int}} = \frac{1}{8\pi c} \text{Re}([E_1 H_2^*] + [E_2 H_1^*]), \tag{2}$$

and angular momentum,

$$\boldsymbol{m}^{\text{int}} = [\boldsymbol{r}\boldsymbol{p}^{\text{int}}]. \tag{3}$$

Here c is the speed of light in vacuum, and $\boldsymbol{r}$ is the radius vector drawn from an arbitrary point to the observation point.

According to Maxwell's equations, when use is made of quasi-monochromatic waves, the first time derivatives t of quantities (1)–(3) in a region of volume V bounded by a closed surface F have the form

$$\frac{\text{d}}{\text{d}t} \int_V w^{\text{int}} \text{d}V = -\oint_F S_i^{\text{int}} \text{d}F_i, \tag{4}$$

$$\frac{\text{d}}{\text{d}t} \int_V p_i^{\text{int}} \text{d}V = -\oint_F (-\sigma_{ik}^{\text{int}}) \text{d}F_k, \tag{5}$$

$$\frac{\text{d}}{\text{d}t} \int_V m_i^{\text{int}} \text{d}V = -\oint_F e_{ijk} r_j (-\sigma_{kl}^{\text{int}}) \text{d}F_l, \tag{6}$$

where $\text{d}F_i = \text{d}F n_i$; $\text{d}F$ is the area of an infinitesimal element of the surface; $\boldsymbol{n}$ is the unit outward normal vector to the surface; e_{ijk} is a unit antisymmetric tensor of the third rank; and $i, j, k, l = x, y, z$. Summation is performed over the repeated indices. On the right-hand sides of (4)–(6) with the 'minus' sign, there are IFs of energy (4), momentum (5) and angular momentum (6), which are determined using the time-averaged Poynting interference vector

$$\boldsymbol{S}^{\text{int}} = \frac{c}{8\pi} \text{Re}([E_1 H_2^*] + [E_2 H_1^*]) \tag{7}$$

and interference Maxwell's stress tensor

$$\sigma_{ik}^{\text{int}} = \frac{1}{8\pi} \text{Re}[E_{1i} E_{2k}^* + E_{2i} E_{1k}^* + H_{1i} H_{2k}^*$$

$$+ H_{2i} H_{1k}^* - \delta_{ik} (E_1 E_2^* + H_1 H_2^*)], \tag{8}$$

where δ_{ik} is the Kronecker symbol. Equations (4)–(6) describe the dynamics of the spatial redistribution of energy, momen-

tum and angular momentum of interfering fields, which is determined by the corresponding IF. For stationary fields in vacuum, all IFs through an arbitrary closed surface are equal to zero. If the area of the spatial overlap of waves is limited, then the lines of stationary IFs are closed.

Expressions (1)–(8) form a complete set of quantities quadratic in the field, sufficient to describe all possible second-order interference phenomena [1]. They contain terms of the form $E_{1i}E_{2j}^*$, $H_{1i}H_{2j}^*$ and $E_{1i}H_{2j}^*$, which determine the interference between different components of not only electric or magnetic fields, but also between different components of electric and magnetic fields. Due to this, the interference is not limited only to the spatial redistribution of energy, momentum and angular momentum.

As an example, we consider the superposition of two evanescent TE and TM waves propagating along the x axis and exponentially decaying along the y axis:

$$\boldsymbol{E}_1 = (0, 0, A_1)\exp[-hy + \mathrm{i}(k_x x - \omega t)],$$
$$\boldsymbol{H}_1 = \left(\mathrm{i}\frac{h}{k}A_1, -\frac{k_x}{k}A_1, 0\right)\exp[-hy + \mathrm{i}(k_x x - \omega t)], \tag{9}$$

$$\boldsymbol{E}_2 = \left(-\mathrm{i}\frac{h}{k}A_2, \frac{k_x}{k}A_2, 0\right)\exp[-hy + \mathrm{i}(k_x x - \omega t)],$$
$$\boldsymbol{H}_2 = (0, 0, A_2)\exp[-hy + \mathrm{i}(k_x x - \omega t)], \tag{10}$$

where A_1 and A_2 are the complex amplitudes; $k_x > k = \omega/c$; ω is the frequency of the waves; and $h = \sqrt{k_x^2 - k^2}$. The equality $\boldsymbol{E}_1\boldsymbol{E}_2 = \boldsymbol{H}_1\boldsymbol{H}_2 = 0$ holds true for waves (9) and (10); therefore, they cannot form an ordinary interference pattern.

In this case, there is interference between the components of the electric and magnetic fields of waves (9) and (10). This leads to the formation of an IF of energy in the direction of the z axis, which is described by a nonzero interference component of the Poynting vector of the total field [2]

$$S_z^{\mathrm{int}} = \frac{ck_x h}{4\pi k^2}|A_1\|A_2|\exp(-2hy)\sin(\varphi_1 - \varphi_2), \tag{11}$$

if the phase difference $\varphi_1 - \varphi_2 \neq \pi n$ ($n = 0, \pm 1, \pm 2, ...$) for the quantities A_1 and A_2. Note that the fluxes of energy transferred by waves (9) and (10) separately are directed along the x axis and are additive. Consequently, the formation of an IF of energy along the z axis cannot be considered as a result of the spatial redistribution of energy fluxes of wave (9) and (10).

A superposition of evanescent waves (9) and (10) is formed in a vacuum under total internal reflection of an elliptically polarised light beam, which is incident from a transparent medium with a refractive index greater than unity onto the flat surface $y = 0$ of the medium–vacuum interface occupying the half-space $y > 0$. The IF of energy is perpendicular to the xy plane of incidence of the beam and leads to a lateral shift of the reflected beam, which, as a result, leaves the plane of incidence [2].

3. IFs in constant electric fields

The presence of electric charges radically changes the properties of the IFs, for which these charges become either sources or drains. Consider an electric field in vacuum produced by two stationary point charges q_1 and q_2,

$$\boldsymbol{E} = \boldsymbol{E}_1 + \boldsymbol{E}_2 = q_1\frac{\boldsymbol{r}_1}{r_1^3} + q_2\frac{\boldsymbol{r}_2}{r_2^3}. \tag{12}$$

Here $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ are radius vectors drawn from charges q_1 and q_2 to the observation point, respectively. For a field of a single charge, the momentum density and momentum flux through an arbitrary closed surface surrounding the charge are equal to zero.

For field (12), the momentum density is still equal to zero, but there arises an IF of the momentum, which describes the Coulomb interaction of charges. If charges q_1 and q_2 are located on the x axis at the points $x_1 = -l/2$ and $x_2 = l/2$, then the IF of the momentum p_x to charge q_2 through the plane $x = 0$ has the form

$$I_{p_x}^{\mathrm{int}} = \iint_{-\infty}^{\infty} \sigma_{xx}^{\mathrm{int}}(0)\,\mathrm{d}y\mathrm{d}z$$
$$= -\frac{q_1 q_2}{4\pi}\iint_{-\infty}^{\infty}\frac{\mathrm{d}y\mathrm{d}z}{(l^2/4 + y^2 + z^2)^2} = -\frac{q_1 q_2}{l^2} \tag{13}$$

and is equal to zero for any plane $x = \mathrm{const} > l/2$. Therefore, the IF of the momentum p_x through an arbitrary closed surface surrounding charge q_2 is equal to (13). It determines the Coulomb force (taken with the opposite sign) acting on charge q_2, which becomes the drain for the IF of the momentum p_x.

The interference component of the energy W^{int} is equal to the potential energy of the Coulomb interaction of two point charges q_1 and q_2 in vacuum [1]:

$$W^{\mathrm{int}} = \frac{1}{4\pi}\int \boldsymbol{E}_1\boldsymbol{E}_2\,\mathrm{d}V$$
$$= \frac{q_1 q_2}{8\pi}\iiint_{-\infty}^{\infty}\frac{r_1^2 + r_2^2 - l^2}{r_1^3 r_2^3}\,\mathrm{d}x\mathrm{d}y\mathrm{d}z = \frac{q_1 q_2}{l}. \tag{14}$$

When charges move along the x axis under the action of Coulomb forces, as can be seen from (13) and (14), the increment in their kinetic energy is equal to the decrease in the interference component of the total electric field energy (14).

A moving charge produces a magnetic field, with the help of which an IF of energy arises, describing a change in the interference component of energy [3]. Let there be a charged empty capacitor formed by two infinite metal plates located in the planes $x_1 = -l/2$ and $x_2 = l/2$. A point charge q moves along the x axis under the action of the capacitor's electric field $\boldsymbol{E} = (E, 0, 0)$. The interference component of the energy of the total field of the capacitor and the charge at the charge rate $v_q \ll c$ has the form

$$W^{\mathrm{int}} = \frac{1}{4\pi}\int \boldsymbol{E}\boldsymbol{E}_q\,\mathrm{d}V$$
$$= \frac{qE}{4\pi}\int_{-l/2}^{l/2}\mathrm{d}x\iint_{-\infty}^{\infty}\frac{(x - x_q)\mathrm{d}y\mathrm{d}z}{[(x - x_q)^2 + y^2 + z^2]^{3/2}} = -qEx_q, \tag{15}$$

where $\boldsymbol{E}_q$ and x_q are the electric field and the coordinate of the charge, respectively [1]. Quantity (15) is the potential energy of the charge q in the constant electric field of the capacitor.

The IF of the momentum p_x to charge q through two planes $x = \pm l/2$ is described by the expression

$$I_{p_x}^{\text{int}} = -2 \iint_{-\infty}^{\infty} \sigma_{xx}^{\text{int}}(l/2)\,\mathrm{d}y\,\mathrm{d}z,$$

$$= -\frac{qEl}{4\pi} \iint_{-\infty}^{\infty} \frac{\mathrm{d}y\,\mathrm{d}z}{r^3} = -qE, \qquad (16)$$

where $r = \sqrt{l^2/4 + y^2 + z^2}$ is equal to the IF of the momentum p_x through an arbitrary closed surface surrounding the charge inside the capacitor, and the force taken with the opposite sign acting on the charge [1]. Hence it follows that the charge is a drain for the IF of the momentum p_x. Note that the momentum density for the total electric field inside the capacitor is zero. The IF of the momentum p_x to the capacitor plates is equal to qE, and the momentum is exchanged between the charged capacitor plates and the charge q.

The kinetic energy ε_q of the charge as it moves along the x axis under the action of the electric field of the capacitor increases due to a decrease in the interference energy (15) of the total electric field. Differentiating W^{int} with respect to time, we obtain

$$-\frac{\mathrm{d}W^{\text{int}}}{\mathrm{d}t} = qEv_q = \frac{\mathrm{d}\varepsilon_q}{\mathrm{d}t}. \qquad (17)$$

If the charge q is surrounded in a capacitor by an arbitrary closed surface, the increment of its kinetic energy is determined by two factors: a decrease in the interference energy inside the selected area and the IF of energy, which describes a decrease in the interference energy outside the selected area [3].

Let us assume that inside the capacitor on the x axis at point $x = 0$ there is a constant electric dipole with the moment $\boldsymbol{d}$, lying in the xz plane. The IF of the angular momentum M_y relative to the y axis for the total electric field of a point dipole and a capacitor to the dipole through two planes $x = \pm l/2$ [1] has the form

$$I_{M_y}^{\text{int}} = -2 \iint_{-\infty}^{\infty} z\sigma_{xx}^{\text{int}}(l/2)\,\mathrm{d}y\,\mathrm{d}z$$

$$= -\frac{3Edl\sin\varphi}{4\pi} \iint_{-\infty}^{\infty} \frac{z^2}{r^5}\mathrm{d}y\,\mathrm{d}z = -Ed\sin\varphi, \qquad (18)$$

where φ is the angle between the x axis and the vector $\boldsymbol{d}$, and $r = \sqrt{l^2/4 + y^2 + z^2}$. The IF of the angular momentum through an arbitrary closed surface bounding the dipole is (18). It determines the moment of forces, taken with the opposite sign, acting on the dipole, which is the drain for the IF of the angular momentum M_y.

The density of the angular momentum for the total field in the capacitor is zero and, therefore, the exchange of the angular momentum is carried out between the dipole and the charged plates of the capacitor. Thus, constant electric fields in a capacitor with zero momentum and angular

momentum densities nevertheless produce interference fluxes of momentum and angular momentum.

The interference component of the electric field energy in a capacitor

$$W^{\text{int}} = \frac{1}{4\pi} \int \boldsymbol{E}\boldsymbol{E}_d\,\mathrm{d}V$$

$$= \frac{Ed\cos\varphi}{4\pi} \iint_{-\infty}^{\infty} \mathrm{d}y\,\mathrm{d}z \int_{-l/2}^{l/2} \frac{3x^2 - r^2}{r^5}\,\mathrm{d}x$$

$$= -Ed\cos\varphi, \qquad (19)$$

where $\boldsymbol{E}_d$ is the electric field of the dipole, is the potential energy of the dipole in the electric field of the capacitor. If the dipole can freely rotate under the action of the moment of forces M_{dy} around the y axis, then the increment of its kinetic energy, as seen from (18) and (19), is equal to the decrease in the energy W^{int}.

The examples considered show that the formation and change in the interference pattern for constant electric fields are associated either with the work on the movement of charges, or with the mutual transformation of the energy of the electric field and the kinetic energy of the charges.

4. Interference in the processes of absorption and stimulated emission of an atom

In 1972, the amplification of light was experimentally observed when it is totally reflected from a medium with an inverted population of atoms [4]. In this regard, it generated a need for generalising the laws of stimulated emission of an atom for the case of its interaction with the field of evanescent waves (9) and (10). The solution of this problem turned out to be possible in the framework of classical electrodynamics with the help of IFs of energy and momentum without quantising evanescent waves [5, 6].

Let us first consider the incidence of a plane monochromatic wave with frequency ω, which propagates in vacuum along the x axis and is polarised along the z axis, on an atom. The wave field

$$\boldsymbol{E} = (0, 0, A)\exp[\mathrm{i}(kx - \omega t)],$$

$$\boldsymbol{H} = (0, -A, 0)\exp[\mathrm{i}(kx - \omega t)], \qquad (20)$$

where A is the complex amplitude, induces the electric dipole moment

$$\boldsymbol{d} = \alpha(\omega)\boldsymbol{E} = \boldsymbol{d}_0\exp(-\mathrm{i}\omega t), \qquad (21)$$

in an atom located at a point with coordinates $x = y = z = 0$, where $\alpha(\omega) = \alpha'(\omega) + \mathrm{i}\alpha''(\omega)$ is the polarisability of the atom; $\alpha'(\omega)$ and $\alpha''(\omega)$ are real functions of frequency ω; and $d_0 = \alpha(\omega)A$ is the complex amplitude of dipole oscillations along the z axis.

In a spherical coordinate system, the field of a point dipole (21) is represented in the form:

$$E_{dr} = 2\cos\theta\left(\frac{1}{k^2r^2} - \frac{\mathrm{i}}{kr}\right)\frac{k^2d_0}{r}\exp[\mathrm{i}(kr - \omega t)], \qquad (22)$$

$$E_{d\theta} = \sin\theta\left(\frac{1}{k^2 r^2} - \frac{i}{kr} - 1\right)\frac{k^2 d_0}{r}\exp[i(kr - \omega t)], \tag{23}$$

$$H_{d\varphi} = \sin\theta\left(-\frac{i}{kr} - 1\right)\frac{k^2 d_0}{r}\exp[i(kr - \omega t)]. \tag{24}$$

Here $E_{d\varphi} = H_{dr} = H_{d\theta} = 0$; r is the radius vector drawn from the dipole to the observation point; θ is the angle between the z axis and the vector r; and φ is the angle between the x axis and the projection of the vector r onto the xy plane.

The wave-field and dipole-field produced IF of energy, arriving at the atom through the plane $x = x_1$, is

$$I_w^{\text{int}}(x_1) = \text{Re}\iint_{-\infty}^{\infty} S_x^{\text{int}}\,dy\,dz$$

$$= \frac{c}{8\pi}\text{Re}\iint_{-\infty}^{\infty}(E_z H_{dy}^* + E_{dz}H_y^*)\,dy\,dz$$

$$= -\frac{1}{2}\,\alpha''\omega|A|^2, \tag{25}$$

if $x_1 > 0$, and equal to zero if $x_1 < 0$ [5]. For an absorbing atom, $\alpha'' > 0$, and the negative IF of energy (25) is subtracted from the wave energy flux (20). In the case of an excited atom, $\alpha'' < 0$, and a positive IF of energy (25) describing the induced emission of the atom is added to the wave energy flux (20).

The IF of energy through an arbitrary closed surface bounding an atom is equal to (25); therefore, an absorbing atom is a drain for this flux, and an excited atom is a source. Such 'focusing' of the IF of energy to the point of location of the atom is due to the unlimited growth of the field of the point dipole at $r \to 0$.

For a closed surface of any area, the ratio of the IF of energy (25) for the absorbing atom to the energy flux density of the incident wave (20), taken with the opposite sign, is the effective absorption cross section of the atom at frequency ω, regardless of the size of the atom [7]. In the general case, the effective absorption cross section does not determine the area of the main localisation of the IP of energy on a closed surface, which depends on the shape and size of this surface.

The IF of momentum p_x arriving at the atom through the plane $x = x_1$ is

$$I_{p_x}^{\text{int}}(x_1) = -\iint_{-\infty}^{\infty}\sigma_{xx}^{\text{int}}\,dy\,dz$$

$$= \frac{1}{8\pi}\iint_{-\infty}^{\infty}(E_z E_{dz}^* + H_y H_{dy}^*)\,dy\,dz = -\frac{1}{2}\alpha''k|A|^2, \tag{26}$$

if $x_1 > 0$, and equal to zero if $x_1 < 0$ [6]. The absorbing atom receives the momentum $p_x > 0$, and the excited atom under stimulated emission receives the momentum $p_x < 0$. Therefore, for $I_{p_x}^{\text{int}}$, the absorbing atom is a drain, and the excited atom is a source.

Expressions (20), (25) and (26) yield the equalities

$$\frac{I_w^{\text{int}}}{I_{p_x}^{\text{int}}} = \frac{S_x}{-\sigma_{xx}} = \frac{\hbar\omega}{\hbar k} = c, \tag{27}$$

where S_x and $-\sigma_{xx}$ are, respectively, the flux densities of energy and momentum p_x in the direction of the x axis for

wave (20); and $\hbar\omega$ and $\hbar k$ are, respectively, the photon energy and momentum of wave (20). Thus, the description of the spatial directionality of the processes of absorption and stimulated emission of an atom by the methods of classical electrodynamics is consistent with the results of quantum theory.

Let us consider the interaction of an atom with the field of an evanescent wave (9), which arises during total internal reflection on the surface $y = 0$ of the vacuum ($y > 0$) – transparent medium ($y < 0$) interface. Let an atom be located at a point with coordinates $x = 0$, $y = l$, $z = 0$. The IF of energy, momentum p_x, parallel to the interface between two media, and momentum p_y, perpendicular to the interface between two media, through the plane $y = y_1$ have the form

$$I_w^{\text{int}} = -\frac{1}{2}\alpha''\omega|A|^2\exp(-2hl), \tag{28}$$

$$I_{p_x}^{\text{int}} = -\frac{1}{2}\alpha''k_x|A|^2\exp(-2hl), \tag{29}$$

$$I_{p_y}^{\text{int}} = \frac{1}{2}\alpha'h|A|^2\exp(-2hl), \tag{30}$$

if $0 < y_1 < l$, and are equal to zero if $y_1 > l$ [5, 6].

For an absorbing atom, the IF of energy is directed along the y axis from the interface between two media to the atom receiving energy from the wave incident on this surface. For an excited atom, the IF of energy is directed along the y axis from the atom to the interface between two media and amplifies the reflected wave due to the stimulated emission of the atom. In contrast to the case of wave (20), the energy is now transferred perpendicular to the direction of propagation of the evanescent wave and the transferred momentum p_x.

The IF of momentum p_y is independent of α'' and determines the time-averaged gradient force acting on the induced dipole in an alternating electromagnetic field, the amplitude of which depends on the coordinates [8].

According to (9), (28) and (29),

$$\frac{I_w^{\text{int}}}{I_{p_x}^{\text{int}}} = \frac{\hbar\omega}{\hbar k_x} \neq \frac{S_x}{-\sigma_{xx}} = \frac{ck_x}{k}, \tag{31}$$

where S_x and $-\sigma_{xx}$ are, respectively, the flux densities of energy and momentum p_x in the direction of the x axis for the evanescent wave. From (28), (29), and (31) it can be seen that the atom receives energy and momentum p_x from a wave incident on the interface between two media, and the evanescent wave only ensures their transfer to the atom.

The calculation and measurements of the momentum received by the absorbing atom, which moves in the direction of propagation of the evanescent wave and interacts with its field, were performed in [9]. In the limiting case of zero atomic velocity, the results of [9] agree with relations (31).

In the superposition of evanescent waves (9) and (10), there arises the IF of angular momentum through the plane $y = y_1$, which, under the condition $A_2 = \pm iA$, is described by the formulae

$$\frac{I_{M_x}^{\text{int}}}{I_w^{\text{int}}} = \pm\frac{k}{k_x\omega}, \quad I_{M_y}^{\text{int}} = 0, \quad \frac{I_{M_z}^{\text{int}}}{I_w^{\text{int}}} = -\frac{h}{k_x\omega}, \tag{32}$$

if $l < y_1 < 0$. For $y_1 > l$, all IFs of angular momentum are equal to zero [6]. The IFs of energy, momentum and angular momentum calculated for the case of interaction of an atom with a plane elliptically polarised monochromatic wave are given in [6].

5. IF of energy and momentum between atoms

Let two identical atoms be located in vacuum on the x axis at points $x_1 = -l/2$ and $x_2 = l/2$. The first atom is excited and radiates spontaneously at frequency ω as a point electric dipole $\boldsymbol{d}_1$ oscillating along the z axis with a complex amplitude d_{10}. In the unexcited second atom, the electric field E_{1z} of the dipole $\boldsymbol{d}_1$ induces the dipole moment

$$d_{2z} = \alpha E_{1z} = -\alpha \frac{k^2 d_{10}}{l}$$

$$\times \left(\frac{1}{k^2 l^2} - \frac{i}{kl} - 1 \right) \exp[i(kl - \omega t)] = d_{20} \exp(-i\omega t), \quad (33)$$

where α is the complex polarisability of the atom, and d_{20} is the complex oscillation amplitude of the dipole $\boldsymbol{d}_2$.

The power of radiation absorbed by the dipole $\boldsymbol{d}_2$ is equal to the work done per unit time by the field E_{1z} over the dipole $\boldsymbol{d}_2$,

$$N_2 = \frac{1}{2} \mathrm{Re}(\dot{d}_{2z} E_{1z}^*) = \frac{1}{2} \alpha'' \omega |d_{10}|^2 \left(\frac{1}{l^6} - \frac{k^2}{l^4} + \frac{k^4}{l^2} \right), \quad (34)$$

where the dot above the letter denotes the time derivative.

The IFs of energy to an unexcited atom through the planes $x = x_1 = 0$ and $x = x_2 > l/2$

$$I_w^{\mathrm{int}}(0) = \frac{1}{2} \omega \left(\frac{1}{l^3} - \frac{k^2}{2l} \right) \mathrm{Re}(i d_{10} d_{20}^*) \quad (35)$$

and

$$I_w^{\mathrm{int}}(x_2) = -N_2 - I_w^{\mathrm{int}}(0) \quad (36)$$

are totally equal to the IFs of energy through an arbitrary closed surface bounding an unexcited atom, and they determine the radiation power absorbed by this atom, taken with the opposite sign. This particular result is consistent with the results of more general calculations performed in [10].

All components of the electromagnetic field (22)–(24) of the dipole participate in the formation of the total IF of energy. If the distance l between atoms is small and $kl \ll 1$, the IF of energy to the unexcited atom is localised on the plane $x = 0$,

$$I_w^{\mathrm{int}}(0) = -\frac{1}{2} \alpha'' \omega \frac{|d_{10}|^2}{l^6}, \quad I_w^{\mathrm{int}}(x_2) = 0, \quad (37)$$

and describes nonradiative energy transfer from an excited atom to an unexcited one [11]. This energy flux arises due to the interference of the reactive (nonradiative) components of the near field of the dipoles, which do not contribute to the dipole emission. Nonradiative energy transfer can be considered as a result of self-action of an excited atom through the near field of the induced dipole $\boldsymbol{d}_2$. Indeed, the work done by the near field $E_{2z} \approx d_{20}/l^3$ of the induced dipole over the dipole $\boldsymbol{d}_1$ per unit time is equal to $I_w^{\mathrm{int}}(0)$ from (37).

The time-averaged force with which the dipole field $\boldsymbol{d}_1$ acts on the induced dipole $\boldsymbol{d}_2$ is directed along the x axis and is described by the formula [8]

$$F_x = \frac{1}{4} \alpha' \frac{\partial |E_1|^2}{\partial x} + \alpha'' k |d_{10}|^2 \left(-\frac{k^2}{l^4} + \frac{k^4}{l^2} \right). \quad (38)$$

The first term on the right-hand side of (38) is the gradient force not related to the atomic absorption. For $kl \ll 1$, this term is the time-averaged force $F_x = -3\alpha' |d_{10}|^2 / (2l^7)$, which describes the attraction of two dipoles performing harmonic oscillations in antiphase along the z axis. It follows from (38) that in the case of nonradiative energy transfer and stimulated emission of the second atom, i.e. if $kl \ll 1$, the momentum of the atom does not change.

The IF of momentum p_x through an arbitrary closed surface bounding the unexcited atom is equal to the sum of the IFs of momentum p_x to this atom through the plane $x = x_1 = 0$,

$$I_{p_x}^{\mathrm{int}}(0) = \left(-\frac{3}{2l^4} + \frac{k^2}{l^2} \right) \mathrm{Re}(i d_{10} d_{20}^*), \quad (39)$$

and through the plane $x = x_2 > l/2$,

$$I_{p_x}^{\mathrm{int}}(x_2) = -F_x - I_{p_x}^{\mathrm{int}}(0). \quad (40)$$

It determines the force taken with the opposite sign (38).

If the distance l between the atoms is large ($kl \gg 1$), then the IFs of energy and momentum p_x through an arbitrary closed surface surrounding the second atom satisfy the equalities

$$\frac{I_w^{\mathrm{int}}}{I_{p_x}^{\mathrm{int}}} = \frac{S_r(\boldsymbol{r})}{-\sigma_{rr}(\boldsymbol{r})} = \frac{\hbar \omega}{\hbar k} = c, \quad (41)$$

similar to equalities (27) for wave (20). Here $S_r(\boldsymbol{r})$ and $-\sigma_{rr}(\boldsymbol{r})$ are, respectively, the flux densities of energy and radial momentum p_r of radiation of the dipole $\boldsymbol{d}_1$ in the direction of the radius vector $\boldsymbol{r}$, drawn from this dipole to the observation point.

If we assume that the radiation power of the first dipole decreases with time according to the exponential law (the case of weak attenuation), then the values of the IFs of energy and momentum decrease with time according to the same law. As a result, the spatial structure of the IF data and their ratio (41) remain unchanged at all times.

6. Absorption and stimulated emission of an atom without a recoil momentum

Analysis of nonradiative energy transfer has shown that the momentum of an atom interacting only with the reactive components of the electromagnetic field does not change during its absorption or stimulated emission. Here we consider another example of this interaction. Let us assume that a transparent medium with a negative dielectric constant ε_d and a magnetic permeability equal to 1 occupies a half-space $y > 0$ and borders a vacuum on the plane $y = 0$. A plane monochromatic wave with frequency ω, polarized along the z axis, is incident from vacuum to the interface between two media along the normal. In a reflecting medium this wave forms a refracted wave

$$E_z = A\exp(-hy - \mathrm{i}\omega t), \quad H_x = \mathrm{i}\frac{h}{k}A\exp(-hy - \mathrm{i}\omega t), \quad (42)$$

where A is the complex amplitude, and $h = \sqrt{|\varepsilon_\mathrm{d}|}\omega/c$.

Under the action of the electric field E_z of the refracted wave, an atom located in a medium at a point with coordinates $x = 0$, $y = l$, $z = 0$ acquires a dipole moment $\boldsymbol{d}$, oscillating at frequency ω along the z axis,

$$d_z = \alpha E_z = \alpha A\exp(-hy - \mathrm{i}\omega t), \quad (43)$$

where $\alpha(\omega)$ is the complex polarisability of the atom. The field of a point dipole (43) is described by formulae (22)–(24), where it is necessary to set $k = \mathrm{i}h$. The fields of the refracted wave and the induced dipole are reactive and, when taken separately, do not transfer either energy or momentum. Nevertheless, superposition of these fields gives rise to IFs of energy and momentum. The IF of energy to an atom through the plane $y = y_1$ is

$$I_w^{\mathrm{int}} = -\iint_{-\infty}^{\infty} S_y^{\mathrm{int}}\mathrm{d}x\mathrm{d}z = -\frac{1}{2}\alpha''\omega|A|^2\exp(-2hl), \quad (44)$$

if $0 < y_1 < l$, and equal to zero if $y_1 > l$ [8]. In the case of an absorbing atom, the energy flux is directed from the interface between two media, i.e. from the incident wave, to the atom. For an excited atom, the energy flux is directed from the atom to the interface between two media and amplifies the reflected wave due to the stimulated emission of the atom.

The IF of momentum to the atom through the plane $y = y_1$ is nonzero only for p_y,

$$I_{p_y}^{\mathrm{int}} = -\iint_{-\infty}^{\infty} \sigma_{yy}^{\mathrm{int}}\mathrm{d}x\mathrm{d}z = \frac{1}{2}\alpha' h|A|^2\exp(-2hl), \quad (45)$$

if $0 < y_1 < l$, and equal to zero if $y_1 > l$ [8]. Expression (45) describes the time-averaged gradient force acting on the induced dipole. This is the force of interaction of the induced dipole with the surface polarisation charges of the medium. It has nothing to do with either the absorption of the atom or its stimulated emission. Thus, absorption and stimulated emission of an atom occur without changing the momentum of the atom. In this case, we can speak of an optical analogue of the Mössbauer effect, since the corresponding momenta from the incident and reflected waves are received by the medium in which the atom is located. The physical nature of this effect is associated with the peculiarity of the energy and momentum transfer by reactive electromagnetic fields. For reactive fields, the time-averaged momentum density is zero and, so they, like constant electric fields, can transfer momentum only between charges.

7. Interference of fields of thermal radiation and zero oscillations

We consider equilibrium radiation in the form of a superposition of a classical thermal field and a quantum field of zero-point oscillations, which is described as a chaotic field. Here, this model of equilibrium radiation is used to derive the Planck formula with the help of interference of thermal radiation fields and zero-point oscillations [12, 13].

Let the mode of equilibrium radiation with frequency ω and given polarisation be a superposition of statistically independent chaotic fields of thermal radiation at temperature T and zero-point oscillations that do not depend on temperature. The average mode energy is expressed as

$$\langle\varepsilon\rangle = \langle\varepsilon_T\rangle + \langle\varepsilon_0\rangle, \quad (46)$$

where $\langle\varepsilon_T\rangle$ is the average energy of the thermal field, and $\langle\varepsilon_0\rangle$ is the average field energy of zero-point oscillations.

Using the well-known properties of classical thermal radiation and zero-point oscillations, we can prove that the dispersion of the mode energy [12, 13] is

$$D_2(\varepsilon) = \langle\varepsilon^2\rangle - \langle\varepsilon\rangle^2 = \langle\varepsilon_T\rangle^2 + 2\langle\varepsilon_0\rangle\langle\varepsilon_T\rangle. \quad (47)$$

In deriving (47), we took into account that

$$D_2(\varepsilon_T) = \langle\varepsilon_T\rangle^2, \quad D_2(\varepsilon_0) = 0. \quad (48)$$

The second term on the right-hand side of equality (47) describes the fourth-order interference in the field between thermal radiation and zero-point oscillations.

Application of the laws of thermodynamics to equilibrium radiation gives a formula for the average energy, expressing Wien's law [14]:

$$\langle\varepsilon\rangle = \omega f(\beta\omega), \quad (49)$$

where f is an unknown function; $\beta = 1/(k_\mathrm{B}T)$; and k_B is the Boltzmann constant. Fluctuations of the energy of a mode in a fixed volume obey the equation

$$D_2(\varepsilon) = -\frac{\mathrm{d}\langle\varepsilon\rangle}{\mathrm{d}\beta}. \quad (50)$$

Applying (48)–(50) separately to the thermal field and the zero-point oscillation field, we obtain

$$\langle\varepsilon_T\rangle = \frac{1}{\beta} = k_\mathrm{B}T, \quad \langle\varepsilon_0\rangle = \omega f(\infty). \quad (51)$$

For a superposition of fields, the solution of equation (50) using (47) and the condition $\langle\varepsilon_T(\infty,\omega)\rangle = 0$ is written in the form [12, 13]

$$\langle\varepsilon_T\rangle = \frac{2\langle\varepsilon_0\rangle}{\exp(2\beta\langle\varepsilon_0\rangle) - 1}. \quad (52)$$

Substituting (52) in (46) and setting $f(\infty) = \hbar/2$, we arrive at the Planck formula for the average energy of one mode of equilibrium radiation

$$\langle\varepsilon\rangle = \frac{\hbar\omega}{\exp[\hbar\omega/(k_\mathrm{B}T)] - 1} + \frac{1}{2}\hbar\omega. \quad (53)$$

This wave derivation of the Planck formula is not rigorous, since the existence of zero-point oscillations with certain statistical properties is postulated. It only demonstrates that, as in the case of IFs, a superposition of linear fields can, due to interference, acquire qualitatively new properties that are absent in interfering fields. Comparison of formulae (51) and (52) clearly shows this, since the superposition of fields changes the statistical properties of thermal radiation.

8. Conclusions

In classical electrodynamics, interference ensures the fulfilment of conservation laws when charges interact with an electromagnetic field. The IFs of energy, momentum and angular momentum, distributed continuously in space serve as an effective tool for analysing the formation of an interference pattern and the processes of interaction of electric charges with an electromagnetic field. They make it possible to reveal the multifunctionality of interference and describe qualitatively new and non-obvious properties of the superposition of both constant and time-variable fields.

The interference fields make it possible to perform a spatial analysis of elementary acts of absorption and emission of atoms interacting with electromagnetic waves of any spatial structure, including evanescent waves and reactive fields. In this case, there is no need to quantise these fields, which is of great importance for near-field fields widely used in nano-optics and plasmonics.

The interference effects considered in the work show that interference is a mechanism for the formation of qualitatively new properties in a linear system of electromagnetic fields, in which an arbitrary combination of these fields is described at a higher level, when the superposition principle is not fulfilled and the system ceases to be linear. In this case, we can speak about a synthesis of elements rather than their addition.

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